

PARAMETER ESTIMATION FOR A LUMPED-PARAMETER GROUND-WATER
MODEL OF THE MESILLA VALLEY, NEW MEXICO

by

C. David Updegraff

and

Lynn W. Gelhar

New Mexico Water Resources Institute

in cooperation with

Department of Geoscience

New Mexico Institute of Mining and Technology

October 1977

The work upon which this report is based was supported in part by funds provided by the United States Environmental Protection Agency under Grant Number S803565-02-0 through the New Mexico Water Resources Research Institute as authorized under the Water Resources Research Act of 1965, Public Law 88-379.

ABSTRACT

A lumped-parameter, linear-reservoir, ground-water model is developed for possible application to the Mesilla Valley, New Mexico. Using a discretized form of the model, specific-yield and response-time parameters are estimated by a least squares technique from water-level and drain-flow data for the Mesilla Valley. Net recharge is then estimated from water-level changes during months that recharge occurs. The estimated parameters are used in simulations which predict water levels and drain flows. The results show that the parameters are physically realizable and similar to values used by others. The simulations are in excellent agreement with the observed data over a five-year period.

orized
Public Law

ACKNOWLEDGMENTS

The work upon which this report is based was supported primarily by funds provided through the New Mexico Water Resources Research Institute (Project Number 3109-314) by the United States Environmental Protection Agency (Grant Number S803565-02-0). Dr. Arthur G. Hornsby, of the Robert S. Kerr Environmental Laboratory in Ada, Oklahoma, served as project officer.

Data on water levels and drain flows in the Mesilla Valley were provided by the U. S. Bureau of Reclamation in El Paso, Texas. The computer Center at New Mexico Tech provided computational facilities.

TABLE OF CONTENTS

	<u>Page</u>
ABSTRACT	ii
ACKNOWLEDGMENTS.	iii
LIST OF FIGURES.	v
LIST OF TABLES	vi
1. INTRODUCTION	1
Scope and Objectives.	2
2. THE DEVELOPMENT OF THE LUMPED-PARAMETER GROUND-WATER MODEL	4
Perched-Stream Case	4
Stream-Connected Aquifer Case	11
Special Cases	12
3. ESTIMATION OF PARAMETERS AND RECHARGE.	14
A Brief Description of the Least Squares Method	14
Estimation of a_d and h_d	15
Estimation of S , t_h , and q_r for Perched-Stream Case	18
Estimation of S , a_r , h_r , and t_h for Stream-Connected Aquifer Case	20
Estimation of New Recharge.	22
4. CASE STUDY - MESILLA VALLEY, NEW MEXICO.	24
5. SUMMARY, RECOMMENDATIONS, AND CONCLUSIONS.	45
REFERENCES	47
APPENDIX A: ALGEBRAIC MANIPULATION TO OBTAIN t_h AND S ESTIMATES FOR THE CASE OF AN AQUIFER WITH A PERCHED STREAM.	49
APPENDIX B: COMPUTER PROGRAM FOR THE CASE OF THE AQUIFER WITH A PERCHED STREAM.	53
APPENDIX C: WATER LEVELS FOR USBR WELLS (1946-1951)	63

LIST OF FIGURES

<u>Figure</u>	<u>Page</u>
2.1 Schematic Vertical Section of an Aquifer with a Perched Stream . .	5
2.2 Idealized Aquifer with a Perched Stream.	5
2.3 Idealized Aquifer with a Fully Penetrating Drain	5
2.4 Schematic Illustration of an Aquifer with a Connected Stream . . .	5
4.1 Location Map of the Mesilla Valley, New Mexico	25
4.2 USBR Well Locations and Thiessen Polygons.	26
4.3 Average Water Level versus Drain flow.	33
4.4 Water-Table Slope.	34
4.5a h_i versus h_{i+1} for First Recession	36
4.5b h_i versus h_{i+1} for Second Recession.	36
4.5c h_i versus h_{i+1} for Third Recession	37
4.5d h_i versus h_{i+1} for Fourth Recession.	37
4.5e h_i versus h_{i+1} for Fifth Recession	38
4.6 Recharge Estimates and Applied Irrigation Water.	40
4.7a Observed and Simulated Average Water Levels.	44
4.7b Observed and Simulated Drain Flows	44

LIST OF TABLES

<u>Table</u>		<u>Page</u>
4.1	THIESSEN POLYGON WEIGHT AND LAND SURFACE ELEVATION AT EACH USBR OBSERVATION WELL.	28
4.2	MONTHLY AVERAGE WATER LEVELS, DRAIN FLOWS AND DIVERTED IRRIGATION WATER	30
4.3	RECHARGE ESTIMATES AND ANNUAL RECHARGE.	39

CHAPTER 1

INTRODUCTION

Mathematical models of ground-water flow and contaminant transport have developed rapidly in the last decade. However, proper application of these models requires various parameters (transmissivity, storage coefficients, dispersion coefficients, etc.), which are generally difficult to estimate. The models fall into two distinct categories: distributed-parameter models and lumped-parameter models. The parameter estimation procedure for one is much different than for the other.

The distributed-parameter models, whose parameters can vary in space and time, are quite complicated. These models are set up in some kind of grid network and are solved using finite difference or finite element techniques. They can take into account heterogeneous and anisotropic transmissivities, spatially varying storage coefficients and complicated boundary conditions.

Because of their complexity, distributed-parameter models require large amounts of input data that are generally hard to obtain or difficult to estimate. Initial conditions of the modeled area are obtained by measuring water levels in wells distributed over the area to be modeled, drawing a water table map and interpolating water levels for various node points. Estimation of transmissivities and storage coefficients is not as easy as the method used to determine initial conditions. Sometimes transmissivities and storage coefficients are determined from pump tests. However, analyses of pump tests assume that the aquifer is at least homogeneous -- an assumption which is generally not valid. At other times, the model is run repeatedly with different sets of parameters until the model produces water levels similar to those observed in wells within the area to be modeled. Hefez et al. (1975) used a finite differ-

ence scheme to estimate transmissivities and storage coefficients. This method requires an orderly location of wells throughout the modeled area and recharge estimates at every node. Frind and Pinder (1974) used a steady state Galerkin finite element approach to estimate transmissivities in a heterogeneous, isotropic aquifer. This method does not require the orderly positioning of wells as in Hefez et al., but it does require that some of the transmissivities be known beforehand. In short, methods of parameter estimation for distributed-parameter models have not obtained the degree of sophistication found in the models themselves.

Lumped-parameter models are essentially volume-averaged representations of the distributed systems. They are described by ordinary differential equations in which time is the only independent variable and are used when one wants a time trend of what is happening in an aquifer. Gelhar and Wilson (1974) used a lumped-parameter model to analyze salt build up in a Massachusetts aquifer because of highway de-icing. Flores and Gelhar (1976) applied a lumped-parameter model to a stochastic water management problem. In a current study, water quality of irrigation return flows is being analyzed by using a lumped-parameter model.

Lumped-parameter models can be described by linear ordinary differential equations and hence several methods have been developed to estimate their parameters (Dooge, 1973). Gelhar (1974) developed a stochastic method that can be used to estimate parameters for a linear-reservoir type aquifer.

Scope and Objectives

The purpose of this report is to estimate the parameters of two lumped-parameter models applied to an irrigated valley in New Mexico. This is done by considering the cyclical nature of irrigation water application. During non-growing months no water is applied to the land and so certain parameters

are estimated while utilizing this knowledge. During the remaining months it is then possible to estimate net recharge to the aquifer.

CHAPTER 2

THE DEVELOPMENT OF THE LUMPED-PARAMETER GROUND-WATER MODEL

The lumped-parameter ground-water model (Gelhar and Wilson, 1974) is based on an aquifer water-balance equation and a discrete form of Darcy's law. The water-balance equation relates the flux of water into and out of an aquifer to the time change of storage of water within that aquifer. In simple terms it is written

$$\frac{dM_a}{dt} = m_i - m_o \quad , \quad (2.1)$$

where

- M_a = mass of water within an aquifer,
- m_i = mass flux of water into an aquifer,
- m_o = mass flux of water out of an aquifer.

Darcy's law governs the transfer of water into and out of the aquifer. Both of these principles are used to develop two lumped-parameter ground-water models: one with a stream perched above the water table, and the other with a stream connected to the aquifer.

Perched-Stream Case

Figure 2.1 shows a schematic cross section for the perched-stream case and the expected types of aquifer inputs and outputs. For simplicity, assume that the surface area of the aquifer is rectangular with length, L , and width, W , and that the drain fully penetrates the aquifer. A cross section of this simplified aquifer is shown in Figure 2.2.

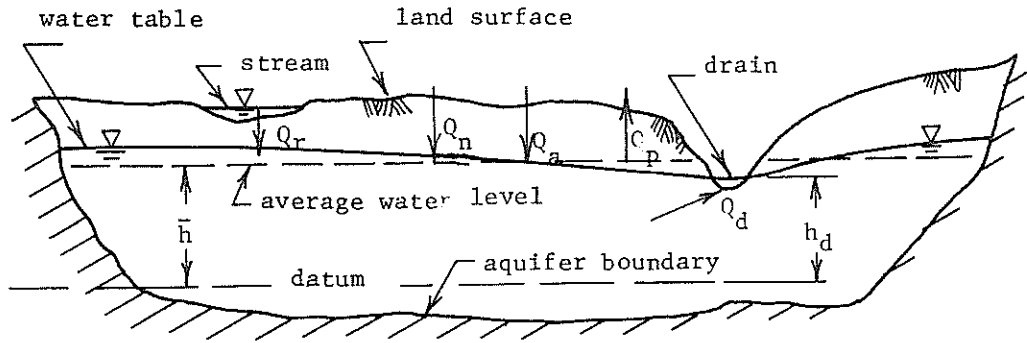


Figure 2.1: Schematic Vertical Section of an Aquifer with a Perched Stream

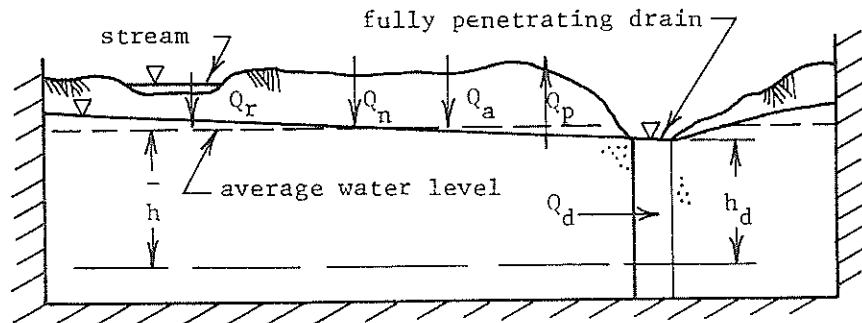


Figure 2.2: Idealized Aquifer with a Perched Stream

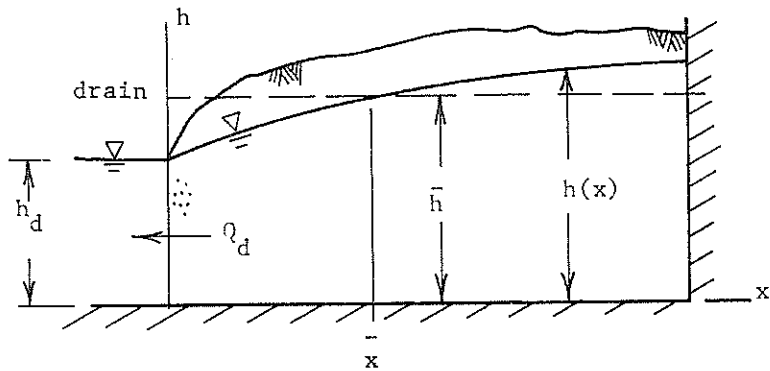


Figure 2.3: Idealized Aquifer with a Fully Penetrating Drain

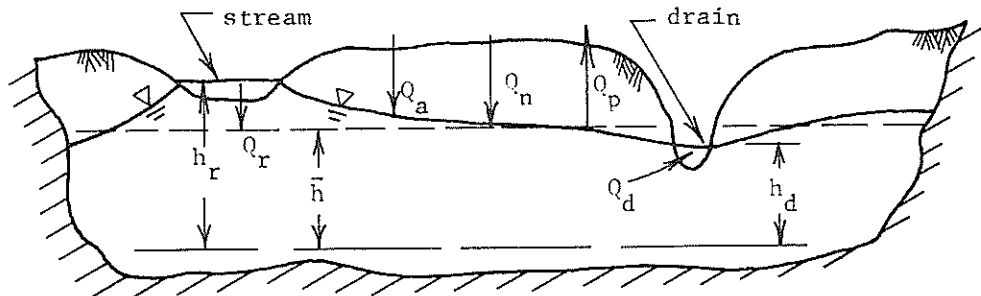


Figure 2.4: Schematic Illustration of an Aquifer with a Connected Stream

Referring to Figure 2.2 we can analyze each term of equation (2.1). The mass of water in the aquifer is simply

$$M_a = \rho SA\bar{h}(t) \quad , \quad (2.2)$$

where

- ρ = density of water,
- S = storage coefficient (specific yield),
- A = area of the aquifer (L x W),
- $\bar{h}(t)$ = average water level of aquifer.

The mass influx is

$$m_i = \rho [Q_n + Q_a + Q_r] \quad , \quad (2.3)$$

where

- Q_n = volumetric rate of natural recharge,
- Q_a = volumetric rate of artificial recharge,
- Q_r = volumetric rate of river recharge,

and the mass efflux is

$$m_o = \rho [Q_p + Q_d] \quad (2.4)$$

where

Q_p = volumetric rate of pumpage,
 Q_d = volumetric rate of drainflow.

Insertion of equations (2.2), (2.3) and (2.4) into the continuity equation (2.1) yields

$$\frac{d}{dt} \rho SA \bar{h}(t) = \rho \left[\bar{Q}_n + Q_a + Q_r - Q_p - Q_d \right] . \quad (2.5)$$

Assuming that the density of water, ρ , the storage coefficient, S , and the area of the aquifer, A , are all independent of time and then manipulating equation (2.5), we obtain

$$S \frac{d\bar{h}}{dt} = q_n + q_a + q_r - q_p - q_d , \quad (2.6)$$

where

$$\begin{aligned}
 q_n &= \frac{Q_n}{A} , & q_a &= \frac{Q_a}{A} , & q_r &= \frac{Q_r}{A} , \\
 q_p &= \frac{Q_p}{A} , & \text{and } q_d &= \frac{Q_d}{A} .
 \end{aligned}$$

Let us now define net recharge as

$$E = q_n + q_a - q_p$$

and substitute this into equation (2.6) to obtain

$$S \frac{d\bar{h}}{dt} = q_r - q_d + E , \quad (2.7)$$

which is the water-balance equation of the lumped-parameter model for the perched-stream case.

Looking through the inputs and outputs shown in Figure 2.2 we find that only the drain flow, Q_d , is dependent on the water level in the aquifer. This drain flow is estimated by applying Darcy's law, and using the notation of Figure 2.3 as follows:

$$Q_d = h_d L_d K \left. \frac{\partial h}{\partial x} \right|_{x=0}, \quad (2.8)$$

where

h_d = water elevation in drain above aquifer bottom,

L_d = length of drain,

K = hydraulic conductivity of aquifer.

