ANALYZING PUMPING TEST DATA

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It is difficult to put into a single phrase the idea that by means of a pumping test a great deal may be learned about an aquifer, a water-bearing formation from which ground water may be obtained through wells. The name applied to this type of quantitative method is aquifer performance test--or simply aquifer test. This distinguishes the aquifer test from a simple pumping test. The latter provides nothing more than the water levels in the well at each of several production rates, to aid in selecting a pump which will perform satisfactorily in the particular well. In many parts of the Rio Grande Valley, for example, the simple pumping test meets the needs of driller and owner, and probably neither would consider the prolonged pumping generally required in the aquifer test to be economical.

Why do we spend time and money learning about the aquifer characteristics? We want to know the hydraulic properties because it is possible to use these characteristics to predict future behavior of water levels in the aquifer. By using mathematical formulas with a given set of values, with assumed or expected pumping conditions and an assumed or predicted time period, we can calculate future water levels in the aquifer. And the calculation of water-level changes allows us to decide whether or not we can afford to operate under those conditions. We can answer such questions as: Will the pumping levels drop below economic pumping lifts? Will the levels drop sufficiently for parts of the aquifer literally to dry up? These are questions that constantly confront large users of ground water. Quantitative methods are also used to learn about discharge from the aquifer, recharge to the aquifer, leakage from other water-bearing beds, and infiltration from surface bodies of water. Among the many other useful applications of the aquifer performance test analyses are some relatively simple problems; for example, how much interference will we have between wells in a particular well field? If a well field is located in an aquifer for which the supply of water is very abundant, the interference may actually be much more important to the operation than the long-range predictions. Interference as used here--or rather as used in groundwater studies generally--is the drawdown in one well caused by pumping another well. Obviously, if a number of pumping wells are located so that each is within the radius of influence of each of the others, that part of the drawdown caused by the

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mutual interference may increase pumping costs to the operator. This would indicate a need for an engineering study to determine the most economical spacing, keeping in mind that greater spacing requires more pipelines, more electric or gas lines, and perhaps more property. Initial cost must be considered together with operating costs.

To help in understanding what is meant by aquifer testing, let us consider first what happens when we pump a well which taps an unconfined aquifer; that is, an aquifer in which the surface of the water is exposed to atmospheric pressure. The water table drops and air replaces water in the voids in, say, 20 percent of the dewatered volume. The gradient, or slope of the water table toward the well, is created causing water in the aquifer to move toward the well. Careful observation of the behavior of the water levels in near-by wells during pumping will permit us to evaluate the coefficient of storage and the coefficient of transmissibility, two terms which are most important in quantitative work. The coefficient of storage, S, is a measure of the amount of water obtained from storage during pumping, and the coefficient of transmissibility, T, is a measure of the rate at which water will flow through the aquifer. (Since the pore spaces retain some water, part of which continues to drain slowly over a period of time and part of which is retained, this description is an over-simplification.)

For a confined aquifer--that is, an artesian aquifer--the concept of obtaining water from storage may be slightly more difficult to visualize. The artesian aquifer has confining beds and the water level in a well penetrating it will rise above the top of the aquifer. When we start pumping we know that as we continue to take water out of the well the aquifer is still saturated; it is still full of water. We are also aware that at some distance from the well the conditions that existed before pumping have not changed. Where, then, did the pumped water come from? First, a very small part of it came from expansion of the water itself as the pressure was reduced near the well. The decrease in pressure also reduces some of the support of the skeleton of the aquifer and the compaction occurs. This is true also of less permeable beds, lying within the aquifer and some water is squeezed out of these beds. The storage coefficient for an artesian aquifer is very small compared to the coefficient for an unconfined (water-table) aquifer. By contrast with the water-table aquifer described before, the entire thickness of the artesian aquifer within the area of a pressure decline contributes water from storage. Of course, just as with the artesian aquifer, some water is released from storage by compaction in an unconfined aquifer below the water table, but this factor is so small that it usually has been neglected. In a very thick aquifer it has been shown to be important.

