# STUDIES ON RAINFALL-RUNOFF MODELING

7.  A Nonlinear Hydrologic Cascade

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STUDIES ON RAINFALL-RUNOFF MODELING

7. A Nonlinear Hydrologic Cascade

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# LIST OF SYMBOLS

| Symbol | Definition |
|--------|-----------|
| $p$ | Inflow to the storage element |
| $q$ | Outflow from the storage element |
| $s$ | Storage in the storage element |
| $t$ | Time |
| $n$ | Number of storage elements in the cascade |
| $k$ | Characteristic parameter of storage element |
| $x$ | Index of nonlinearity |
| $\dot{S}$ | The vector form of rate of change of storage in the element |
| $P$ | The vector form of inflow |
| $K$ | The vector form of parameter $k$ |
| $B$ | Matrix |
| $S$ | The vector form of storage element |
| $Q$ | The vector form of outflow |
| $C$ | Matrix |
| $\dot{S}_o$ | The vector of $\dot{S}$ at $t = 0$ |
| $S_o$ | The vector of $S$ at $t = 0$ |
| $S_1$ | The vector of $S$ at $t = 1$ |
| $\Delta t$ | The time interval |
| $k_1$ | The parameter $k$ of the 1st storage element |
| $k_2$ | The parameter $k$ of the 2nd storage element |
| $k_3$ | The parameter $k$ of the 3rd storage element |
| $f$ | Infiltration rate of Philip's equation |
| $\beta$ | A parameter of Philip's equation for infiltration |
| $\alpha$ | A parameter of Philip's equation for infiltration |
| $F$ | Objective function |
| $Q_{p_o}$ | Observed hydrograph peak |

| Symbol | Definition |
|--------|------------|
| $Q_{p_e}$ | Estimated hydrograph peak |
| M | Number of rainfall-runoff events in the optimization set |
| SLOPE | Average watershed slope |
| CSLOPE | Average mainstream slope |
| AREA, A | Watershed area |
| XLR, L | Length of mainstream |
| WIDTH | Width of watershed |
| DD | Drainage density |
| SHAPE | Shape factor |
| SO | Stream order |

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## ABSTRACT

A nonlinear hydrologic cascade is developed for simulation of watershed runoff. The cascade consists of three nonlinear storage elements. The parameters of these elements are determined by optimization by the modified Rosenbrock-Palmer algorithm. An attempt is made to correlate these parameters with the watershed physiography but the correlations are not encouraging.

Using the optimized parameter values hydrographs are predicted for a number of events on a number of natural agricultural watersheds. The predicted hydrographs are in good agreement with the observed hydrographs and indicate that the cascade performs well in prediction of runoff.

CHAPTER 1

INTRODUCTION

1.1  General Remarks

The growing interest in the development, calibration, testing,
and application of nonlinear runoff models primarily stems from recog-
nition of the nonlinearity of rainfall-runoff process.  The past two
decades have witnessed evolution of a wide variety of nonlinear ap-
proaches to runoff modeling.  Of them all  storage models appear to
be most prominent, in a large measure for their simplicity.  Some of
the well-known and more recent models are briefly reviewed here.

Diskin (1964) developed a model consisting of two parallel cas-
cades, each having a number of equal reservoirs.  Laplace transforms
were employed to solve the equations.  The nonlinearity in surface
runoff was established by showing variability in the shape of the in-
stantaneous unit hydrograph with rainfall storm characteristics.

Kulandaiswamy (1964) derived a general storage equation and applied it
to natural watersheds.  His results indicated that the model parameters
decreased exponentially with increase in the peak discharge, thus indi-
cating basin nonlinearity.

Singh (1964) studied the departure from actual basin behavior of
the results of essentially linear or linearlized models.  His approach
accounted for apparent variation in instantaneous unit hydrographs with
rainfall-excess characteristics.

Prasad (1967) developed a nonlinear hydrologic system response
model by using the continuity equation and a nonlinear storage reser-
voir.  The model was applied to natural watersheds and yielded good
results.  The model parameters were estimated from watershed and rain-
fall-excess characteristics by multiple linear regression and correla-

tion analysis. Watershed characteristics such as area, length of main-stream and slope of mainstream were considered. The rainfall-excess characteristics included the volume of rainfall-excess, its duration and its average intensity.

Mein, et al (1974) developed a simple nonlinear model with a single parameter that was evaluated by fitting the recorded data. The watershed storage effects were represented in the model by nonlinear storage elements.