Assume that

$$L_d = c_1 L, \quad (2.9)$$

where

c_1 = a constant;

and

$$\left. \frac{\partial h}{\partial x} \right|_{x=0} = c_2 \frac{\bar{h}(t) - h_d}{\bar{x}}, \quad (2.10)$$

where

$$\begin{aligned}\bar{h}(t) &= \text{average water level in aquifer,} \\ \bar{x} &= \text{point where average water level occurs,} \\ c_2 &= \text{a constant;}\end{aligned}$$

and

$$\bar{x} = c_3 W \quad , \quad (2.11)$$

where

$$c_3 = \text{a constant.}$$

By keeping c_1 , c_2 and c_3 constant, we make three implicit assumptions in the model: (1) the water table is always above the drain elevation throughout the aquifer, (2) the slope of water table at the drain is always proportional to the average slope of the water table in the aquifer and (3) the location of the average water table always stays at the same point. Inserting equations (2.9), (2.10), and (2.11) into (2.8) and recalling that $A = LW$, we obtain

$$q_d = \frac{c_1 c_2 h_d L K (\bar{h} - h_d)}{c_3 L W^2} \quad , \quad (2.12)$$

or more simply,

$$q_d = a_d (\bar{h} - h_d) \quad , \quad (2.13)$$

where

$$a_d = \frac{c_1 c_2 T}{c_3 W^2} = \text{drain discharge constant,}$$

$$T = Kh_d = \text{a transmissivity of the aquifer.}$$

Gelhar and Wilson (1974) have shown, using a simple steady flow solution for the distributed system, that $c_1 c_2 / c_3 = 3$. Equation (2.13) is now substituted into (2.7) yielding

$$S \frac{dh}{dt} = -a_d (h - h_d) + q_r + E \quad , \quad (2.14)$$

where the bars indicating average water levels are now dropped for convenience. Systems described by a linear first-order ordinary differential equation of the form of equation (2.14) are commonly called linear reservoirs.

Two solutions of equation (2.14), one general and the other for the special case where $E = 0$, are easily obtained. The general solution (recharge case) for constant q_r and h_d is

$$h = h_o e^{-t/t_h} + \left[h_d + \frac{q_r}{a_d} \right] \left[1 - e^{-t/t_h} \right] + \frac{1}{S} \int_0^t E(\tau) e^{-(t-\tau)/t_h} d\tau \quad , \quad (2.15)$$

where

$$h_o = \text{water level at } t = 0,$$

$t_h = S/a_d =$ hydraulic response time of the aquifer,
 $\tau =$ dummy integration variable.

The equation for the special case where $E = 0$ (recession case) for constant q_r and h_d is

$$h = h_R e^{-t/t_h} + \left[h_d + \frac{q_r}{a_d} \right] \left[1 - e^{-t/t_h} \right] , \quad (2.16)$$

where

$h_R =$ water level at start of recession,

and time, t , is measured from the beginning of the recession.

Stream-Connected Aquifer Case

Figure 2.4 shows an example of an aquifer connected to a stream. Its continuity equation is the same as that for the perched-stream case. Following the same analysis as for drain flow, the river discharge term is obtained:

$$q_r = -a_r(h - h_r) , \quad (2.17)$$

where

$a_r =$ river discharge constant,

$h_r =$ average level of water in river.

Substituting equations (2.13) and (2.17) into equation (2.7), yields

$$S \frac{dh}{dt} = -a_d(h - h_d) - a_r(h - h_r) + E . \quad (2.18)$$

The solution for the recharge case for constant h_d and h_r is

$$h = h_o e^{-t/t_h} + \left[\frac{a_r h_r + a_d h_d}{a_r + a_d} \right] \left[1 - e^{-t/t_h} \right] + \frac{1}{S} \int_0^t E(\tau) e^{-(t-\tau)/t_h} d\tau, \quad (2.19)$$

where

$$t_h = \frac{S}{a_r + a_d} = \text{hydraulic response time of the aquifer.}$$

For the recession case, the solution is

$$h = h_R e^{-t/t_h} + \left[\frac{a_r h_r + a_d h_d}{a_r + a_d} \right] \left[1 - e^{-t/t_h} \right]. \quad (2.20)$$

Special Cases

Due to heavy pumping, it is possible that the average water level in the aquifer, h , may fall below the drain elevation h_d . Since the aquifer provides the only source of water to the drain, a situation such as this causes $q_d = 0$. This requires that equation (2.14) become

$$S \frac{dh}{dt} = q_r + E$$

and that equation (2.18) become

$$S \frac{dh}{dt} = -a_r (h - h_r) + E.$$

Similarly, water levels in the aquifer may fall below the river elevation, h_r . Since rivers usually have an upstream source of water, equation (2.18) needs no modification. In essence, the equation

$$q_r = -a_r(h - h_r)$$

applies for both positive and negative $(h - h_r)$. Of course, if the river dries up, then $q_r = 0$, and the appropriate modifications are made to equation (2.18). Finally, it is also possible that h may fall so far below h_r that the river is essentially perched and equation (2.14) then applies.

CHAPTER 3

ESTIMATION OF PARAMETERS AND RECHARGE

In order to apply the models derived in Chapter 2, certain parameters (storage coefficient S , drain discharge constant a_d , river discharge constant a_r , drain elevation h_d , and river elevation h_r or river discharge q_r) must be obtained. These parameters are estimated by using the least squares method. Also the net recharge, E , is calculated by using an input-output analysis.

A Brief Description of the Least Squares Method

Define a residual, δ_i , as the difference between the observed value of a dependent variable and its "true" value. Then we can write

$$\delta_i = \hat{y}_i - y(\vec{x}_i; \vec{\theta}) \quad , \quad (3.1)$$

where

- δ_i = i^{th} residual,
- \hat{y}_i = observed value of a dependent variable,
- y = "true" value of a dependent variable,
- \vec{x}_i = vector of observed independent variables,
- $\vec{\theta}$ = vector of parameters,
- i = index of i^{th} data point.

The "true" value of the dependent variable is described by some theoretical or empirical mathematical model. We now define the sums of squares of the residuals as

$$SSq = \sum_{i=1}^N \delta_i^2$$

$$= \sum_{i=1}^N \left(\hat{y}_i - y(\vec{x}_i; \vec{\theta}) \right)^2, \quad (3.2)$$

where

SSq = sums of squares of the residuals,

N = number of data points.

For different values of $\vec{\theta}$, SSq attains different values; we want to find the $\vec{\theta}$ that minimizes SSq. This is done by setting the derivative of SSq, with respect to each of its parameters, to zero. Mathematically, this yields

$$\frac{\partial \text{SSq}}{\partial \theta_j} = 2 \sum_{i=1}^N \left(\hat{y}_i - y(\vec{x}_i; \vec{\theta}) \right) \left(- \frac{\partial y}{\partial \theta_j} \right) = 0, \quad (3.3)$$

where

θ_j = j^{th} parameter.

There is an equation of the form of equation (3.3) for each parameter. Depending on the degree of non-linearity of the function, the solution of equation (3.3) can be quite complicated. For more information concerning the least squares technique and methods of solution, see Draper and Smith (1966) and Bard (1974).

Estimation of a_d and h_d

Suppose we are given a set of observed drain flows, $\hat{q}_{d_{ji}}$, and water levels h_{ji} ; then, using equation (3.2), we can set up the sum of squares of the residuals as

$$SSq = \sum_{j=1}^{NY} \sum_{i=1}^{NM_j} \left(\hat{q}_{d_{ji}} - q_d (h_{ji} ; a_d, h_d) \right)^2, \quad (3.4)$$

where

$\hat{q}_{d_{ji}}$ = observed drain flow for the i^{th} month of the j^{th} recharge-recession period,

q_d = mathematical equation describing drain flow,

h_{ji} = observed average water level in aquifer for the i^{th} month of the j^{th} recharge-recession period,

NM_j = number of months in the j^{th} recharge-recession period,

NY = number of recharge-recession periods.

In this report a recharge-recession period is defined as the period from the first month of recharge following a recession to the last month of the following recession. These are usually twelve months long, but sometimes they are shorter or longer. The double summation of equation (3.4) results from the use of the double subscripting system.

Substituting equation (2.13) into equation (3.4) yields

$$SSq = \sum_{j=1}^{NY} \sum_{i=1}^{NM_j} \left(\hat{q}_{d_{ji}} - a_d (h_{ji} - h_d) \right)^2. \quad (3.5)$$

Taking derivatives of equation (3.5) with respect to a_d and h_d and setting them to zero yields

$$\frac{\partial SSq}{\partial a_d} = 2 \sum_{j=1}^{NY} \sum_{i=1}^{NM_j} \left(\hat{q}_{dji} - a_d (h_{ji} - h_d) \right) \left(- (h_{ji} - h_d) \right) = 0 \quad (3.6a)$$

and

$$\frac{\partial SSq}{\partial h_d} = 2 \sum_{j=1}^{NY} \sum_{i=1}^{NM_j} \left(\hat{q}_{dji} - a_d (h_{ji} - h_d) \right) (a_d) = 0 \quad (3.6b)$$

The parameters a_d and h_d are then solved for:

$$a_d = \frac{\sum_{j=1}^{NY} \sum_{i=1}^{NM_j} \hat{q}_{dji} h_{ji} - \sum_{j=1}^{NY} \sum_{i=1}^{NM_j} \hat{q}_{dji} \sum_{j=1}^{NY} \sum_{i=1}^{NM_j} h_{ji}}{\sum_{j=1}^{NY} \sum_{i=1}^{NM_j} h_{ji}^2 - \left[\sum_{j=1}^{NY} \sum_{i=1}^{NM_j} h_{ji} \right]^2} \quad (3.7a)$$

and

$$h_d = \frac{\sum_{j=1}^{NY} \sum_{i=1}^{NM_j} h_{ji} - \left[\sum_{j=1}^{NY} \sum_{i=1}^{NM_j} \hat{q}_{dji} \right] / a_d}{\sum_{j=1}^{NY} \sum_{i=1}^{NM_j} 1} \quad (3.7b)$$

If equations (3.7a) and (3.7b) are to be programmed into a computer and if the values of h_{ji} are large, then the computer, because of its limited digital capacity, may produce erroneous results because of the loss of significant digits in the denominator of equation (3.7a). This is remedied by introducing a suitable datum; equations (3.7a) and (3.7b) then become

$$a_d = \frac{\sum_{j=1}^{NY} \sum_{i=1}^{NM_j} \hat{q}_{d,ji} h'_{ji} - \sum_{j=1}^{NY} \sum_{i=1}^{NM_j} \hat{q}_{d,ji} \sum_{j=1}^{NY} \sum_{i=1}^{NM_j} h'_{ji}}{\sum_{j=1}^{NY} \sum_{i=1}^{NM_j} h'_{ji}{}^2 - \left[\sum_{j=1}^{NY} \sum_{i=1}^{NM_j} h'_{ji} \right]^2} \quad (3.8a)$$

and

$$h_d = \frac{\sum_{j=1}^{NY} \sum_{i=1}^{NM_j} h'_{ji} - \left[\sum_{j=1}^{NY} \sum_{i=1}^{NM_j} \hat{q}_{d,ji} \right]}{\sum_{j=1}^{NY} \sum_{i=1}^{NM_j}} + h_b, \quad (3.8b)$$

where

$$h'_{ji} = h_{ji} - h_b,$$

$$h_b = \text{base elevation.}$$

Estimation of S, t_h and q_r for Perched-Stream Case

To estimate the parameters S, t_h and q_r for the perched-stream case, we make use of the recession curve of each recharge-recession period and the recession equation (2.16). We can discretize equation (2.16) as

$$h'_{ji+1} = h'_{ji} e^{-\Delta t/t_h} + \hat{h}_j \left(1 - e^{-\Delta t/t_h} \right),$$

$$R_j \leq i \leq NM_j - 1 \quad (3.9)$$

where

$$\hat{h}_j = h_d + \frac{q_r}{a_d} j - h_b$$

= steady-state water level for the j^{th} recharge-recession period,

Δt = a constant time interval between h'_{ji} and h'_{ji+1} ,

R_j = index of the first month of recession for the j^{th} recharge-recession period.

Note that we now assume that, while q_{rj} is constant within a recharge-recession period, it changes from period to period. We set up the sum of squares as

$$SSq = \sum_{j=1}^{NY} \sum_{i=R_j}^{NM_j} \left[h'_{ji+1} - h'_{ji} e^{-\Delta t/t_h} - \hat{h}_j \left(1 - e^{-\Delta t/t_h} \right) \right]^2. \quad (3.10)$$

Treating $e^{-\Delta t/t_h}$ and \hat{h}_j as parameters, differentiating equations (3.10) with respect to these parameters, setting the resulting equations to zero and solving yields (see Appendix A)

$$e^{-\Delta t/t_h} = \frac{\sum_{j=1}^{NY} \sum_{i=R_j}^{NM_j-1} h'_{ji+1} h'_{ji} - \sum_{j=1}^{NY} \frac{1}{NM_j - R_j} \sum_{i=R_j}^{NM_j-1} h'_{ji+1} \sum_{i=R_j}^{NM_j-1} h'_{ji}}{\sum_{j=1}^{NY} \sum_{i=R_j}^{NM_j-1} h'_{ji}{}^2 - \sum_{j=1}^{NY} \frac{1}{NM_j - R_j} \left[\sum_{i=R_j}^{NM_j-1} h'_{ji} \right]^2} \quad (3.11a)$$

and

$$\hat{h}_j = \frac{e^{-\Delta t/t_h} \sum_{i=R_j}^{NM_j-1} h'_{ji} - \sum_{i=R_j}^{NM_j-1} h'_{ji+1}}{\left(NM_j - R_j \right) \left(e^{-\Delta t/t_h} - 1 \right)}, \quad (3.11b)$$

from which we calculate

$$t_h = - \frac{\Delta t}{\ln \left[e^{-\Delta t/t_h} \right]} , \quad (3.12a)$$

$$q_{r_j} = a_d \left(\hat{h}_j + h_b - h_d \right) , \quad j = 1, NY \quad (3.12b)$$

and

$$S = a_d t_h . \quad (3.12c)$$

Estimation of S , a_r , h_r and t_h for Stream-Connected Aquifer Case

The estimation of parameters for the stream-connected aquifer case involves a two-step process that utilizes the recession curve and characteristics for that case. From equation (2.20) the discretized recession equation is

$$h_{ji+1} = h_{ji} e^{-\Delta t/t_h} + \hat{h} \left(1 - e^{-\Delta t/t_h} \right) , \quad (3.13)$$

where

$$\hat{h} = \frac{a_r h_r + a_d h_d}{a_r + a_d} ,$$

from which the parameters $e^{-\Delta t/t_h}$ and \hat{h} are solved using the least squares technique. We then have the equations

$$S = \left(a_r + a_d \right) t_h \quad (3.14a)$$

and

$$a_r h_r + a_d h_d = (a_r + a_d) \hat{h} \quad (3.14b)$$

where a_d , h_d , t_h and \hat{h} are known. Equation (2.17) is substituted into equation (2.7) with the E term deleted and integrated over the entire recession period to yield

$$SI_{j1} + a_r I_{j2} = a_r h_r I_{j3} - I_{j4} \quad , \quad (3.15)$$

where

$$I_{j1} = h_{jNM_j} - h_{jR_j} \quad ,$$

$$I_{j2} = \int_{t_{R_j}}^{t_{NM_j}} h dt \quad ,$$

$$I_{j3} = t_{NM_j} - t_{R_j} \quad ,$$

$$I_{j4} = \int_{t_{R_j}}^{t_{NM_j}} q_d dt \quad .$$

The quantities I_{j2} and I_{j4} are obtained by numerical integration using the discrete values of h_{ji} and \hat{q}_{dji} . We now set up the sum of squares as

$$\begin{aligned} SSq = & \sum_{j=1}^{NY} \left(SI_{j1} + a_r I_{j2} - a_r h_r I_{j3} + I_{j4} \right)^2 \\ & + \lambda_1 \left[S - (a_r + a_d) t_h \right] + \lambda_2 \left[a_r h_r + a_d h_d - (a_r + a_d) \hat{h} \right] \quad (3.16) \end{aligned}$$

where λ_1 and λ_2 are Lagrange multipliers, and solve for S , a_r , h_r , λ_1 and λ_2 . The use of Lagrange multipliers forces the estimates to satisfy equations (3.14a) and (3.14b), which are called constraints.