As pumping continues in either a water-table or artesian aquifer, the effects of pumping continue to spread until sufficient natural discharge or formerly rejected recharge is intercepted to offset the quantity pumped. Leakage through adjacent beds may supply the recharge. More recently, Hantush has found that partial penetration, particularly of very thick aquifers, tends to cause the water levels in observation wells to behave similarly to those resulting from recharge.

In medicine, when the doctor has diagnosed your case, he writes his diagnosis in some mysterious symbolic language that probably most of the old Romans would have had trouble deciphering. In ground-water work, the hydrologist does something very similar. He probes around to find all the facts that apply to a particular situation, then applies a mysterious formula or two (really more complex than mysterious) from which he draws conclusions about the values of the coefficients of transmissibility and storage.

Many of the facts he obtains are gathered by conducting aquifer performance tests. These data, generally discharge measurements and water levels measured periodically in one pumped well together with water levels measured in one or more observation wells, are used to obtain particular solutions to equations of ground-water flow. There are innumerable conditions under which ground water can occur, and for each situation the hydrologist, geologist, or engineer must be aware of the limitations of the formula he has decided to use. He must understand the assumptions that were made when the formula was derived; such simplifying assumptions have to be made for any derivation to keep from obtaining an equation which would simply be too cumbersome to apply.

A little more than a hundred years ago, in 1856, a Frenchman named Henry Darcy published the law of flow which has since come to bear his name. A few years later, in 1863, DuPuit, also a Frenchman, applied Darcy's law to well hydraulics, using an ideal example of a well located at the center of a circular island. In 1906, Thiem, a German, modified DuPuit's work to apply to more general problems. From then until 1935, a number of investigators presented modified forms of the DuPuit relationship. These are all forms of the "equilibrium method" in which the rate of discharge of the pumped well is equal to the rate of flow of water toward the well, for all concentric cylinders about the well. But in 1935, C. V. Theis, of the U.S. Geological Survey, Ground-Water Branch, published a paper entitled The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using ground-water storage. This nonequilibrium method, as it has come to be known, contains an exponential integral, a little mathematical complexity which made it fairly difficult to solve. Theis suggested a graphical method to Jacob and Wenzel, both of whom subsequently published descriptions of such a method in 1940 and 1942, respectively.

In the Theis nonequilibrium method, a number of assumptions were made in its derivation. It was assumed that the aquifer was infinte in extent--in a practical sense, this means that it covered a very large area compared to its thickness; it was assumed that the hydraulic properties of the aquifer were the same in all directions; it assumed that water was released instantaneously from the formation. Since Theis' publication in 1935 there have been numerous contributions, some of which modified the nonequilibrium method, some of which presented methods and formulas for handling situations other than the ideal. Techniques have been developed for dealing with boundaries of aquifers, partial penetration of the aquifer by wells, leakage into or out of the aquifer through the confining beds, special recharge or discharge conditions, effect of sloping beds, effect of varying thicknessess, and combinations of these. Several of these conditions will usually be present, and the practicing hydrologist must understand the conditions if he hopes to define the ground-water situation with reliability.

The lower of the two curves in Figure 1 shows the behavior of water levels in a pumped well in an ideal aquifer. When pumping starts the water level drops rapidly, then the rate of lowering decreases as pumping continues. The upper curve in Figure 1 shows water levels in a typical observation well, which in this instance is located 120 feet from the pumped well. Shape of the curves will vary depending on the conditions of pumping and on the aquifer characteristics. Note that these curves have been plotted on plain coordinate graph paper and that each unit is represented by an equal spacing.

In Figure 2 the same observation well data used in Figure 1 are shown plotted on log-log (logarithmic scale) graph paper for a type-curve graphical solution of the Theis nonequilibrium formula. The drawdown (change in water level) is plotted on one scale, and time, or rather the reciprocal of time, is plotted on the other scale. Note that equal spacings on a logarithmic scale represent the logarithm of the units plotted and not the units themselves. Each cycle represents 10 times the previous cycle. A type-curve is plotted on the same scale log paper, in which W(u), the well function containing the exponential integral, is plotted against tabulated values of u, which contains distance, time and the coefficients of transmissibility and storage. The field data generally are plotted on semitransparent tracing paper which is positioned above the type-curve until the "best-fit" position is obtained, keeping the axes of both sheets parallel. The coordinates of any point common to both sheets are read and these values inserted into the equations to obtain the formation constants.