Reed, et al (1975) considered Nash's model (Nash, 1957) which represents a watershed as a cascade of n linear reservoirs, each with storage coefficient or lag time k. For each reservoir the lag time k was nonlinearly estimated. By applying it to natural catchments it was found that the lag model gave a much improved representation of the hydrograph, particularly its peak characteristics.

Singh (1976) developed a uniformly nonlinear hydrologic cascade. It represented a watershed by a cascade of n nonlinear storage elements or reservoirs, each with storage coefficient k. The model parameter k was estimated by optimization. Further, it was shown that it could be estimated from watershed characteristics. The model was applied to several natural agricultural watersheds and a close agreement between observed and predicted hydrographs was found.

These studies indicate that nonlinear effects are significant in the rainfall-runoff process. The present study considers a nonlinear hydrologic cascade for simulation of watershed response.

1.2  Objectives

The objectives of this study are:

(1)  To develop a nonlinear hydrologic cascade for prediction of watershed runoff.

(2)  To estimate the model parameters from watershed characteristics.

CHAPTER 2

NONLINEAR HYDROLOGIC CASCADE

## 2.1 Model Formulation

The nonlinear cascade represents a watershed by a series of un-equal nonlinear storage elements or reservoirs as shown in Fig. 2.1. The governing equations for a nonlinear element are a spatially lumped continuity equation and a nonlinear storage-discharge relation, which can be written respectively as:

$$p = q + \frac{ds}{dt} \qquad (2\text{-}1)$$

$$q = ks^x \qquad (2\text{-}2)$$

where $p$ = inflow to the storage element in cm/hr;

$q$ = outflow from the storage element in cm/hr;

$s$ = storage in the storage element in cm;

$t$ = time in hour;

$\frac{ds}{dt}$ = rate of change of storage in the element;

$k$ = storage characteristic parameter; and

$x$ = an index of nonlinearity.

Equations (2-1) and (2-2) can be combined to yield a single differential equation relating inflow and storage:

$$\frac{ds}{dt} = p - ks^x \qquad (2\text{-}3)$$

The nonlinear cascade of Fig. 2.1 can then be represented by a system of equations:

$$\frac{ds_1}{dt} = p_1 - k_1 s_1^x$$

$$\frac{ds_2}{dt} = p_2 + k_1 s_1^x - k_2 s_2^x$$

Fig. 2.1. A nonlinear hydrologic cascade with distributed input.
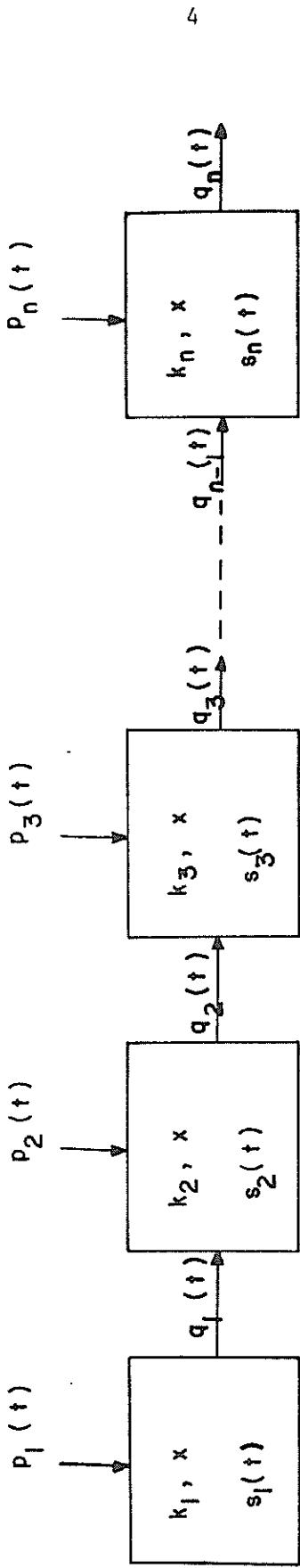
$$\frac{ds_3}{dt} = p_3 + k_2 s_2^x - k_3 s_3^x$$

$$\frac{ds_j}{dt} = p_j + k_{j-1} s_{j-1}^x - k_j s_j^x \qquad\qquad (2\text{-}4)$$

$$\frac{ds_{n-1}}{dt} = p_{n-1} + k_{n-2} s_{n-2}^x - k_{n-1} s_{n-1}^x$$

$$\frac{ds_n}{dt} = p_n + k_{n-1} s_{n-1}^x - k_n s_n^x$$

where n denotes the number of elements in the cascade. Note that the parameter n does not change from one element to another whereas k does.