Estimation of Net Recharge

Given the parameters and equation (2.7), estimates of net recharge, E , can be obtained for the perched-stream case. Equation (2.7) is integrated over two time intervals using Simpson's Rule to yield

$$\begin{aligned}
 S \left(h_{j,i+1} - h_{j,i-1} \right) &= 6q_{rj} \frac{\Delta t}{3} - \frac{\Delta t}{3} \left(\hat{q}_{d,j,i+1} + 4\hat{q}_{d,j,i} + \hat{q}_{d,j,i-1} \right) \\
 &\quad + \frac{\Delta t}{3} \left(E_{j,i+1} + 4E_{j,i} + E_{j,i-1} \right) \quad , \\
 i &= 1, R_j - 1 \quad (3.17)
 \end{aligned}$$

which we rearrange as

$$\begin{aligned}
 E_{j,i-1} + 4E_{j,i} + E_{j,i+1} &= \frac{3S}{\Delta t} \left(h_{j,i+1} - h_{j,i-1} \right) - 6q_{rj} \\
 &\quad + \hat{q}_{d,j,i+1} + 4\hat{q}_{d,j,i} + \hat{q}_{d,j,i-1} \\
 &= A_{ji} \quad , \quad i = 1, R_j - 1 \quad . \quad (3.18)
 \end{aligned}$$

This produces a set of $R_j - 1$ equations in $R_j + 1$ unknowns for each recharge-recession period j . For $i = 1$ we know that the preceding month is the last month of recession, implying $E_{j0} = 0$. Similarly, for $i = R - 1$ we know that the following month is the first month of the next recession, implying $E_{jR} = 0$. We lose two of our unknowns; thence, equation (3.18) produces for each j a triadiagonal set of $R - 1$ equations with $R - 1$ unknowns of the form

$$\begin{array}{rcl}
4E_1 + E_2 & & = A_1 \\
\vdots & & \\
E_{i-1} + 4E_i + E_{i+1} & & = A_i \\
\vdots & \vdots & \vdots \\
E_{R-2} + 4E_{R-1} & = & A_{R-1} \quad , \quad (3.19)
\end{array}$$

which is easily solved using the algorithm given in Smith (1965, p. 20). Using a similar analysis we can estimate the net recharge for the stream-connected case.

CHAPTER 4

CASE STUDY - MESILLA VALLEY, NEW MEXICO

The Mesilla Valley, a rich agricultural area located along the Rio Grande in south-central New Mexico (Figure 4.1), was chosen for evaluation and estimation of the parameters of the two models. The valley covers 108,000 acres of which approximately 80,000 acres is irrigated farmland. An 80 to 100 foot-thick alluvial-fill aquifer underlies the valley and extends beyond its boundaries. Another aquifer, that of the Santa Fe formation, lies beneath the alluvial aquifer and extends thousands of feet below it. This study deals primarily with the shallow alluvial aquifer.

The data required for the study was obtained from two sources. Monthly water-level measurements from 1946 through 1971 at 39 United States Bureau of Reclamation (USBR) observation wells were obtained from Richardson (1971). The river bed of the Rio Grande at the Leasburg Dam at the northern-most point of the valley was used as an additional water-level observation because observation wells were lacking in that area. The valley was then divided into two parts: one east and the other west of the Rio Grande. Each part was subdivided into Thiessen polygons according to the location of the 39 observation wells and the river bed at the Leasburg Dam (see Figure 4.2). Each polygon was assigned a weighting factor calculated by dividing the area of the polygon by the area of the valley (Table 4.1). The water-level measurements and river-bed elevation were then referenced to sea level and the monthly average water levels were calculated as

$$\bar{h} = \sum_{k=1}^{40} w_k h_k \quad ,$$

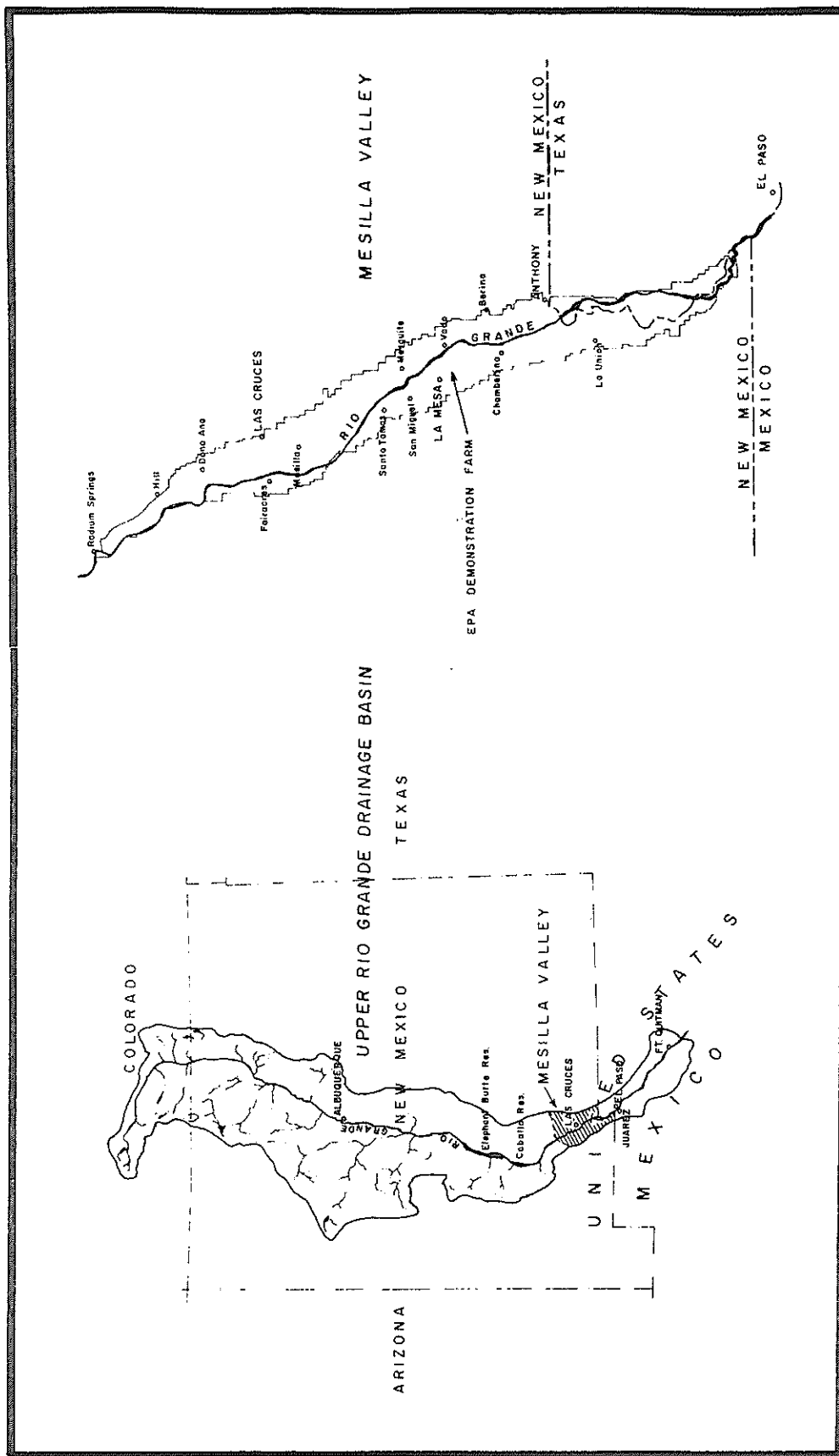


Figure 4.1: Location Map of the Mesilla Valley, New Mexico

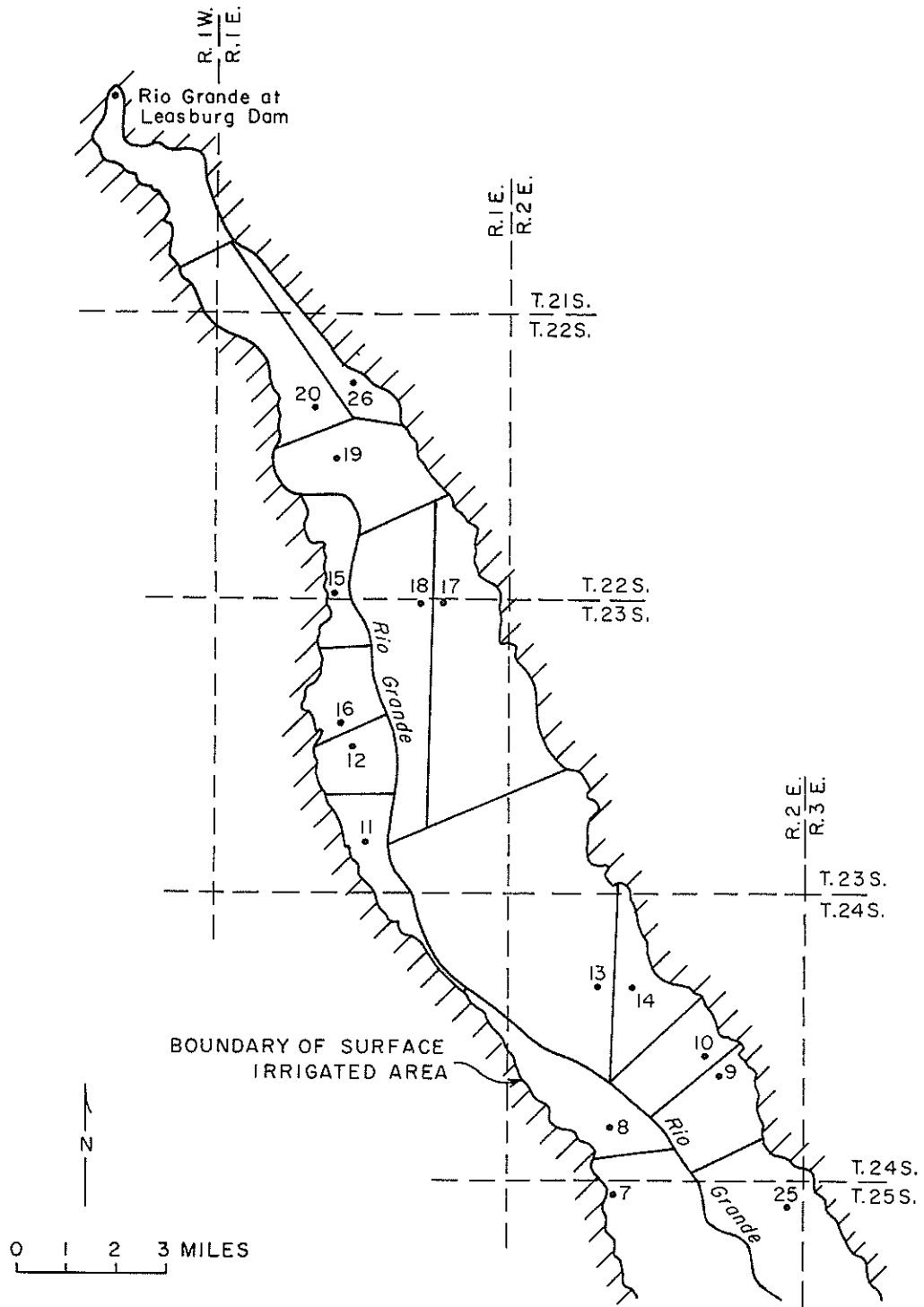


Figure 4.2: USBR Well Locations and Thiessen Polygons

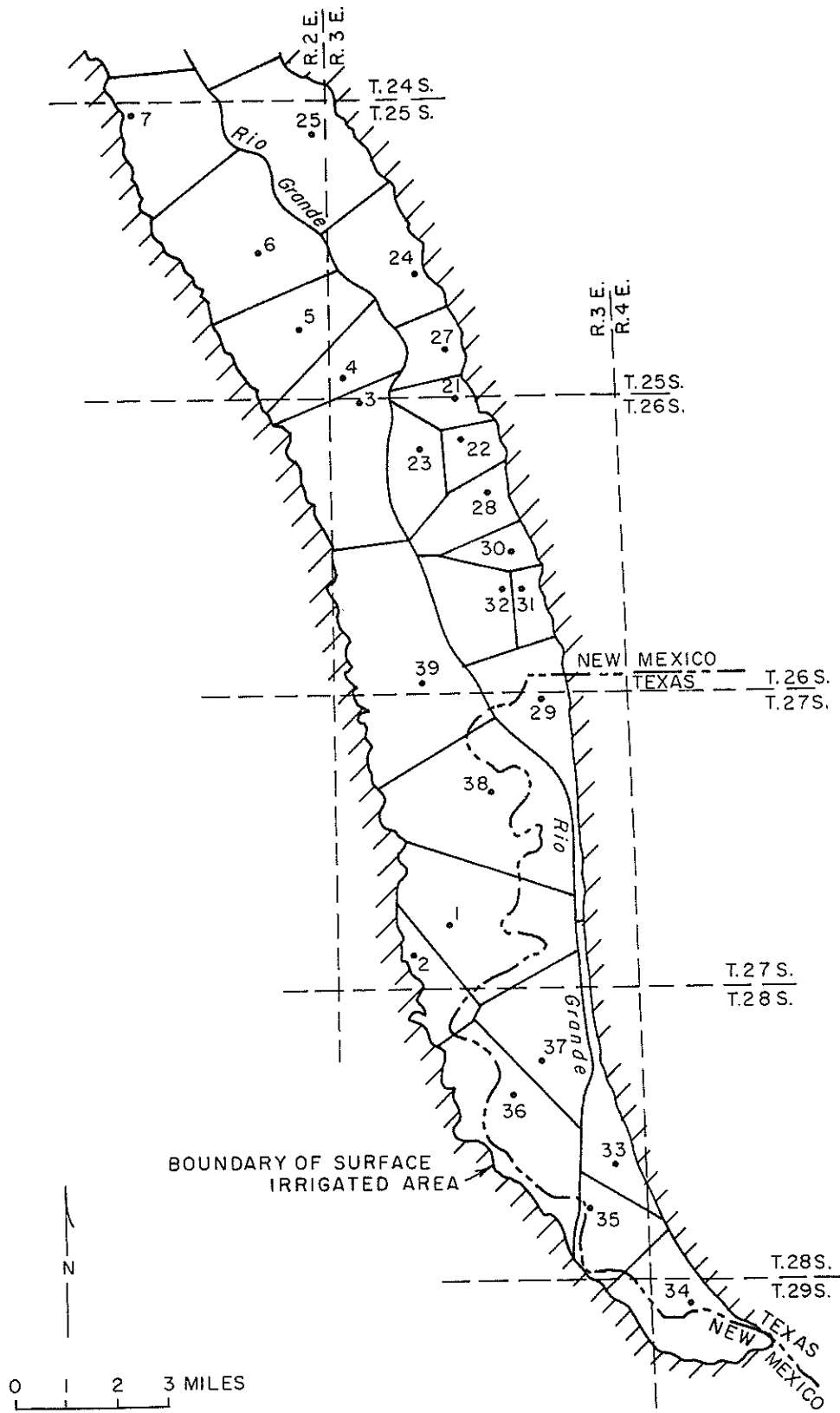


Figure 4.2: (Continued)

TABLE 4.1

THIESSEN POLYGON WEIGHT AND LAND SURFACE ELEVATION
AT EACH OBSERVATION WELL

USBR WELL NO.	LAND ELEVATION	POLYGON WEIGHT	USBR WELL NO.	LAND ELEVATION	POLYGON WEIGHT
Rio Grande at Leasburg Dam	3960.00	0.01983	4	3812.76	0.01657
			27	3813.06	0.00845
26	3928.52	0.01190	21	3808.98	0.00789
20	3926.59	0.02833	23	3807.50	0.01223
19	3921.84	0.02838	22	3803.18	0.00756
15	3904.81	0.01421	3	3811.01	0.03282
18	3908.35	0.04524	28	3804.95	0.01237
17	3908.21	0.05813	32	3793.08	0.01672
16	3893.13	0.01162	31	3791.35	0.00614
12	3894.19	0.01483	30	3792.96	0.00661
11	3880.79	0.01417	39	3788.53	0.05288
13	3861.36	0.10548	29	3786.84	0.02248
14	3860.40	0.01757	38	3777.54	0.04900
10	3849.85	0.01761	1	3769.70	0.04056
9	3847.08	0.01851	2	3771.68	0.01105
8	3849.00	0.02507	37	3753.92	0.02725
25	3833.74	0.03626	36	3753.15	0.03376
7	3845.47	0.02710	35	3746.57	0.01516
6	3827.05	0.04042	33	3745.79	0.01577
5	3819.78	0.02521	34	3733.97	0.02399
24	3817.41	0.02087	Valley Mean	3834.07	1.00000

where

$$\begin{aligned}\bar{h} &= \text{average water level,} \\ h_k &= \text{water level of } k^{\text{th}} \text{ well,} \\ w_k &= \text{weighting factor for } k^{\text{th}} \text{ well,}\end{aligned}$$

and plotted against time. Monthly total valley drain flows were available from the USBR in El Paso, Texas. These drain flows, divided by valley area, were used for this study. It has been assumed that the observed drain flow results entirely from outflow from the aquifer. Also obtained from the USBR in El Paso, Texas, were water distribution sheets. These sheets list the amounts and months of diverted irrigation water. They were used principally to determine the dates and length of recharge-recession periods and recession periods. Table 4.2 shows monthly average water levels, drain flows and diverted irrigation water.