The Jacob straight-line modification, in common use, simplifies the analytical procedures for the ideal aquifer since after an initial period of pumping, the drawdown vs. time plot becomes a straight line on semilog graph paper. In Figure 3 the drawdown is plotted on the arithmetic scale as in Figure 1 and time is plotted on a logarithmic scale as in Figure 2. In an ideal aquifer the water level continues to decline along the straight line. The slop of the straight line portion is used in an equation to evaluate the coefficient of transmissiblity, T. The intercept of the straight line extrapolation with the zero-drawdown line is used to obtain the coefficient of storage, S.

The following figures illustrate aquifer test results for some of the field conditions other than the ideal:

The curves in Figure 4[1/] illustrate variations in the time-drawdown relation that will occur for observation wells partially penetrating an aquifer. Curve 2 indicates that the water level would behave quite differently for one-fourth penetration than for complete penetration, as shown by Curve 1, the Jacob straight-line modification. Curve 2 for an infinitely thick aquifer illustrates a condition in which water-levels would appear to approach equilibrium. Similarly-shaped curves may be obtained when recharge is induced as a result of pumping, or when leakage occurs through confining beds, illustrating the importance of knowing the geologic conditions in the area.

In Figure 5 are shown log-log and semilog plots of a partial penetration analysis of an aquifer test. The log-log analysis developed by Hantush involves matching field data with a family of type-curves as shown in Figure 6.

Leakage through a confining bed is shown in Figure 7. Note here that the shape of the curve is somewhat similar to that developed by partial penetration. Under conditions of pumping, the original head relationship between upper and lower aquifer is changed and leakage occurs. The quantity leaking through a single square foot of the base of the confining bed may be very small, but when multiplied by the area of influence of the pumping well this quantity

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[1/] Figures 4 through 8 are from published works of Dr. Mahdi S. Hantush, formerly of the New Mexico Institute of Mining and Technology, now on the staff of the University of Baghdad.