Equation (2-4) can be written more conveniently and concisely in the matrix form as:

$$\dot{S} = P + KBS \qquad\qquad (2\text{-}5)$$

where

$$\dot{S} = \begin{bmatrix} \dfrac{ds_1}{dt} \\[2ex] \dfrac{ds_2}{dt} \\[2ex] \dfrac{ds_3}{dt} \\[1ex] \vdots \\[1ex] \dfrac{ds_j}{dt} \\[1ex] \vdots \\[1ex] \dfrac{ds_n}{dt} \end{bmatrix} \; ;$$

$$P \;=\; \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ \vdots \\ \\ \\ \\ p_j \\ \\ \\ p_n \end{bmatrix} \;;$$

$$K \;=\; \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ \vdots \\ \\ \\ \\ k_j \\ \\ k_n \end{bmatrix} \;;$$

$$S = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ \vdots \\ s_j \\ \vdots \\ s_n \end{bmatrix} \quad ; \quad \text{and}$$

$$B = \begin{bmatrix} s_1^{x-1} & 0 & 0 & ------- & 0 & 0 \\ s_1^{x-1} - s_2^{x-1} & 0 & ------ & 0 & 0 \\ 0 & s_2^{x-1} - s_3^{x-1} & ----- & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \\ 0 & 0 & 0 & & s_{n-1}^{x-1} & -s_n^{x-1} \end{bmatrix}$$

Our main purpose is to obtain the relationship between the final outflow $q_n$ and the lateral inflows $p_1$, $p_2$, $p_3$------$p_j$---$p_n$. The storage-discharge relationship for the cascade can then be expressed in the matrix form as:

$$Q = KCS \qquad (2\text{-}6)$$

where

$$Q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ \vdots \\ \vdots \\ \vdots \\ q_j \\ \vdots \\ q_n \end{bmatrix} \quad ;$$

$$C = \begin{bmatrix} s_1^{x-1} & 0 & ------------ & 0 & 0 \\ 0 & s_2^{x-1} & ----------- & 0 & 0 \\ \vdots & \vdots & & & \\ \vdots & \vdots & & & \\ \vdots & \vdots & & & \\ \vdots & \vdots & & & \\ \vdots & \vdots & & & \\ \vdots & \vdots & & s_{n-1}^{x-1} & 0 \\ 0 & 0 & & 0 & s_n^{x-1} \end{bmatrix} \quad ; \text{ and}$$

K and S are as defined previously.

Equations (2-5) and (2-6) provide a state variable representation of the cascade. For simulation of runoff, however, the cascade with lumped input was considered as shown in Fig. 2.2.; that is, the inflows $p_2$, $p_3$, $p_4$ --------$p_n$ in P are zero and $p_1$ is positive. The cascade operation can now be summarized as follows:

(1) Specify parameters $k_1$, $k_2$, --------$k_n$; x and n.

(2) Select the time interval $\Delta t$.

(3) Given the input pattern (rainfall-excess) use Eq. (2-5) to compute $\dot{S}$. At the beginning of time, $t = 0$, B and S are zero. The inputs $p_2$, $p_3$--------$p_n$ in the matrix P are all zero; only $p_1$ is positive. So the value of $\dot{S}_o$ at $t = 0$ is obtained. This is the initial condition.

(4) Input into jth storage element at the beginning of a particular time interval $\Delta t$ is an impulse of input, equal to the sum of input $p_j$, the surface inflow $q_{j-1}$ and the surface outflow of $q_j$. The impulse input causes an instantaneous change in storage of the jth element. Responses of the elements to the impulse inputs determine the outputs and the states of the elements at the end of the time interval $\Delta t$; the new states are the initial conditions for the following time increment.

(5) For given initial states of the system, $B_o$, $S_o$, and $P_o$ at the beginning of a particular time interval $\Delta t$, $\dot{S}_o$ can be obtained from Eq. (2-5). The state of the system, at the end of this time interval $\Delta t$, is the initial state at the beginning of the next time interval $\Delta t$ and is given by $S_1 = S_o + \dot{S}_o$. Outflows from the n elements at the end of the time $\Delta t_o$ can then be obtained from Eq.(2-6).
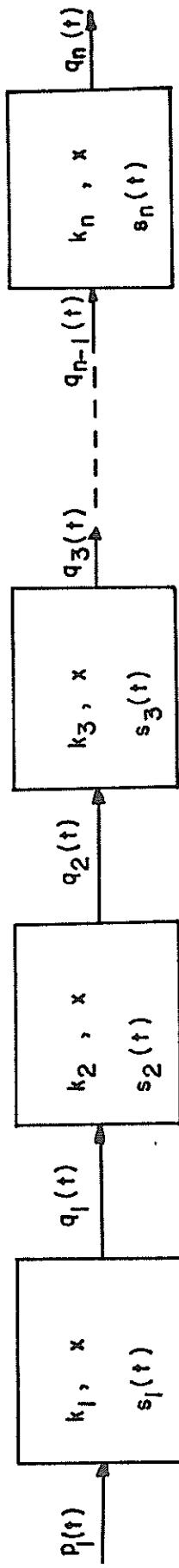
Fig. 2.2. A nonlinear hydrologic cascade with lumped input.