An examination of a temporal plot of water levels showed that suitable recessions occurred from March, 1946, to February, 1951. After this period, heavy pumping resulting from a drought lowered the water table so much that recessions could not occur. Analysis of the water distribution sheets showed five recharge-recession periods: March, 1946, to February, 1947; March, 1947, to March, 1948; April, 1948, to February, 1949; March, 1949, to February, 1950; and March, 1950 to February, 1951. Respective recession periods began in October for each recharge-recession period. Having obtained the water levels and drain flows for this five-year period, the parameter estimation process was begun.

Using equation (3.8a) and (3.8b) the drain flow equation (2.13) was estimated as

TABLE 4.2

MONTHLY AVERAGE WATER LEVELS, DRAIN FLOWS,
AND DIVERTED IRRIGATION WATER

DATE	AVERAGE WATER LEVEL (ft above MSL)	MONTHLY DRAIN FLOW (ft/mo)	DIVERTED IRRIGATION WATER (ft/mo)	TOTAL ANNUAL DIVERSION (ft)
JAN 1946	3826.22	0.1204	0.0	
FEB 1946	3826.12	0.0900	0.115	
MAR 1946	3826.38	0.1270	0.393	
APR 1946	3827.34	0.2034	0.725	
MAY 1946	3827.54	0.2312	0.487	
JUN 1946	3828.01	0.2489	0.704	
JUL 1946	3828.36	0.2858	0.856	
AUG 1946	3828.42	0.3016	0.811	
SEP 1946	3827.68	0.2446	0.309	
OCT 1946	3826.86	0.1616	0.103	
NOV 1946	3826.52	0.1256	0.082	
DEC 1946	3826.17	0.1152	0.060	4.63
JAN 1947	3825.95	0.0975	0.0	
FEB 1947	3825.89	0.0775	0.053	
MAR 1947	3826.18	0.1105	0.385	
APR 1947	3827.18	0.1831	0.718	
MAY 1947	3827.50	0.1995	0.502	
JUN 1947	3827.81	0.2101	0.607	
JUL 1947	3828.32	0.2503	0.851	
AUG 1947	3828.32	0.2640	0.697	
SEP 1947	3827.77	0.2182	0.452	
OCT 1947	3826.84	0.1356	0.042	
NOV 1947	3826.38	0.1061	0.023	
DEC 1947	3826.19	0.0915	0.018	4.35
JAN 1948	3825.86	0.0830	0.007	
FEB 1948	3825.79	0.0719	0.006	
MAR 1948	3825.74	0.0744	0.265	
APR 1948	3826.79	0.1485	0.683	
MAY 1948	3827.21	0.1875	0.406	
JUN 1948	3827.38	0.1943	0.540	

TABLE 4.2 (cont'd)

DATE	AVERAGE WATER LEVEL (ft above MSL)	MONTHLY DRAIN FLOW (ft/mo)	DIVERTED IRRIGATION WATER (ft/mo)	ANNUAL DIVERSION (ft)
JUL 1948	3828.06	0.2539	0.889	
AUG 1948	3828.24	0.2775	0.851	
SEP 1948	3827.69	0.2428	0.469	
OCT 1948	3826.95	0.1620	0.096	
NOV 1948	3826.58	0.1204	0.065	
DEC 1948	3826.25	0.1134	0.048	4.33
JAN 1949	3825.89	0.1045	0.0	
FEB 1949	3825.87	0.0823	0.0	
MAR 1949	3826.17	0.1182	0.481	
APR 1949	3827.01	0.1961	0.611	
MAY 1949	3827.28	0.2192	0.458	
JUN 1949	3827.43	0.2284	0.611	
JUL 1949	3828.06	0.2760	0.835	
AUG 1949	3828.11	0.2844	0.819	
SEP 1949	3827.62	0.2230	0.327	
OCT 1949	3826.87	0.1481	0.141	
NOV 1949	3826.52	0.1286	0.080	
DEC 1949	3826.24	0.1106	0.034	4.40
JAN 1950	3826.02	0.0947	0.0	
FEB 1950	3825.95	0.0776	0.042	
MAR 1950	3826.20	0.1384	0.655	
APR 1950	3826.98	0.1887	0.504	
MAY 1950	3827.40	0.2040	0.566	
JUN 1950	3827.59	0.2089	0.531	
JUL 1950	3827.80	0.2634	0.635	
AUG 1950	3828.00	0.2640	0.794	
SEP 1950	3827.38	0.2301	0.434	
OCT 1950	3826.68	0.1609	0.073	
NOV 1950	3826.33	0.1178	0.023	
DEC 1950	3826.11	0.1019	0.010	4.27
JAN 1951	3825.89	0.0924	0.0	
FEB 1951	3825.84	0.0659	0.0	

$$q_d = 0.0812(\bar{h} - 3824.85) \quad . \quad (4.1)$$

This equation along with the corresponding drain flows and water levels are plotted in Figure 4.3. As can be seen, the equation predicting drain flows fits the data very well, indicating that the linear outflow assumption is appropriate and that the estimates of a_d and h_d are suitable. Indeed, h_d compares reasonably with the average valley elevation of 3,834 feet. During the drought years in the early 1950's, the average water level dropped below the average drain elevation, but contrary to equation (4.1), drain flow still occurred. By referring to Figures 4.4a and 4.4b we can visualize a possible reason for this discrepancy. During the high water-table years of the late 1940's, drain flow occurred throughout the whole valley. This, as shown in Figure 4.4a, implied that the water table was higher than the drain elevation throughout the Mesilla Valley. During the drought, heavier pumpage in the northern part of the valley than in the southern part lowered and changed the slope of the water table to that shown in Figure 4.4b. The water table was higher than the drains in the southern part of the valley and lower in the northern part, violating one of the assumptions used to develop the drain flow equation (2.13). This implies that equation (4.1) is valid only for the condition illustrated in Figure 4.4a.

Using equations (3.11a), (3.11b), (3.12a), (3.12b) and (3.12c), the parameters for the perched-river case were estimated, for the five recharge-recession periods, as

$$S = 0.210, \quad t_h = 2.59 \text{ months,}$$

$$q_{r1} = 0.0622 \text{ ft./mo.,}$$

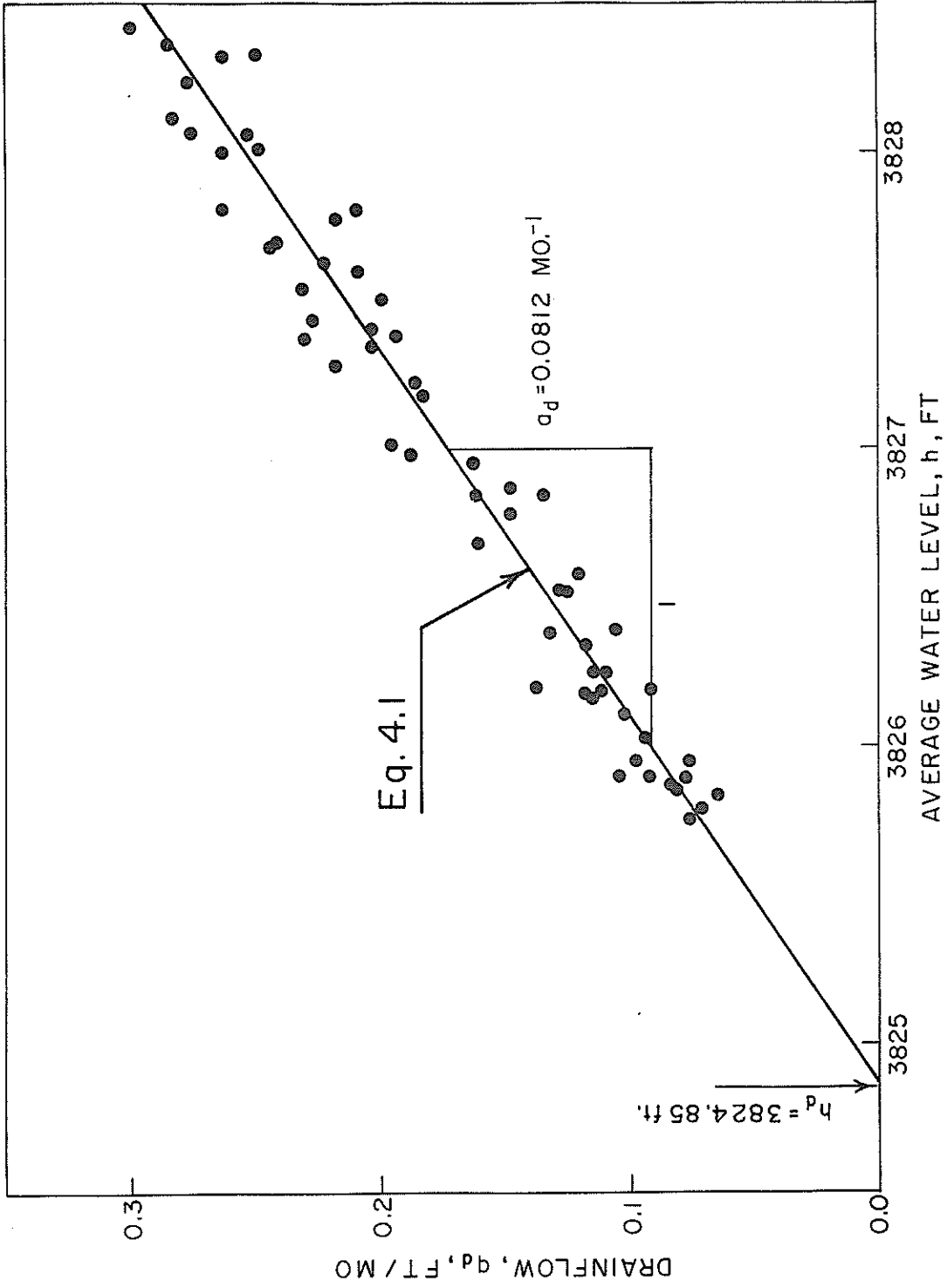


Figure 4.3: Average Water Level versus Drain Flow

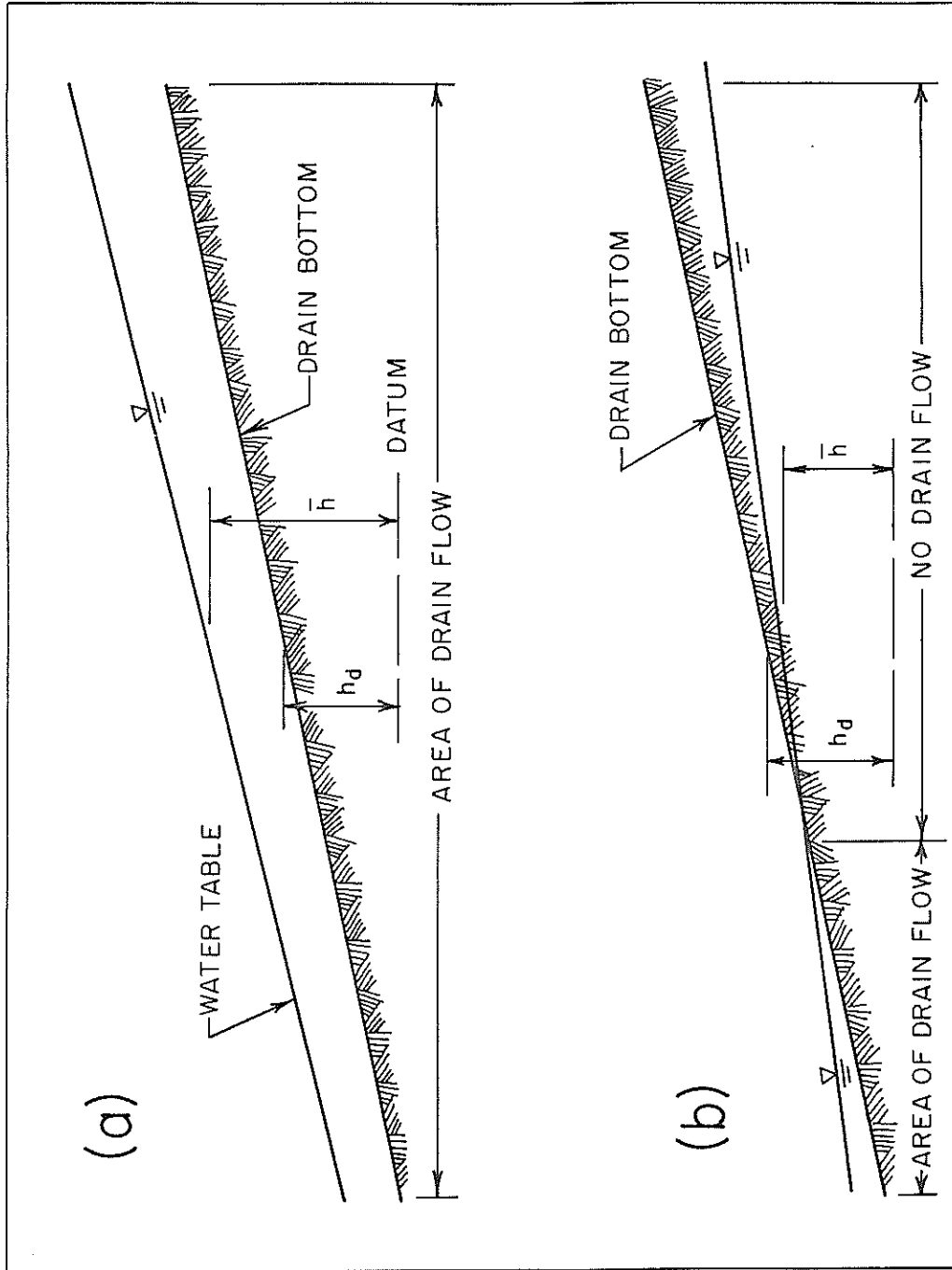


Figure 4.4: Water-Table Slope. (a): before Heavy Pumping. (b): after Heavy Pumping

$$q_{r_2} = 0.0546 \text{ ft./mo.},$$

$$q_{r_3} = 0.0585 \text{ ft./mo.},$$

$$q_{r_4} = 0.0682 \text{ ft./mo.},$$

$$q_{r_5} = 0.0601 \text{ ft./mo.}$$

The values of h_{ji} and h_{ji+1} for each j along with the best fit lines are plotted on Figures 4.5a-e. The real data deviates systematically from the best fit line for each recession, indicating either that the river discharge, q_{r_j} , is not constant from month to month as assumed, or recession is not precisely exponential. The value of storage coefficient compares favorably with the value of 0.20 that Richardson (1971) used in his study of the Mesilla Valley and the value of 0.25 suggested by Conover (1954). Based on a monthly input-output analysis of the Rio Grande within the valley boundaries, the values of q_{r_j} are usually about three to four times too large and definitely vary from month to month. It is quite possible that the estimated values of q_{r_j} contain, in addition to river discharge, leakage from the Santa Fe aquifer and irrigation canals.