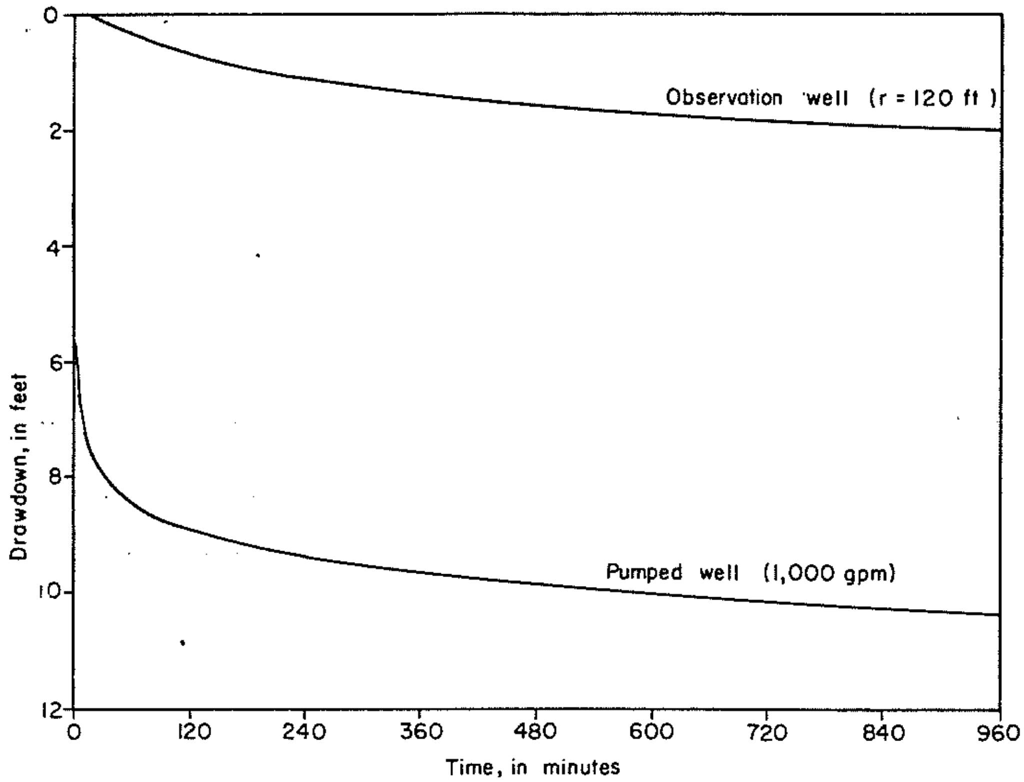


Figure 1. Water levels vs. time. Observation well at a distance of 120 feet from the pumped well.

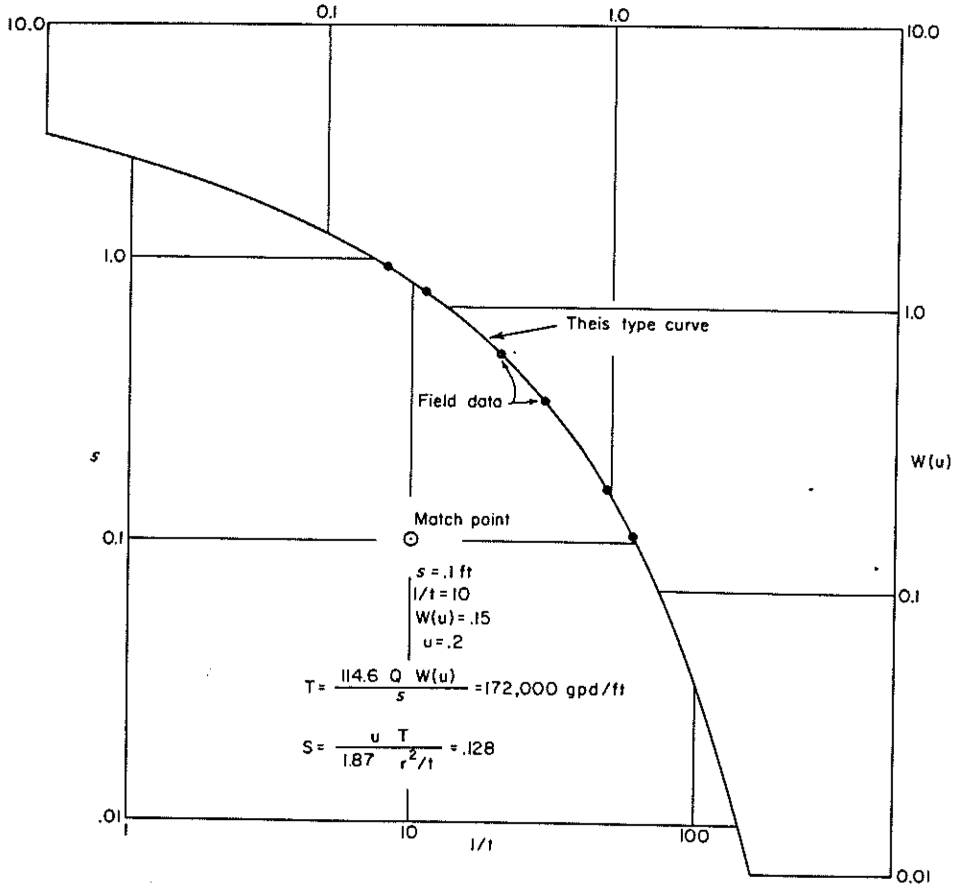


Figure 2. Theis type curve "matched" with field data on log-log plots. Same data as shown in Figure 1.