CHAPTER 3

APPLICATION TO NATURAL WATERSHEDS

For model application, thirty-eight natural agricultural water-
sheds were selected. These watersheds represent several regions of
the United States. Of them, 16 watersheds are near Riesel (Waco),
Texas; 5 near Hastings, Nebraska; 5 near Coshocton, Ohio; 8 near Ox-
ford, Mississippi; and 1 each in Watkinsville, Georgia; McCredie, Mis-
souri; Fennimore, Wisconsin; and Ralston Creek, Iowa. These are small
watersheds varying in area from 0.5 ha to 3,055 ha. For complete in-
formation on watershed characteristics see Shelburne and Singh(1976)
or USDA publications entitled "Hydrologic Data for Experimental Agri-
cultural Watersheds in the United States."

3.1 Determination of mean Areal Rainfall

Rainfall-runoff data of all 38 watersheds were obtained from the
USDA publications,"Hydrologic Data For Experimental Agricultural Water-
sheds in the United States." These publications are released almost
every year and contain for each watershed one largest event a year.
Although there may be more than one raingage on a watershed, data is
normally available in the USDA publications for a centrally located
raingage indicating that this represents the mean areal rainfall. For
consistency, this practice was followed for each watershed. The guide-
lines for selection of hydrologic data are given by Shelburne and Singh
(1976).

3.2 Determination of Rainfall-Excess

Rainfall-excess is inflow to the model and is determined by sub-
stracting infiltration from rainfall. Philip's equation (Philip, 1957)
was used to estimate infiltration which can be written as:

$$f = \tfrac{1}{2} \beta t^{-\frac{1}{2}} + \alpha \tag{3-1}$$

where f = infiltration rate in cm/hr;

$\beta$ = a parameter, dependent on soil characteristics and initial

soil moisture content;

t = time in hours; and

$\alpha$ = a parameter, dependent on soil characteristics.

The parameter $\alpha$ is related to the saturated hydraulic conductivity

(Schulz, 1973) and, therefore, can be determined from soil characteris-

tics (Gray, 1970). To account for antecedent soil moisture conditions,

the parameter $\beta$ was allowed to vary with each rainfall event and was

determined by preserving the volume continuity of water. For a sample

rainfall event on watershed 4-H, Hastings, Nebraska, rainfall-excess

determination by this method is shown in Fig. 3.1.

3.3  Choice of Objective Function

The following objective function was used in this study:

$$F = \sum_{j=1}^{M} \left[ Q_{p_o}(j) - Q_{p_e}(j) \right]^2 \implies \min \tag{3-2}$$

where F = objective function;

$Q_{p_o}(j)$ = observed hydrograph peak in cm/hr for the jth event;

$Q_{p_e}(j)$ = estimated hydrograph peak in cm/hr for the jth event; and

M = number of rainfall-runoff events in the optimization set.