Net recharge for the perched-river case was estimated using equation (3.19), and the results, along with irrigation diversion, are presented in Table 4.3 and plotted in Figure 4.6. The irrigation diversion is always larger than the estimated recharge, reflecting consumptive use through evapo-transpiration of vegetation. The temporal pattern of variation of recharge is very similar to that of the irrigation diversion; the lower diversions in May are reflected in the smaller recharge amount for that month. The difference between the annual amounts of diversion and recharge (Table 4.2 and Table 4.3) ranges from 2.98 to 3.21 feet, indicating the annual water consumption based on the total valley

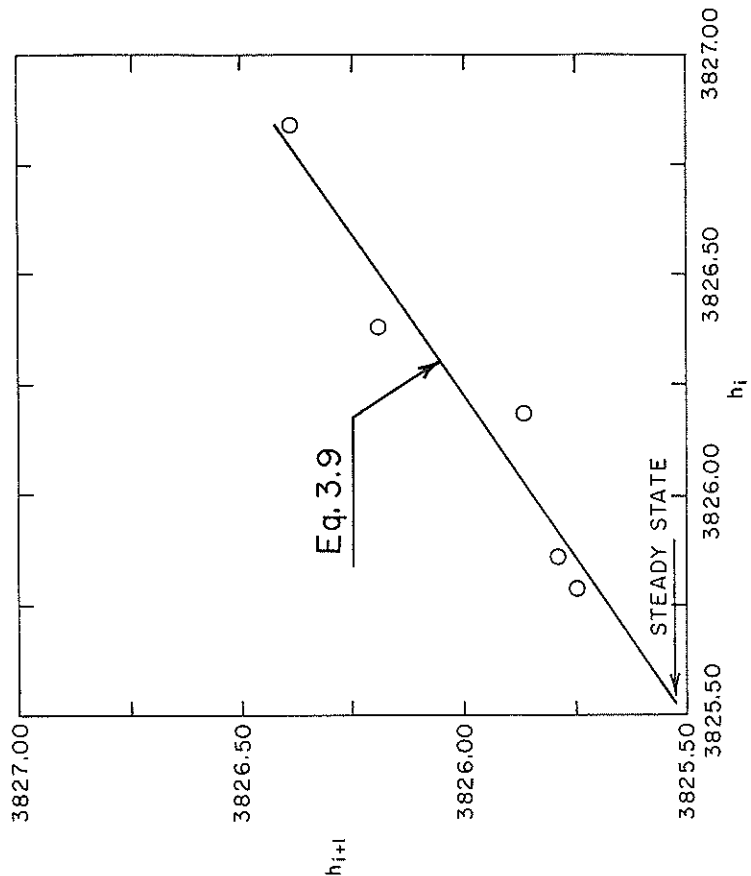


Figure 4.5a: h_i versus h_{i+1} for First Recession

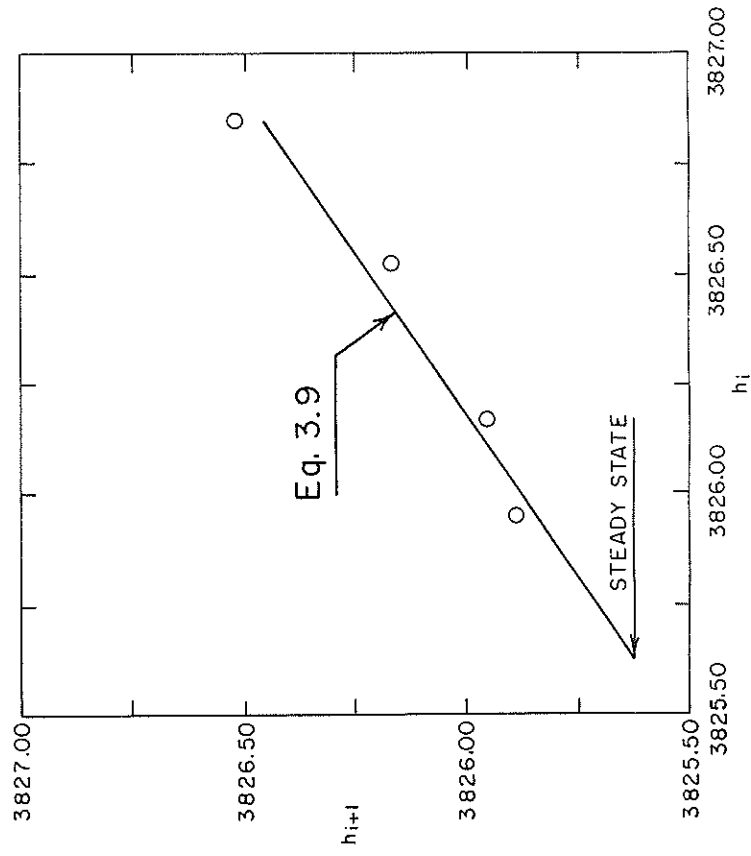


Figure 4.5b: h_i versus h_{i+1} for Second Recession

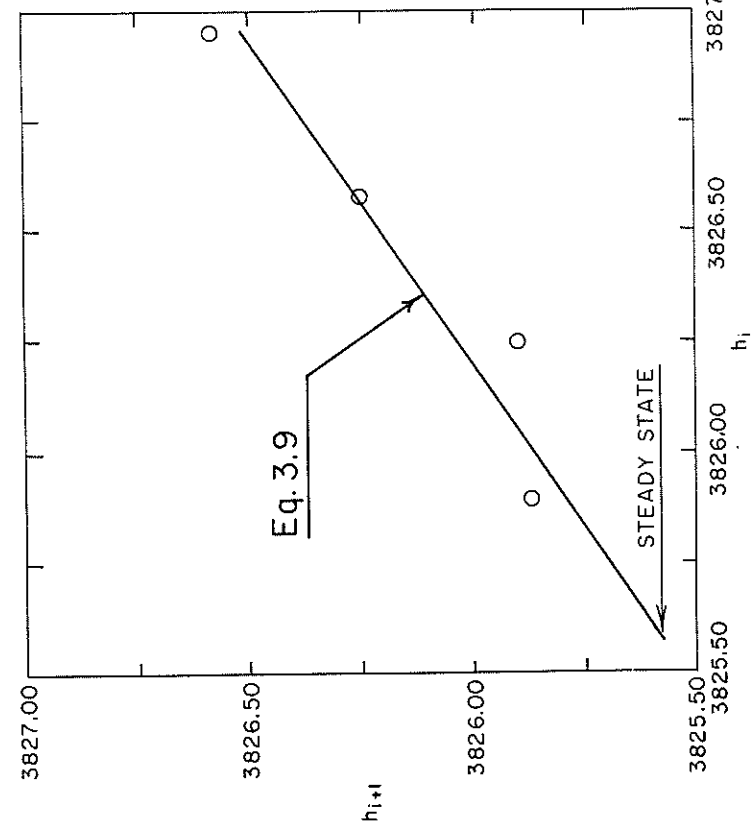


Figure 4.5c: h_i versus h_{i+1} for Third Recession

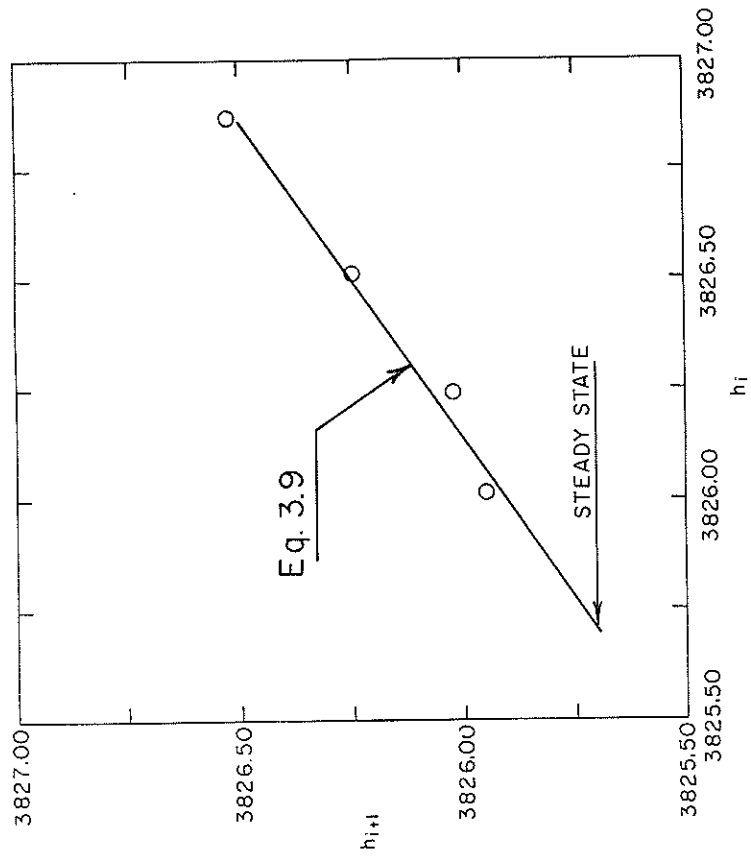


Figure 4.5d: h_i versus h_{i+1} for Fourth Recession

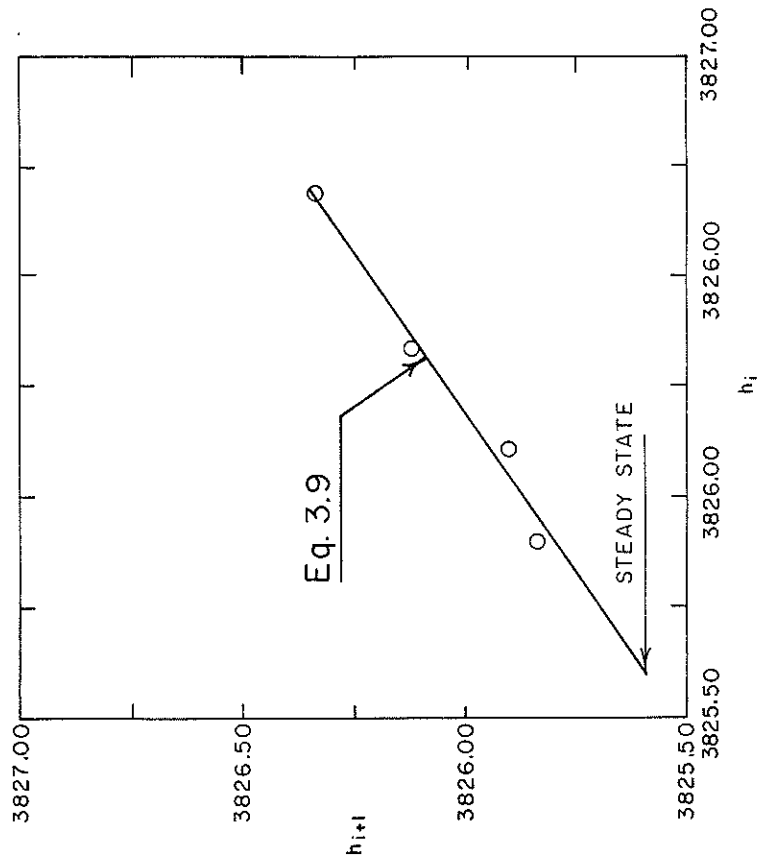


Figure 4.5e: h_i versus h_{i+1} for Fifth Recession

TABLE 4.3

RECHARGE ESTIMATES AND ANNUAL RECHARGE

DATE	RECHARGE ESTIMATES (ft/mo)	ANNUAL RECHARGE (ft)	DATE	RECHARGE ESTIMATES (ft/mo)	ANNUAL RECHARGE (ft/mo)
MAR 1946	0.2313		SEP 1948	0.0188	
APR 1946	0.2697		OCT 1948	0.0	
MAY 1946	0.2145		NOV 1948	0.0	
JUN 1946	0.2910		DEC 1948	0.0	1.21
JUL 1946	0.2791		JAN 1949	0.0	
AUG 1946	0.1680		FEB 1949	0.0	
SEP 1946	-0.0238		MAR 1949	0.2003	
OCT 1946	0.0		APR 1949	0.2623	
NOV 1946	0.0		MAY 1949	0.1591	
DEC 1946	0.0	1.43	JUN 1949	0.2581	
JAN 1947	0.0		JUL 1949	0.3036	
FEB 1947	0.0		AUG 1949	0.1636	
MAR 1947	0.2255		SEP 1949	-0.0077	
APR 1947	0.2862		OCT 1949	0.0	
MAY 1947	0.1778		NOV 1949	0.0	
JUN 1947	0.2596		DEC 1949	0.0	1.34
JUL 1947	0.2644		JAN 1950	0.0	
AUG 1947	0.1541		FEB 1950	0.0	
SEP 1947	-0.0347		MAR 1950	0.2105	
OCT 1948	0.0		APR 1950	0.2720	
NOV 1948	0.0		MAY 1950	0.1956	
DEC 1948	0.0	1.33	JUN 1950	0.1826	
JAN 1948	0.0		JUL 1950	0.2685	
FEB 1948	0.0		AUG 1950	0.1633	
MAR 1948	0.0		SEP 1950	-0.0014	
APR 1948	0.3223		OCT 1950	0.0	
MAY 1948	0.1400		NOV 1950	0.0	
JUN 1948	0.2358		DEC 1950	0.0	1.29
JUL 1948	0.3164		JAN 1951	0.0	
AUG 1948	0.1729		FEB 1951	0.0	