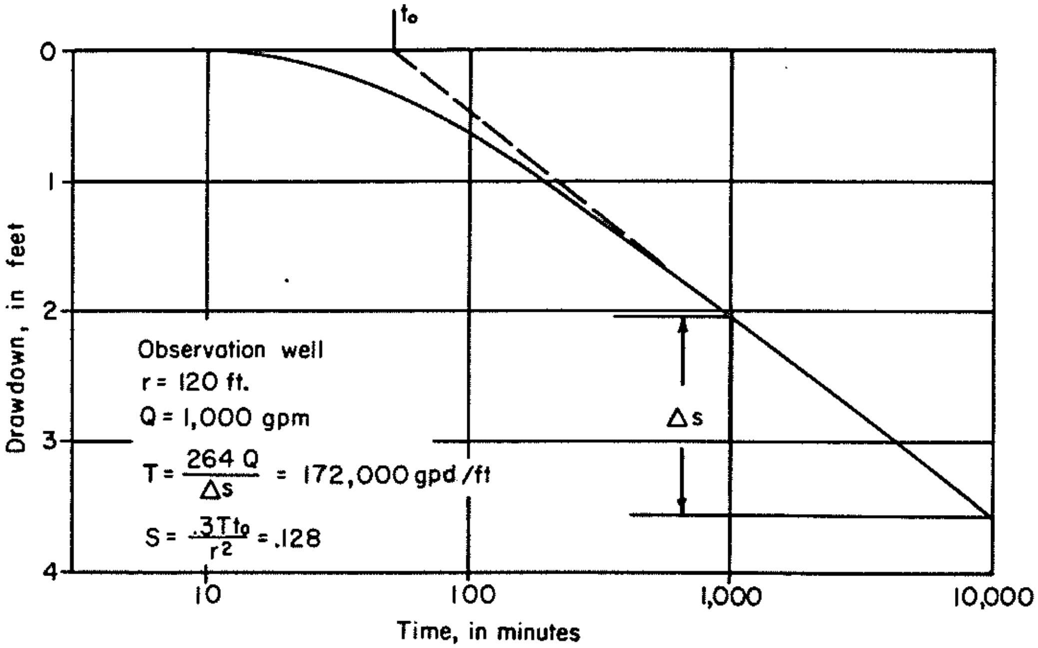


Figure 3. Jacob's modified straight-line plot. Same data as shown in Figures 1 and 2.

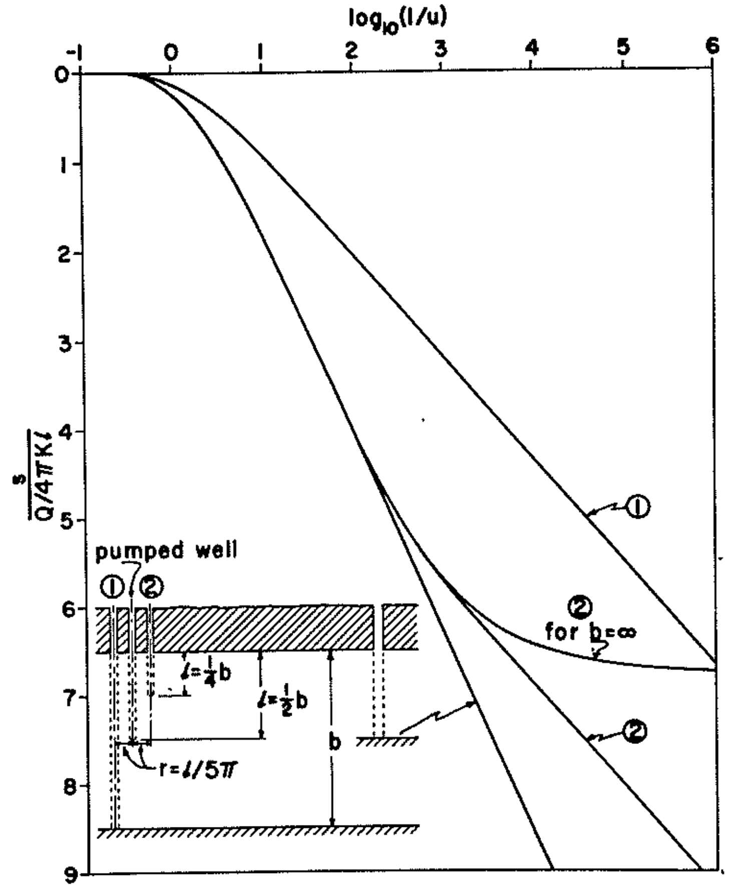


Figure 4. Time-drawdown variation in partially penetrating wells.