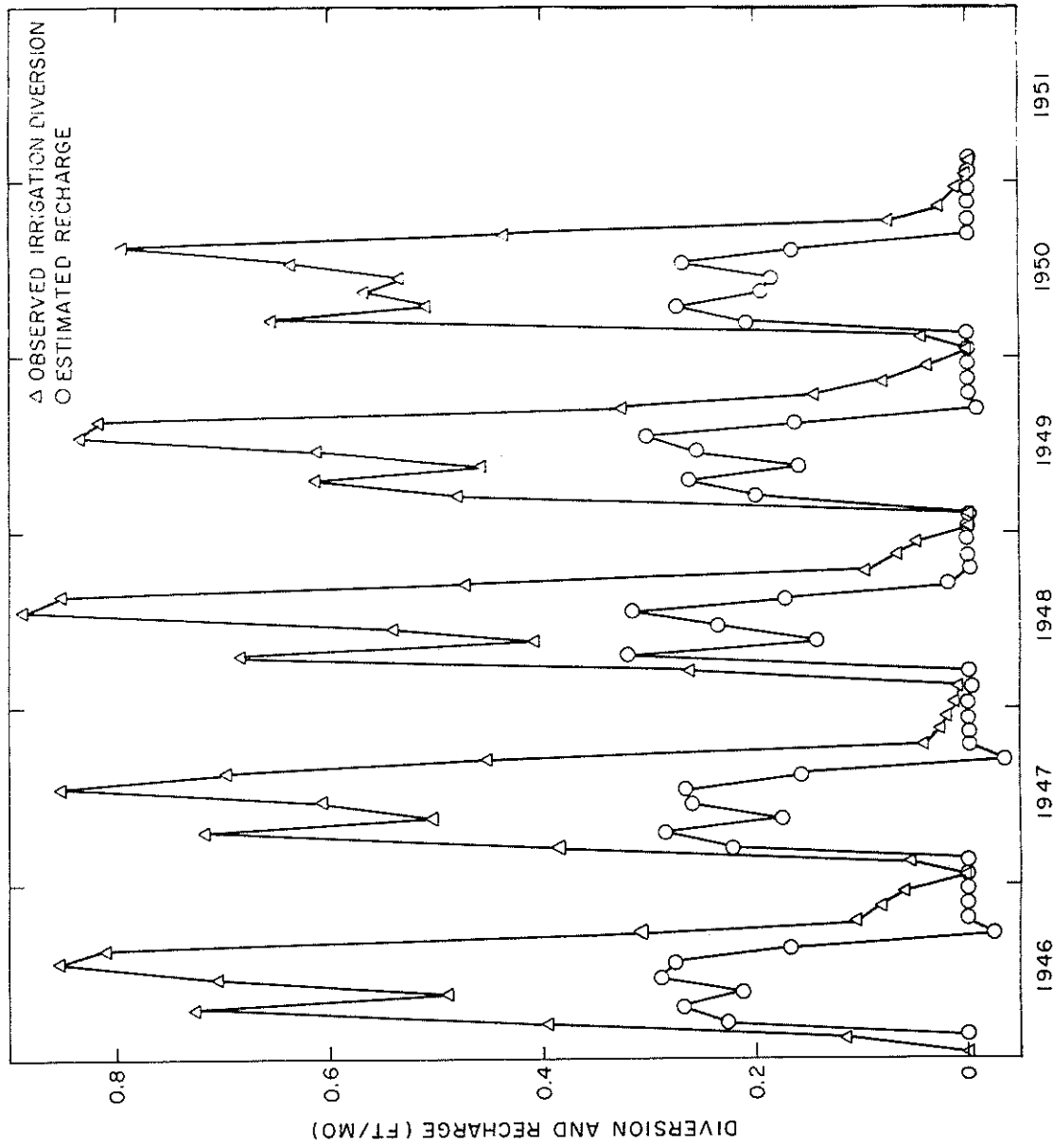


Figure 4.6: Recharge Estimates and Applied Irrigation Water

area. These consumption figures incorporate water use by agriculture as well as that of phreatophytes and other natural vegetation, and that associated with municipal water supply. It should also be recognized that some of the recharge resulting from the irrigation diversion (e.g., canal seepage) may be included in the q_{r_j} terms. If the entire q_{r_j} term, which averages about 0.06 ft./mo., were recharge from this source, then this water applied over a seven month irrigation season would increase the annual recharge by about 0.4 feet with a corresponding reduction in the estimated consumption. Thus, the over-all range for the annual consumptive use for the valley could be from 2.6 to 3.2 feet; we consider this range to be reasonable in relation to data on crop consumptive use (Henderson and Sorensen, 1968; Gregory and Hanson, 1976).

A simulation based on the parameter and net recharge estimates for the perched-stream case was run. This simulation was based on a discretized version of the general solution to the perched-stream case (equation (2.15)). This discretization yields

$$h_{i+1} = h_i e^{-\Delta t/t_h} + \left[h_d + \frac{q_r}{a_d} \right] \left[1 - e^{-\Delta t/t_h} \right] + \frac{1}{S} \int_0^{\Delta t} E(\tau) e^{-(t-\tau)/t_h} d\tau \quad (4.2)$$

Water levels, h_i and h_{i+1} , were assumed to be measured in the middle of the i^{th} and $i+1^{\text{th}}$ months and net recharge, E , was treated as a pulse recharge, E_i , during the i^{th} month. The last term of equation (4.2) was then rewritten as

$$\begin{aligned}
\frac{1}{S} \int_0^{\Delta t} E(\tau) e^{-(t-\tau)/t_h} d\tau &= \frac{1}{S} \int_0^{\Delta t/2} E_i e^{-(t-\tau)/t_h} d\tau \\
&+ \frac{1}{S} \int_{\Delta t/2}^{\Delta t} E_{i+1} e^{-(t-\tau)/t_h} d\tau \\
&= \frac{E_i e^{-t/t_h}}{a_d} \left(e^{\Delta t/2t_h} - 1 \right) \\
&+ \frac{E_{i+1} e^{-t/t_h}}{a_d} \left(e^{\Delta t/t_h} - e^{\Delta t/2t_h} \right) . \quad (4.3)
\end{aligned}$$

When discretized, the right-hand side of equation (4.3) becomes

$$\frac{E_i}{a_d} \left(e^{-\Delta t/2t_h} - e^{-\Delta t/t_h} \right) + \frac{E_{i+1}}{a_d} \left(1 - e^{-\Delta t/2t_h} \right) . \quad (4.4)$$

Substitution of equation (4.4) into the last term on the right hand side of equation (4.2) yields the simulation equation

$$\begin{aligned}
h_{i+1} &= h_i e^{-\Delta t/t_h} + \left(h_d + \frac{q_r}{a_d} \right) \left(1 - e^{-\Delta t/t_h} \right) \\
&+ \frac{E_i}{a_d} \left(e^{-\Delta t/2t_h} - e^{-\Delta t/t_h} \right) + \frac{E_{i+1}}{a_d} \left(1 - e^{-\Delta t/t_h} \right) . \quad (4.5)
\end{aligned}$$

Drain flow is then simulated from equation (2.13).

Simulated water levels and drain flows are shown in Figure 4.7a and 4.7b, respectively. As can be seen, the simulation matches (except for a couple of peaks) the observed data very well. The root-mean-squared error (RMSE) for water levels and drain flows is 0.13 feet and 0.012 ft./mo., respectively. It seems that the perched-stream linear-reservoir model can simulate water levels and drainflows very well for the Mesilla Valley.

The parameter estimation and simulation computations for the Mesilla Valley were carried out using the IBM 360-44 at the New Mexico Tech Computer Center; Appendix B gives a brief description of the computer program, plus listing of the code and the results.

An attempt was made to estimate the parameters for the stream-connected aquifer case. However, the value of the storage coefficient was less than zero which is physically impossible. This result implies that the assumption of the stream-connected aquifer as developed herein is not appropriate for the field conditions in the Mesilla Valley.

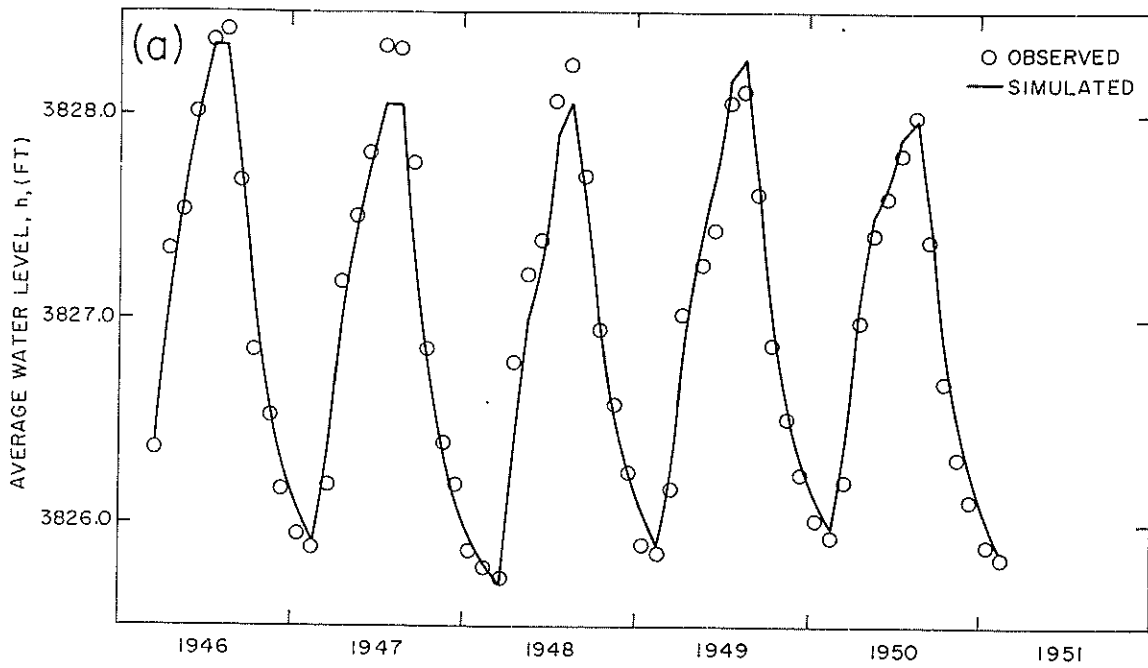


Figure 4.7a: Observed and Simulated Average Water Levels

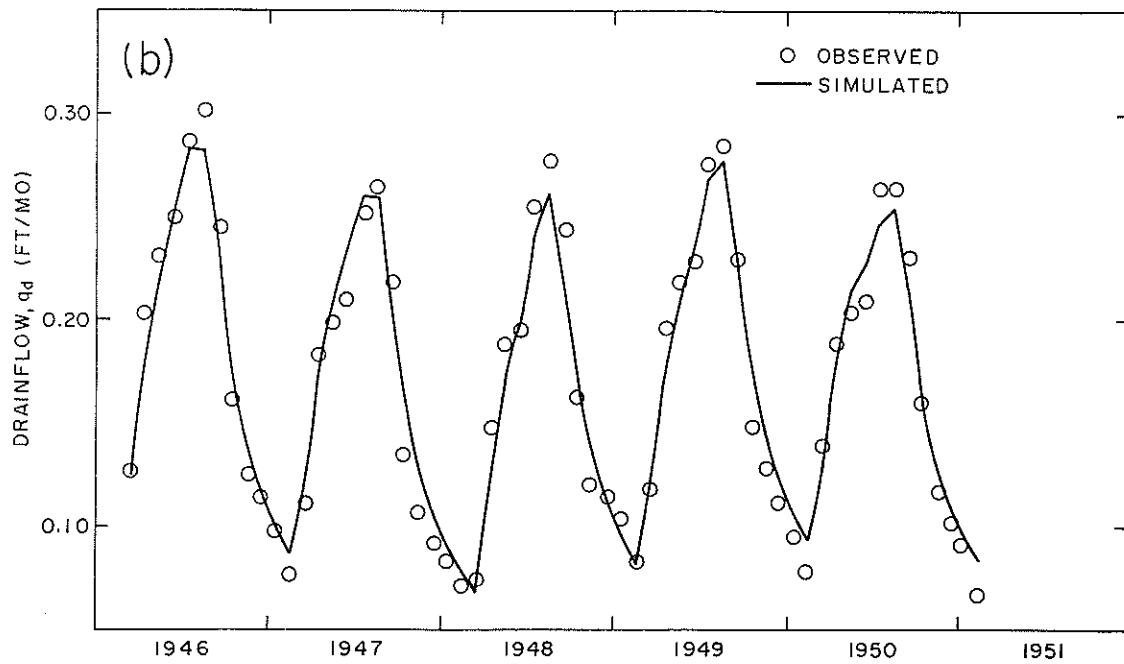


Figure 4.7b: Observed and Simulated Drain Flows

CHAPTER 5

SUMMARY, RECOMMENDATIONS, AND CONCLUSIONS

In the previous chapters, two lumped-parameter ground-water models were developed and their respective parameters were estimated by using the least squares technique. This method was applied to the Mesilla Valley, an irrigation area in south-central New Mexico. The data required for parameter estimation consisted of average monthly water levels, monthly drain flows and dates of recession. For the model aquifer with the perched stream, the parameter estimates were quite reasonable; however, when the stream-connected aquifer model was chosen, some of the parameter estimates were physically unreasonable ($S < 0$). Estimates of leakage from the river appeared to be high. It is likely that there were additional inputs into the aquifer not accounted for in the models but which were computationally included in the river discharge and net recharge estimates by the recharge estimation routine. A simulation using the parameters and net recharge estimates for the Mesilla Valley produced good agreement with the observed water levels and drain flows.

Conclusions drawn from this study include:

- (1) if the water levels follow a yearly pattern of recharge and recession, reasonable parameter and net recharge estimates can easily be obtained for the lumped-parameter ground-water model by using the least squares method, and
- (2) the results of the application to the Mesilla Valley show that a lumped-parameter, linear-reservoir ground-water model with appropriate stream-aquifer interaction can provide accurate predictions of average water levels and drain flows.

Recommendations for further research include:

- (1) a study to determine the extent to which lumped-parameter ground-water

models are suitable for both flow and water quality simulations,

(2) the development of methods to decompose net recharge into its various components,

(3) an evaluation of the possible nonlinearity of the drain flow-water level relationship (equation (2.13)), and

(4) the application of the parameter estimation technique to other irrigated areas similar to the Mesilla Valley.

REFERENCES

- Bard, Yonathan, Nonlinear Parameter Estimation, 341 pp., Academic Press, New York, 1974.
- Conover, C. S., Ground Water Conditions of the Rincon and Mesilla Valleys and Adjacent Areas in New Mexico, 200 pp., Geological Survey Water-Supply Paper 1230, United States Government Printing Office, Washington, D.C., 1954.
- Dooge, James C. I., Linear Theory of Hydrologic Systems, 327 pp., Technical Bulletin No. 1468, Agricultural Research Service, United States Department of Agriculture, Washington, D.C., 1973.
- Draper, N. R., and H. Smith, Applied Regression Analysis, 407 pp., John Wiley and Sons, New York, 1966.
- Flores W., Adán E. Z., and Lynn W. Gelhar, "A Stochastic Management Model for the Operation of a Stream-Aquifer System," New Mexico Water Resources Research Institute Report No. 075, 209 pp., New Mexico Water Resources Research Institute, Las Cruces, 1976.
- Frind, Emil O., and George F. Pinder, "Finite Elements in the Solution of the Inverse Problem in Ground Water Flow," in Finite Element Methods in Flow Problems, edited by J. T. Oden, O. C. Zienkiewicz, R. H. Gallagher and C. Taylor, pp. 559-572, University of Alabama Press, Huntsville, 1974.
- Gelhar, L. W., "Stochastic Analysis of Phreatic Aquifers," Water Resources Research, vol. 10, no. 3, pp. 539-545, June, 1974.
- Gelhar, Lynn W., and John L. Wilson, "Ground Water Quality Modeling," Ground Water, vol. 12, no. 6, pp. 399-408, November-December 1974.
- Gregory, E. J., and E. G. Hanson, "Predicting Consumptive Use from Climatological Data," Water Resources Research Institute, Report No. 66, New Mexico State University, 1976, 43 pp.

Hefez, E., U. Shamir, and J. Bear, "Identifying the Parameters of an Aquifer Cell Model," Water Resources Research, vol. 11, no. 6, pp. 993-1104, December, 1975.

Henderson, D. C., and E. F. Sorensen, "Consumptive Use Requirements of Selected Irrigated Areas in New Mexico," Agricultural Experiment Station, Bulletin 531, New Mexico State University, 1968.

Richardson, Gary L., Water Table Investigation of the Mesilla Valley, 206 pp., M. S. Thesis, New Mexico State University, Las Cruces, 1971.

Smith, G. D., Numerical Solution of Partial Differential Equations, Oxford University Press, 129 pp., 1965.

APPENDIX A

ALGEBRAIC MANIPULATION TO OBTAIN
 t_h AND S ESTIMATES FOR THE CASE OF
AN AQUIFER WITH A PERCHED STREAM

The objective function which is used to estimate t_h and S for the aquifer with a perched stream is equation (3.10); for ease of calculation let $\alpha = e^{-\Delta t/t_h}$. Differentiating equation (3.10) with respect to α and \hat{h}_j , setting to zero and dividing off constants yield

$$\frac{\partial S S q}{\partial \alpha} = \sum_{j=1}^{NY} \sum_{i=R_j}^{NM_j-1} \left(h'_{ji+1} - h'_{ji} \alpha - \hat{h}_j + \hat{h}_j \alpha \right) \left(-h'_{ji} + \hat{h}_j \right) = 0, \quad (A1)$$

$$\frac{\partial S S q}{\partial \hat{h}_j} = \sum_{i=R_j}^{NM_j-1} \left(h'_{ji+1} - h'_{ji} \alpha - \hat{h}_j + \hat{h}_j \alpha \right) = 0 \quad . \quad (A2)$$

There are NY equations (A2), one for each recharge-recession period. Equation (A1) can be rewritten as

$$\begin{aligned} & \sum_j \sum_i \left(-h'_{ji+1} h'_{ji} + h'_{ji}{}^2 \alpha + \hat{h}_j h'_{ji} - \hat{h}_j h'_{ji} \alpha \right) \\ & + \sum_j \hat{h}_j \sum_i \left(h'_{ji+1} - h'_{ji} \alpha - \hat{h}_j + \hat{h}_j \alpha \right) = 0 \quad . \end{aligned} \quad (A3)$$

where i always runs from R_j to NM_j-1 and j runs from one to NY . Multiply each equation (A2) by \hat{h}_j and sum over all j to obtain

$$\sum_j \hat{h}_j \sum_i h'_{j+1} - h'_{ji} \alpha - \hat{h}_j + \hat{h}_j \alpha = 0 \quad (A4)$$

which can be subtracted from equation (A3) to yield

$$\sum_j \sum_i \left(-h'_{ji+1} h'_{ji} + h'_{ji}{}^2 \alpha + \hat{h}_j h'_{ji} - \hat{h}_j h'_{ji} \alpha \right) = 0 \quad (A5)$$

and can be rewritten

$$\sum_j \sum_i -h'_{ji+1} h'_{ji} + \alpha \sum_j \sum_i h'_{ji}{}^2 + \sum_j \hat{h}_j \sum_i h'_{ji} - \alpha \sum_j \hat{h}_j \sum_i h'_{ji} = 0. \quad (\text{A6})$$

Multiply equation (A2) by $\sum_i h'_{ji} / (NM_j - R_j)$ and sum over all j to yield

$$\sum_j \left\{ \frac{\sum_i h'_{ji+1} \sum_i h'_{ji}}{(NM_j - R_j)} - \alpha \frac{\left[\sum_i h'_{ji} \right]^2}{(NM_j - R_j)} - \frac{\sum_i \hat{h}_j \sum_i h'_{ji}}{(NM_j - R_j)} + \alpha \frac{\sum_i \hat{h}_j \sum_i h'_{ji}}{(NM_j - R_j)} \right\} = 0. \quad (\text{A7})$$

Note now that $\hat{h}_j = \frac{\sum_i \hat{h}_j}{(NM_j - R_j)}$ which can be substituted into equation (A7) to yield

$$\sum_j \frac{\sum_i h'_{ji+1} \sum_i h'_{ji}}{(NM_j - R_j)} - \alpha \sum_j \frac{\left[\sum_i h'_{ji} \right]^2}{(NM_j - R_j)} - \sum_j \hat{h}_j \sum_i h'_{ji} + \alpha \sum_j \hat{h}_j \sum_i h'_{ji} = 0 \quad (\text{A8})$$

which can be added to equation (A6) to yield

$$\sum_j \sum_i -h'_{ji+1} h'_{ji} + \alpha \sum_j \sum_i h'_{ji}{}^2 + \sum_j \frac{\sum_i h'_{ji+1} \sum_i h'_{ji}}{(NM_j - R_j)} - \alpha \sum_j \frac{\left[\sum_i h'_{ji} \right]^2}{(NM_j - R_j)} = 0 \quad (\text{A9})$$

Equation (A9) can be solved for α to yield

$$\alpha = \frac{\sum_j \sum_i h'_{ji+1} h'_{ji} - \sum_j \frac{\sum_i h'_{ji+1} \sum_i h'_{ji}}{(NM_j - R_j)}}{\sum_j \sum_i h'_{ji}{}^2 - \sum_j \frac{\left(\sum_i h'_{ji}\right)^2}{(NM_j - R_j)}} = e^{-\Delta t/t_h} \quad (\text{A10})$$

where

$$\begin{aligned} i &= R_j, NM_j-1, \\ j &= 1, NY. \end{aligned}$$

The set of equations (A2) can be rewritten as

$$\sum_i h'_{ji+1} - \alpha \sum_i h'_{ji} - (NM_j - R_j) \hat{h}_j + \alpha (NM_j - R_j) \hat{h}_j = 0 \quad (\text{A11})$$

which can be solved for \hat{h}_j to yield

$$\hat{h}_j = \frac{\alpha \sum_i h'_{ji} - \sum_i h'_{ji+1}}{(NM_j - R_j)(\alpha - 1)} \quad (\text{A12})$$

where

$$\begin{aligned} \alpha &= e^{-\Delta t/t_h}, \\ i &= R_j, NM_j-1. \end{aligned}$$

APPENDIX B

COMPUTER PROGRAM FOR THE CASE OF THE
AQUIFER WITH A PERCHED STREAM

The following program estimates h_d and a_d from water-level and drain flow data, then estimates S , t_h and q_r from water-level data from months that recession occurs, then estimates E (net recharge) for each month that recharge occurs. Finally the program runs a simulation using the estimated parameters and results. The program is set up to run on a monthly basis.

Running the program requires four different types of data cards. The first data card is the TITLE CARD and it contains the name of the project. After the TITLE CARD comes the BASIC DATA CARD which contains data required to run the program that cannot fit on other cards. Next come the RECHARGE-RECESSION CARDS. There is one of these cards for each recharge-recession period. Following these cards come the WATER LEVEL-DRAIN FLOW CARDS. There is one card for each month. The following tables give a list of the more important symbols used in the program and a list of what is contained on the data cards.

DATA CARDS

CARD NAME	NO. OF CARDS	DATA NAME	COLUMNS	FIELD
TITLE CARD	1	TITLE	1-80	20A4
BASIC DATA CARD	1	NY	1-10	I10
		HBASE	11-20	F10.0
		HS	21-30	F10.0
		DRS	31-40	F10.0
RECHARGE - RECESSION CARDS	NY (1 FOR EACH RECHARGE - RECESSION PERIOD)	NM	1-10	I10
		IBEG	11-20	I10
WATER LEVEL - DRAINFLOW CARDS	NY $\sum_{J=1} NM(J)$ J=1 (1 FOR EACH MONTH)	DATE	3-10	2A4
		H	11-20	F10.0
		DR	21-30	F10.0

LIST OF IMPORTANT SYMBOLS

SYMBOLS READ ON INPUT	
SYMBOL	MEANING
DR(J,I)	Observed drainflow for the i^{th} month of the j^{th} recharge-recession period
DRS	Observed drain flow for month before the first month of the first recharge-recession period
DATE(K,J,I)	Month (K=1) and year (K=2) for the i^{th} month of the j^{th} recharge-recession period
HBASE	Base average water level or datum of aquifer
HS	Observed average water level of aquifer for month before the first month of the first recharge-recession period
IBEG(J)	Index of month that recession starts for j^{th} recharge-recession period. (First month of a recharge-recession period is indexed 1).
M(J)	Total number of months in j^{th} recharge-recession period
NY	Total number of recharge-recession periods
TITLE	Title of problem
SYMBOLS WRITTEN ON OUTPUT	
AD	Drain discharge constant
DR(J,I)	Observed drain flow for i^{th} month of the j^{th} recharge-recession period
DRPRED(J,I)	Predicted drain flow from regression equation or simulated drain flow for the i^{th} month of the j^{th} recharge-recession period
E(J,I)	Estimated net recharge for the i^{th} month of the j^{th} recharge-recession period

LIST OF IMPORTANT SYMBOLS (cont'd)

H(J,I)	Observed average water level of aquifer for the i^{th} month of the j^{th} recharge-recession period
HDP	Average drain elevation minus HBASE
HPPRED(J,I)	Simulated average water level of aquifer for the i^{th} month of the j^{th} recharge-recession period
J	Index of recharge-recession period
N	Storage coefficient (specific yield, S)
NEND	Index of last month of a recharge-recession period
NR	Number of months of recession in a recharge-recession period
QR(J)	Discharge per month from river to aquifer for j^{th} recharge-recession period
RES(J,I)	$DR(J,I) - DRPRED(J,I)$
RHOSQU	Correlation coefficient between observed drain flows and observed average water levels
RMSD	Mean squared error between observed and simulated drain flows
RMSH	Mean squared error between observed and simulated water levels
SERES	Standard error of the residuals from the drain flow-average water level regression
TH	Hydraulic response time of the aquifer
VARRES	Variance of the residuals from the drain flow-average water level regression

```

0001      DIMENSION TITLE(20),H(25),IREG(25),DATE(2,25,15),H(25,15),
&HP(25,15),DR(25,15),QR(25),E(25,15),HR(25),HI(25),HPI(25),
&H(25),HPI2(25),HIHPI(25),ALPHA(15),RES(25,15),DRPRED(25,15),
&HPPRED(25,15)
0002      REAL N
0003      READ(5,100) TITLE
0004      100 FORMAT(20A4)
0005      READ(5,101) NY,HBASE,HS,DRS
0006      101 FORMAT(1I0,3F10.0)
0007      READ(5,102) (M(J),IBEG(J),J=1,NY)
0008      102 FORMAT(2I0)
0009      DO 1 J=1,NY
0010      NM=M(J)
0011      READ(5,103) (DATE(1,J,I),DATE(2,J,I),H(J,I),DR(J,I),I=1,NM)
0012      1 CONTINUE
0013      103 FORMAT(2X,2A4,2F10.0)
0014      WRITE(6,104) TITLE, NY, HBASE
0015      104 FORMAT(1H0,10X, 'ESTIMATION OF PARAMETERS AND RECHARGE FOR LUMPED P
PARAMETER GROUNDWATER MODEL'//
&11X,20A4, '///11X, 'BASIC INPUT DATA : '//
&15X, 'NUMBER OF YEARS IS ', I2//
&15X, 'THE BASE ELEVATION IS ', F8.3//
&11X, 'WATER YEAR DATA : '//
&15X, 'WATER YEAR          BEGINNING      START OF          ENDING          TOTAL MO
ENTHS IN          TOTAL MONTHS IN'//17X, 'NUMBER          DATE          RECES
&SIGN          DATE          WATER YEAR
DO 20 J=1,NY
NR=M(J)-IBEG(J)+1
IPEC=IBEG(J)
NEND=M(J)
WRITE(6,105) J,DATE(1,J,1),DATE(2,J,1),DATE(1,J,IREC),
&DATE(2,J,IREC),DATE(1,J,NEND),DATE(2,J,NEND),NEND,NR
20 CONTINUE
105 FORMAT(1H0,18X,I2,10X,2A4,6X,2A4,4X,2A4,11X,I2,18X,I2//)
DO 2 J=1,NY
NM=M(J)
DO 3 I=1,NM
HP(J,I)=H(J,I)-HBASE
2 CONTINUE
DRSUM=0.0
DR2SUM=0.0
HPSUM=0.0
HP2SUM=0.0
HPPRSU=0.0
RESSQ=0.0
DRP2SU=0.0
NMT=0
DO 3 J=1,NY
NM=M(J)
NMT=NMT+NM
DO 3 I=1,NM
DRSUM=DRSUM+DR(J,I)
DR2SUM=DR2SUM+DR(J,I)*DR(J,I)
HPSUM=HPSUM+HP(J,I)
HP2SUM=HP2SUM+HP(J,I)*HP(J,I)
HPPRSU=HPPRSU+HP(J,I)*DR(J,I)
3 CONTINUE
RNMT=NMT
AD=(RNMT*HPPRSU-DRSUM*HPSUM)/(RNMT*HP2SUM-HPSUM*HPSUM)
HPP=(HPSUM-DRSUM/AD)/RNMT
HPPRSU=(RNMT*HPPRSU-DRSUM*HPSUM)*2.0/
&RNMT*HP2SUM-DRSUM*DRSUM)/(RNMT*HP2SUM-HPSUM*HPSUM)
DO 4 J=1,NY
NM=M(J)
DO 4 I=1,NM
DRPRED(J,I)=AD*(HP(J,I)-HPP)
RES(J,I)=DR(J,I)-DRPRED(J,I)
RESSQ=RESSQ+RES(J,I)*RES(J,I)
4 CONTINUE
VARRES=RESSQ/(RNMT-2.0)
SERES=SQRT(VARRES)
WRITE(6,106) AD,HPP
106 FORMAT(1H1,10X, 'WATER LEVEL - DRAINFLOW PARAMETERS'//
&16X, 'AD = ', F8.5//
&16X, 'HPP = ', F8.5, ' + BASE ELEVATION'//22X, 'H
&RESIDUAL'//31X, '(OBSERVED)          (PREDICTED)'//)
DO 21 J=1,NY
NM=M(J)
DO 21 I=1,NM
WRITE(6,107) H(J,I),DR(J,I),DRPRED(J,I),RES(J,I)
21 CONTINUE
107 FORMAT(1H0,17X,F8.3,6X,F8.6,6X,F8.6,7X,F8.6)
WRITE(6,108) RHOSQ,VARRES,SERES
108 FORMAT( '///25X, 'CORRELATION BETWEEN DR AND H IS ', F7.6//
&25X, 'VARIANCE OF THE RESIDUALS IS ', F8.6//
&25X, 'STANDARD ERROR OF THE RESIDUALS IS ', F8.6)
DO 5 J=1,NY
NM=M(J)-1
ISTART=IBEG(J)
HI(1J)=0.0
HPI(1J)=0.0
HI2(1J)=0.0
HPI2(1J)=0.0
HIHPI(1J)=0.0
DO 5 I=ISTART,NM
HI(IJ)=HI(IJ)+HP(J,I)
HPI(IJ)=HPI(IJ)+HP(J,I)
HI2(IJ)=HI2(IJ)+HP(J,I)*HP(J,I)
HPI2(IJ)=HPI2(IJ)+HP(J,I)*HP(J,I)
HIHPI(IJ)=HIHPI(IJ)+HP(J,I)*HP(J,I)
5 CONTINUE
SUM1=0.0
SUM2=0.0
SUM3=0.0
SUM4=0.0
DO 6 J=1,NY
SUM1=SUM1+HIHPI(1J)
SUM2=SUM2+HPI(1J)*HI(1J)/FLCAT(NY(J)-IBEG(J))
SUM3=SUM3+HI2(1J)
SUM4=SUM4+HI(1J)*HI(1J)/FLCAT(NY(J)-IBEG(J))
6 CONTINUE
EXPINT=(SUM1-SUM2)/(SUM3-SUM4)