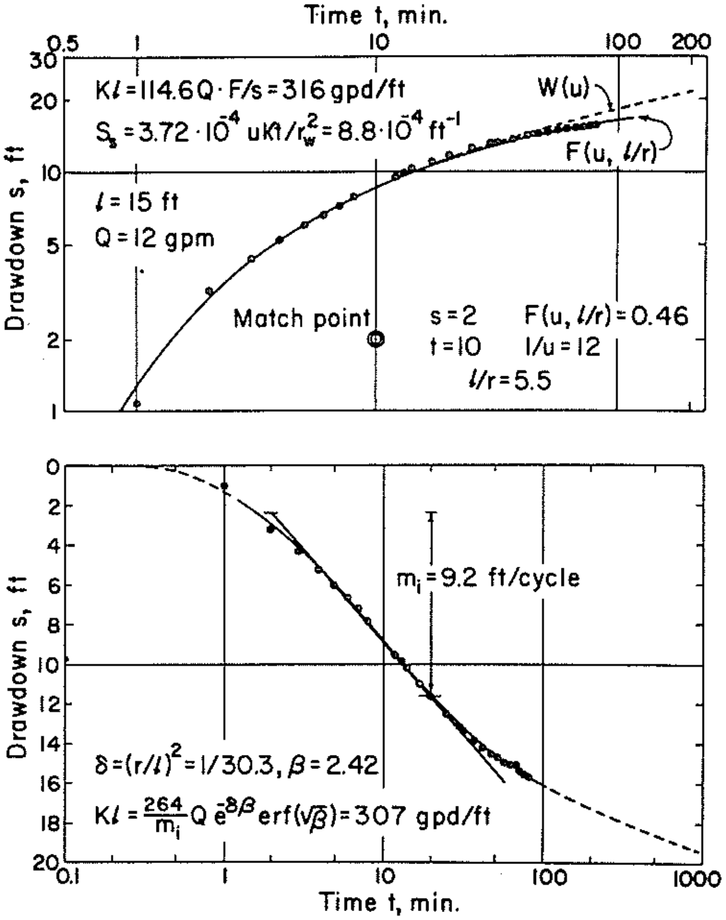


Figure 5. Analysis of partial penetration, observational data.

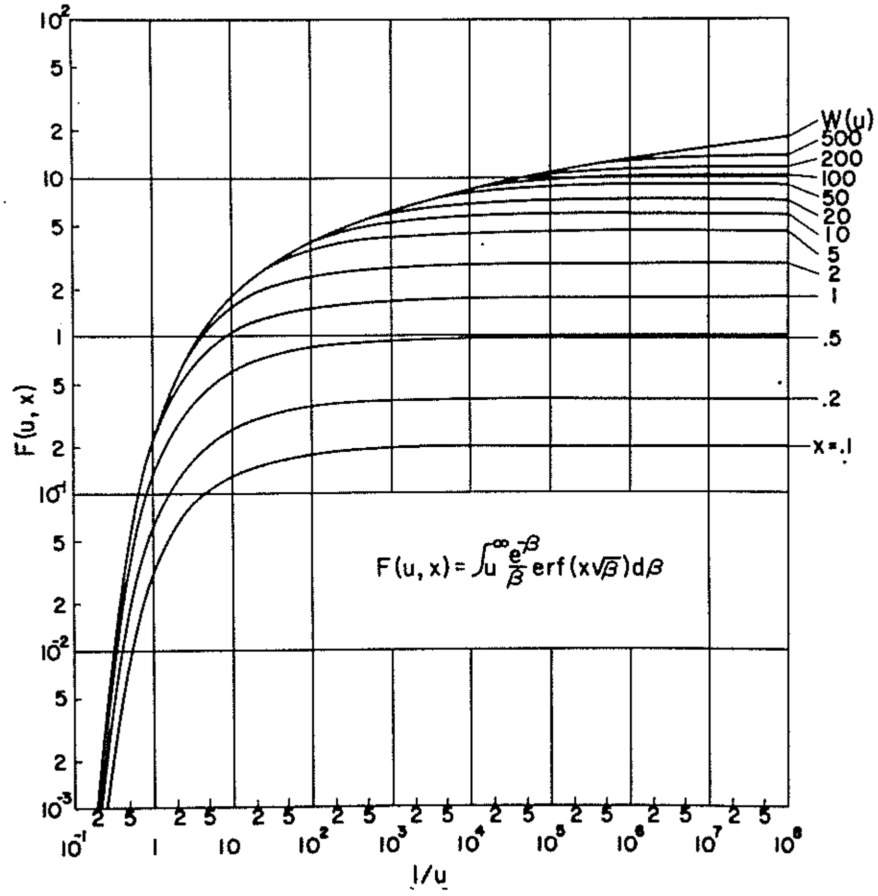


Figure 6. Type-curves of the function F(u,x).

is very large and, as indicated by the flattening of the water-level curve, a condition may be reached when no further drawdown will occur in wells penetrating the pumped aquifer.

Yet another condition which occurs frequently and which results in data varying from the behavior of water levels in the ideal aquifer is a very thick water-bearing formation with the lower part responding like a semipervious layer. Permeabilities often decrease with depth because of compaction resulting from the added weight of the greater thickness of deposits.

Figure 8 is included to show the complexities arising from sloping beds.

It is important that aquifer conditions must be included in the analyses of aquifer tests. Without such recognition it is obvious that analyses may be misleading and incorrect to such an extent as to cause gross errors in the prediction of future conditions.

This is not an attempt to sell, either directly or indirectly, the idea that quantitative analysis is the only tool of value in ground-water hydrology. Every discipline that can contribute even the smallest amount of knowledge in this work must be used. Without a doubt, the geology must be understood to appreciate the occurrence of ground water. Geophysics can help tremendously, and soil science will help toward understanding the movement in the zone above the water table so that the recharge mechanism will be more readily explained. The ground-water hydrologist must look for help from many of his colleagues and must himself become a sort of jack-of-all-trades.

Under the right conditions, or any conditions, with the right technique and proper analysis, the ground-water situation can be adequately defined. Many of the conditions in the field are so complex that workable methods have not yet been derived. In some instances, we still do not have the right technique and are using other techniques with the realization that although they may not be entirely accurate, the answers found are nearly always better than mere guesses.

The selected reference list, though by no means complete, includes some of the recent developments in ground-water hydrology.

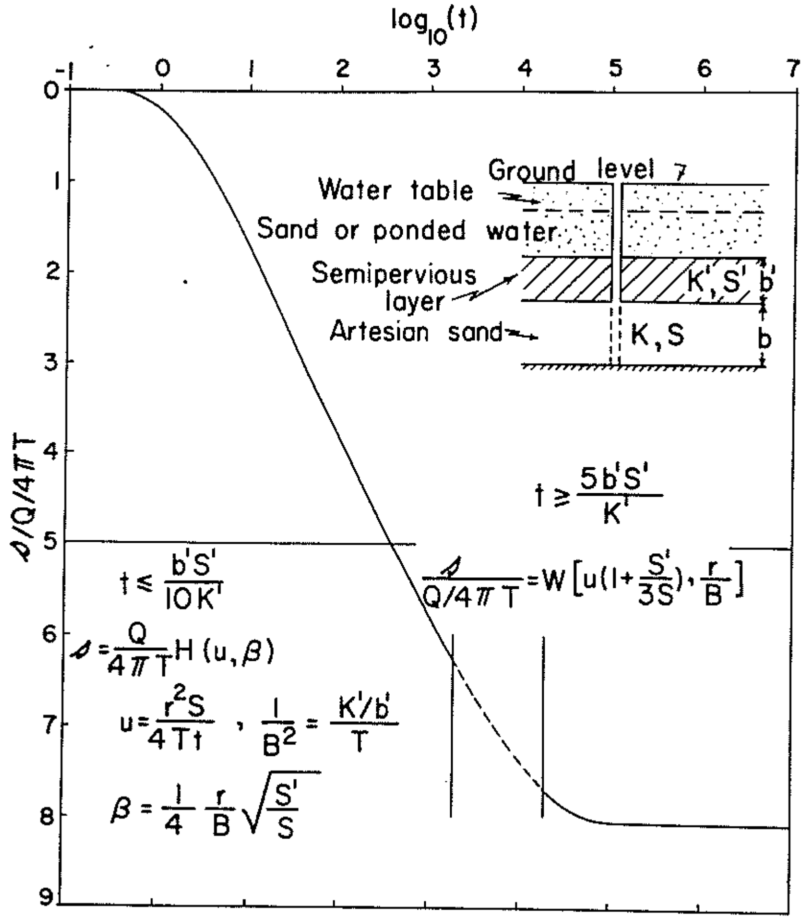


Figure 7. Analysis of leaky aquifer observational data.