```



```

0095      DO 7 J=1,NY
0096      HR(J)=EXPNNNT*HI(J)-HTPL(J)/(EXPNNNT-1.0)/FLOAT(M(J)-IREG(J))
0097      DR(J)=AD*(HR(J)-HDP)
0098      7 CONTINUE
0099      TH=-1.0/ALOG(EXPNNNT)
0100      N=AD*TH
0101      WRITE(6,109) TH,N,(J,QR(J),J=1,NY)
0102      109 FORMAT(1H1,10X,'RIVER LOSS',RESPONSE TIME AND POROSITY'//
      &21X,TH=' ',F9.5,/'21X,N=' ',F8.5,/'//
      &16X,'YEAR',RIVER LOSS,/'(17X,12,12X,F8.6)')
0103      ALPHA(I)=4.0
0104      DO 8 I=2,15
0105      ALPHA(I)=4.0-1.0/ALPHA(I-1)
0106      8 CONTINUE
0107      HS=HS-HBASE
0108      DO 9 J=1,NY
0109      IREGJ=IREG(J)
0110      IREGM1=IREGJ-1
0111      F(J,1)=DRS+4.0*DR(J,1)+DR(J,2)-6.0*QR(J)+3.0*N*(HP(J,2)-HS)
0112      DO 10 I=2,IREGM1
0113      F(J,I)=DR(J,I-1)+4.0*DR(J,I)+DR(J,I+1)-6.0*QR(J)+
      &3.0*N*(HP(J,I+1)-HP(J,I-1))
0114      E(J,I)=E(J,I)-E(J,I-1)/ALPHA(I-1)
0115      10 CONTINUE
0116      F(J,IREGM1)=E(J,IREGM1)/ALPHA(IREGM1)
0117      DO 11 I=2,IREGJ
0118      II=IREGJ-I
0119      E(J,II)=(E(J,II)-E(J,II+1))/ALPHA(II)
0120      11 CONTINUE
0121      NM=M(J)
0122      HS=HP(J,NM)
0123      DRS=DR(J,NM)
0124      DO 12 I=IREGJ,NM
0125      F(J,I)=0.0
0126      12 CONTINUE
0127      9 CONTINUE
0128      WRITE(6,110)
0129      110 FORMAT(1H1,10X,'RECHARGE ESTIMATES'//
      &21X,'DATE',RECHARGE')
0130      DO 22 J=1,NY
0131      NM=M(J)
0132      DO 22 I=1,NM
0133      WRITE(6,111) DATE(1,J,I),DATE(2,J,I),E(J,I)
0134      22 CONTINUE
0135      111 FORMAT(1H0,18X,2A4,8X,F8.6)
0136      RMSH=0.0
0137      DE=AD*(HPPRED(1,1)-HDP)-DR(1,1)
0138      RMSD=DE*DE
0139      TM=0.0
0140      C1=EXPNNNT
0141      C2=(SQRT(EXPNNNT)-EXPNNNT)/AD
0142      C3=(1.0-SQRT(EXPNNNT))/AD
0143      HS=HP(1,1)
0144      ES=F(1,1)
0145      HPPRED(1,1)=HP(1,1)
0146      ISTART=2
0147      DO 30 J=1,NY
0148      C4=HB(J)*(1.0-EXPNNNT)
0149      NM=M(J)
0150      DO 31 I=ISTART,NM
0151      HPPRED(J,I)=HS*C1+ES*C2+E(J,I)*C3+C4
0152      DRPRED(J,I)=AD*(HPPRED(J,I)-HDP)
0153      HS=HPPRED(J,I)
0154      ES=E(J,I)
0155      TM=TM+1.0
0156      HE=HPPRED(J,I)-HP(J,I)
0157      DE=AD*(HPPRED(J,I)-HDP)-DR(J,I)
0158      RMSH=RMSH+HE*HE
0159      RMSD=RMSD+DE*DE
0160      31 CONTINUE
0161      ISTART=1
0162      30 CONTINUE
0163      RMSH=RMSH/TM
0164      RMSD=RMSD/(TM+1.0)
0165      DO 40 J=1,NY
0166      NM=M(J)
0167      DO 40 I=1,NM
0168      HPPRED(J,I)=HPPRED(J,I)+HBASE
0169      40 CONTINUE
0170      WRITE(6,114)
0171      114 FORMAT(1H1,10X,'PREDICTION OF WATER LEVELS AND DRAINFLWS'////
      &24X,'DATE',17X,'H',14X,'H',18X,'DR',14X,'DR'/'
      &41X,'(OBSERVED)',4X,'(PREDICTED)',9X,'(OBSERVED)',5X,'(PREDICTED)'
      &///)
0172      DO 41 J=1,NY
0173      NM=M(J)
0174      WRITE(6,112) (DATE(1,J,I),DATE(2,J,I),H(J,I),HPPRED(J,I),
      &DR(J,I),DRPRED(J,I),I=1,NM)
0175      41 CONTINUE
0176      WRITE(6,113) RMSH,RMSD
0177      112 FORMAT(1H0,21X,2A4,12X,F8.3,7X,F8.3,11X,F8.6,8X,F8.6)
0178      113 FORMAT(1H0,///,25X,'MEAN SQUARED ERROR OF WATER LEVELS IS ',F8.5,/'0000600
      &25X,'MEAN SQUARED ERROR OF DRAINFLWS IS ',F8.5)
0179      END

```

ESTIMATION OF PARAMETERS AND RECHARGE FOR LUMPED PARAMETER GROUNDWATER MODEL
 MESILLA VALLEY, NEW MEXICO AND TEXAS - MARCH, 1946, TO FEBRUARY, 1951

BASIC INPUT DATA :

NUMBER OF YEARS IS 5
 THE BASE ELEVATION IS 3820.000

WATER YEAR NUMBER	BEGINNING DATE	START OF RECESSION	ENDING DATE	TOTAL MONTHS IN WATER YEAR	TOTAL MONTHS IN RECESSION
1	MAR 1946	OCT 1946	FEB 1947	12	5
2	MAR 1947	OCT 1947	MAR 1948	13	6
3	APR 1948	OCT 1948	FEB 1949	11	5
4	MAR 1949	OCT 1949	FEB 1950	12	5
5	MAR 1950	OCT 1950	FEB 1951	12	5

RIVER LOSS, RESPONSE TIME AND POROSITY

TH = 2.58663
 V = 0.20999

YEAR	RIVER LOSS
1	0.062816
2	0.054633
3	0.058538
4	0.064208
5	0.060073

WATER LEVEL - DRAINFLOW PARAMETERS

AD = 0.08119

HD = 4.95229 + BASE ELEVATION

CORRELATION BETWEEN DR AND H IS .957931
 VARIANCE OF THE RESIDUALS IS 0.000202
 STANDARD ERROR OF THE RESIDUALS IS 0.014224

H	DR (OBSERVED)	DR (PREDICTED)	RESIDUAL	RECHARGE ESTIMATES	RECHARGE
				DATE	
3826.377	0.126993	0.123779	0.003214	MAR 1946	0.231311
3827.344	0.203447	0.202287	0.001160	APR 1946	0.269693
3827.535	0.231206	0.217787	0.013419	MAY 1946	0.214497
3828.008	0.248913	0.256199	-0.007286	JUN 1946	0.290951
3828.363	0.285803	0.285018	0.000785	JUL 1946	0.279163
3829.419	0.301573	0.299556	0.012017	AUG 1946	0.168066
3827.677	0.244579	0.279322	0.015257	SEP 1946	-0.023764
3826.845	0.161577	0.161774	-0.000197	OCT 1946	0.0
3826.522	0.125609	0.135552	-0.009943	NOV 1946	0.0
3826.168	0.115188	0.106812	0.008376	DEC 1946	0.0
3825.951	0.097481	0.089192	0.008289	JAN 1947	0.0
3825.989	0.077468	0.084158	-0.006690	FEB 1947	0.0
3826.177	0.110485	0.107546	0.002939	MAR 1947	0.225459
3827.180	0.183065	0.188968	-0.005903	APR 1947	0.286157
3827.500	0.199481	0.214953	-0.015472	MAY 1947	0.177804
3827.805	0.210087	0.239705	-0.029621	JUN 1947	0.259642
3828.323	0.250297	0.281767	-0.031470	JUL 1947	0.264428
3828.319	0.264038	0.281450	-0.017412	AUG 1947	0.154070
3827.766	0.218202	0.236557	-0.018355	SEP 1947	-0.034685
3826.844	0.135570	0.161695	-0.026125	OCT 1947	0.0
3826.384	0.106058	0.124354	-0.018296	NOV 1947	0.0
3826.185	0.091486	0.108200	-0.016714	DEC 1947	0.0
3825.863	0.083002	0.092057	0.000945	JAN 1948	0.0
3825.787	0.071935	0.075893	-0.003958	FEB 1948	0.0
3825.742	0.074425	0.072226	0.002199	MAR 1948	0.0
3826.785	0.148481	0.156899	-0.008418	APR 1948	0.322348
3827.210	0.187492	0.191406	-0.003914	MAY 1948	0.140043
3827.383	0.194316	0.205459	-0.011142	JUN 1948	0.235834
3828.055	0.253893	0.263004	-0.006111	JUL 1948	0.316356
3828.237	0.277503	0.274799	0.002713	AUG 1948	0.172907
3827.692	0.242826	0.230531	0.012295	SEP 1948	0.019809
3826.949	0.162038	0.170218	-0.008180	OCT 1948	0.0
3826.579	0.120353	0.140190	-0.019837	NOV 1948	0.0
3826.252	0.113436	0.113631	-0.000195	DEC 1948	0.0
3825.894	0.104490	0.084574	0.019916	JAN 1949	0.0
3825.865	0.082264	0.092215	0.000049	FEB 1949	0.0
3826.170	0.118231	0.106971	0.011260	MAR 1949	0.200259
3827.010	0.196069	0.175173	0.020896	APR 1949	0.262316
3827.275	0.219217	0.196678	0.022539	MAY 1949	0.159071
3827.430	0.228439	0.209264	0.019175	JUN 1949	0.258074
3829.061	0.276027	0.260500	0.015527	JUL 1949	0.303637
3828.110	0.284419	0.264484	0.019935	AUG 1949	0.163595
3827.621	0.222998	0.224783	-0.001785	SEP 1949	-0.007719
3826.868	0.148112	0.163638	-0.015526	OCT 1949	0.0
3826.519	0.128561	0.135314	-0.006753	NOV 1949	0.0
3826.243	0.110577	0.112897	-0.002320	DEC 1949	0.0
3826.018	0.094714	0.094663	0.000071	JAN 1950	0.0
3825.945	0.077644	0.088716	-0.011072	FEB 1950	0.0
3825.201	0.138392	0.139488	0.002894	MAR 1950	0.210476
3826.984	0.198691	0.173052	0.015639	APR 1950	0.272000
3827.403	0.204000	0.207094	-0.003094	MAY 1950	0.195575
3827.593	0.208888	0.222504	-0.013616	JUN 1950	0.182582
3827.803	0.263392	0.239550	0.023842	JUL 1950	0.268535
3827.996	0.264038	0.255228	0.008810	AUG 1950	0.163261
3827.376	0.230053	0.204884	0.025169	SEP 1950	-0.001426
3826.683	0.160904	0.148633	0.012271	OCT 1950	0.0
3826.330	0.117761	0.119973	-0.002212	NOV 1950	0.0
3826.107	0.101871	0.101857	0.000014	DEC 1950	0.0
3825.894	0.097409	0.094574	0.002835	JAN 1951	0.0
3825.935	0.065940	0.079777	-0.013837	FEB 1951	0.0

DEFLECTION OF WATER LEVELS AND DRAINFLOWS

DATE	H (OBSERVED)	H (PREDICTED)	DR (OBSERVED)	DR (PREDICTED)
MAR 1946	3826.377	3826.377	0.126993	0.123779
APR 1946	3827.344	3827.133	0.203447	0.185145
MAY 1946	3827.535	3827.595	0.231206	0.222694
JUN 1946	3828.709	3827.977	0.248913	0.253645
JUL 1946	3829.363	3828.346	0.285803	0.283676
AUG 1946	3828.419	3828.336	0.301573	0.282840
SEP 1946	3827.677	3827.716	0.244579	0.237460
OCT 1946	3826.845	3827.003	0.161577	0.174622
NOV 1946	3826.522	3826.562	0.125609	0.138773
DEC 1946	3826.169	3826.261	0.115188	0.114418
JAN 1947	3825.951	3826.058	0.097481	0.097872
FEB 1947	3825.889	3825.919	0.077468	0.086632
MAR 1947	3826.177	3826.281	0.110485	0.116000
APR 1947	3827.183	3827.061	0.183065	0.179283
MAY 1947	3827.500	3827.464	0.199481	0.212023
JUN 1947	3827.805	3827.722	0.210787	0.232953
JUL 1947	3828.323	3828.053	0.250797	0.259869
AUG 1947	3828.319	3828.048	0.264038	0.259451
SEP 1947	3827.766	3827.439	0.218202	0.210002
OCT 1947	3826.844	3826.763	0.135570	0.155160
NOV 1947	3826.384	3826.366	0.106058	0.122927
DEC 1947	3826.185	3826.097	0.091486	0.101029
JAN 1948	3825.863	3825.913	0.083002	0.086153
FEB 1948	3825.787	3825.789	0.071935	0.076046
MAR 1948	3825.742	3825.704	0.074425	0.069180
APR 1948	3826.785	3826.360	0.148481	0.122426
MAY 1948	3827.210	3826.986	0.187492	0.173255
JUN 1948	3827.383	3827.294	0.194316	0.198212
JUL 1948	3828.055	3827.848	0.253893	0.243198
AUG 1948	3828.237	3828.057	0.277503	0.260211
SEP 1948	3827.692	3827.610	0.242826	0.223902
OCT 1948	3826.949	3826.990	0.162038	0.173605
NOV 1948	3826.579	3826.536	0.120353	0.136710
DEC 1948	3826.252	3826.227	0.113436	0.111645
JAN 1949	3825.894	3826.018	0.104490	0.094617
FEB 1949	3825.865	3825.875	0.082264	0.083049
MAR 1949	3826.170	3826.250	0.118231	0.113489
APR 1949	3827.012	3826.997	0.196269	0.174088
MAY 1949	3827.275	3827.391	0.219217	0.206100
JUN 1949	3827.430	3827.699	0.228439	0.230292
JUL 1949	3828.061	3828.167	0.276027	0.269078
AUG 1949	3828.110	3828.269	0.284419	0.277414
SEP 1949	3827.621	3827.718	0.222998	0.232678
OCT 1949	3826.868	3827.055	0.148112	0.178824
NOV 1949	3826.519	3826.618	0.128561	0.143356
DEC 1949	3826.243	3826.321	0.110577	0.119260
JAN 1950	3826.019	3826.120	0.094714	0.102891
FEB 1950	3825.945	3825.993	0.077644	0.091770
MAR 1950	3826.201	3826.313	0.138392	0.118601
APR 1950	3826.994	3827.046	0.188691	0.178135
MAY 1950	3827.403	3827.499	0.204000	0.214060
JUN 1950	3827.593	3827.625	0.208889	0.225111
JUL 1950	3827.803	3827.889	0.263392	0.245844
AUG 1950	3827.996	3827.979	0.264038	0.253877
SEP 1950	3827.376	3827.502	0.230053	0.215137
OCT 1950	3826.683	3826.887	0.160904	0.165211
NOV 1950	3826.330	3826.472	0.117761	0.131499
DEC 1950	3826.107	3826.190	0.101871	0.108597
JAN 1951	3825.894	3825.998	0.092409	0.093038
FEB 1951	3825.815	3825.868	0.076594	0.082468

APPENDIX C

WATER LEVELS FOR USBR WELLS (1946 - 1951)

1946

WELL NO.	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
26	3919.82	3919.32	3919.12	3919.02	3919.42	3919.32	3922.12	3922.92	3922.12	3921.82	3920.62	3919.92
27	3919.74	3919.34	3919.54	3919.14	3919.34	3919.04	3922.04	3922.54	3921.54	3920.54	3919.44	3918.74
28	3919.74	3919.34	3919.54	3919.14	3919.34	3919.04	3922.04	3922.54	3921.54	3920.54	3919.44	3918.74
29	3919.74	3919.34	3919.54	3919.14	3919.34	3919.04	3922.04	3922.54	3921.54	3920.54	3919.44	3918.74
30	3919.74	3919.34	3919.54	3919.14	3919.34	3919.04	3922.04	3922.54	3921.54	3920.54	3919.44	3918.74
31	3919.74	3919.34	3919.54	3919.14	3919.34	3919.04	3922.04	3922.54	3921.54	3920.54	3919.44	3918.74
32	3919.74	3919.34	3919.54	3919.14	3919.34	3919.04	3922.04	3922.54	3921.54	3920.54	3919.44	3918.74
33	3919.74	3919.34	3919.54	3919.14	3919.34	3919.04	3922.04	3922.54	3921.54	3920.54	3919.44	3918.74
34	3919.74	3919.34	3919.54	3919.14	3919.34	3919.04	3922.04	3922.54	3921.54	3920.54	3919.44	3918.74
VALLEY	3826.222	3826.121	3826.377	3827.344	3827.535	3828.008	3828.363	3828.419	3827.677	3826.845	3826.522	3826.168
MEAN												
NODE 1	3886.433	3886.158	3886.319	3887.161	3887.449	3888.011	3888.514	3888.597	3887.932	3887.176	3886.745	3886.382
MEAN												
NODE 2	3789.503	3789.510	3789.825	3790.869	3790.999	3791.419	3791.683	3791.727	3790.933	3790.055	3789.707	3789.448
MEAN												