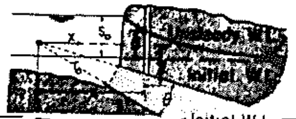
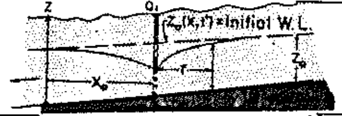
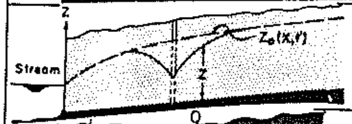
	$\bar{s} = s_o A(\rho,\alpha)$, $A(\bar\rho,\alpha) = 1 - \frac{2}{\pi}\int_0^\infty e^{-\alpha\mu^2} \frac{J_o(\mu)Y_o(\mu\bar\rho) - Y_o(\mu)J_o(\mu\bar\rho)}{J_o^2(\mu)+Y_o^2(\mu)} \frac{d\mu}{\mu}$ $q = K\theta s_o G(\alpha)$ $G(\alpha) = \frac{4}{\pi^2}\int_0^\infty \frac{e^{-\alpha\mu^2}}{J_o^2(\mu)+Y_o^2(\mu)}\frac{d\mu}{\mu}$ $\alpha = Kt/r_o^2 S_s$, $\bar\rho = \frac{1}{r_o}\sqrt{X^2+Z^2}$
	$\bar{s} = s_o\left\{1 - 2\sum_{n=1}^{\infty}\frac{J_o(\beta_n\bar\rho)}{\beta_n J_1(\beta_n)} e^{-\alpha\beta_n^2}\right\}$ $q = 2K\theta s_o \sum_{n=1}^\infty e^{-\alpha\beta_n^2}$ β_n are zeros of J_o
	$\bar s = \frac{1}{2} s_o\left\{\mathrm{erfc}\left(\frac{X}{\sqrt{4\nu t}} - \frac{\sqrt{\nu t}}{a}\right) + e^{2X/a}\mathrm{erfc}\left(\frac{a}{\sqrt{4\nu t}} + \frac{\sqrt{\nu t}}{a}\right)\right\}$ $q = K s_o\left(\frac{b_o}{a}\right)\left\{\frac{1}{\sqrt\pi}\frac{e^{-\nu t/a^2}}{(\sqrt{\nu t/a})} - \mathrm{erfc}\left(\frac{\sqrt{\nu t}}{a}\right)\right\}$, $\nu = K/S_s$
	$Z_o^2 - Z^2 = \frac{Q}{2\pi K}\exp\left(-\frac{x-x_o}{\beta}\right)W\left(u,\frac{r}{\beta}\right)$, $W(u,\delta) = \int_u^\infty \frac{e^{-y-\frac{\delta^2}{4y}}}{y}dy$ $= r^2 S_y/4K\bar{Z}t$, $\beta = (2\bar{Z}/m)$
	$Z_o^2 - Z^2 = \frac{Q}{2\pi K}\exp\left(-\frac{x-x_o}{\beta}\right)\left\{W\left(u,\frac{r}{\beta}\right) - W\left(u,\frac{r'}{\beta}\right)\right\}$ $r = \sqrt{(x-x_o)^2+y^2}$, $r' = \sqrt{(x+x_o)^2+y^2}$
	$s = \frac{Q}{4\pi K b_o}\exp\left(\frac{x-x_o}{a}\right)W\left(u,\frac{r}{a}\right)$ $u = r^2 S_s/4Kt$, $b = b_o\exp\left(-2\frac{x-x_o}{a}\right)$

Figure 8. Flow in sands of nonuniform thickness.

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