The choice of this objective function is based on the findings of Singh

(1975a, 1975b, 1975c, 1976) and Shelburne and Singh (1976). Because it

only requires the hydrograph peak from each rainfall-runoff event, com-

putationally it is more efficient. When F is divided by the number of

events in the optimization set, the mean square error can be obtained

which reflects on the average error occurring in the performing of

optimization.
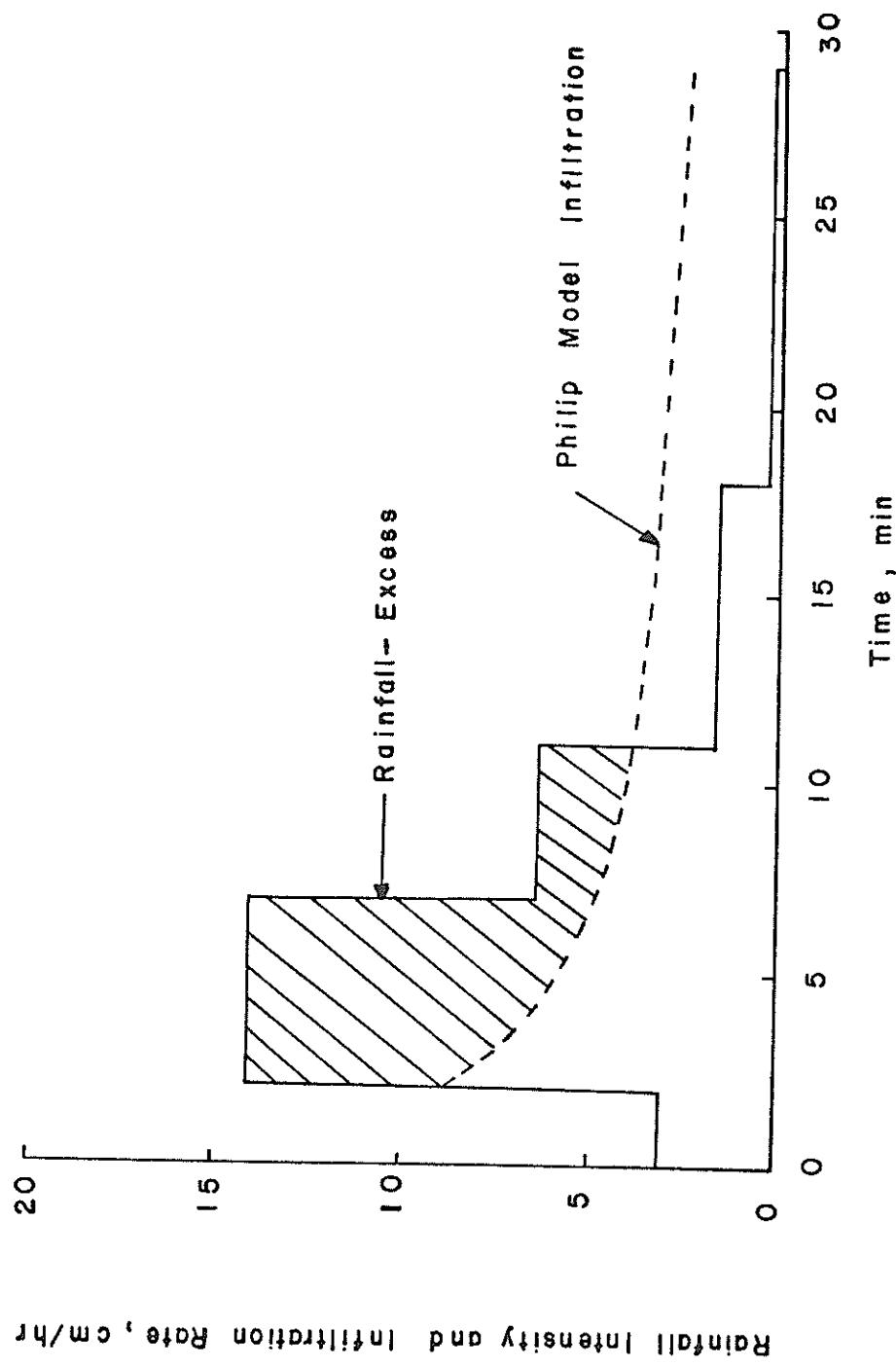
Fig. 3.1. Determination of rainfall-excess by Philip equation for the event of 8-11-1939, Watershed 4-H, Hastings, Nebraska.

3.4  Determination of Computation Time Interval

To study the effect of the computation time interval on model parameters and its predictive performance a watershed  4-H, Hastings, Nebraska,was selected.  This watershed had 18 events.  These events were divided into two sets; one set, called the optimization set, consisted of 10 events; the other set, called the prediction set, consisted of 8 events.  These two sets did not have any events in common.  The number of elements, n, in the cascade was restricted to three and x was fixed at 1.5 (Singh, 1976).  Then parameters $k_1$, $k_2$, and $k_3$ were optimized for the optimization set of events by the modified Rosenbrock-Palmer algorithm (Rosenbrock, 1960; Palmer, 1969; Himmelblau, 1972) using the objective function of Eq. (3-2).  The optimization was performed for different time intervals of 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0 and 4.5 minutes.  Table 3.1 shows the optimized values of $k_1$, $k_2$, $k_3$ and the objective function values for different time intervals. It is evident from Table 3.1 and Fig. 3.2 that the parameters $k_1$, $k_2$, and $k_3$ change with time interval and the objective function value increases with the increase in the time interval.  To see the effect of time interval on model performance runoff hydrographs were predicted for the events in the prediction set, using optimized values of $k_1$, $k_2$ and $k_3$ corresponding to each computation time interval.  The predicted hydrograph peak and its time are shown in Table 3.2 and Figs. 3.3-3.11. It is clear that the relative error increases with the increase in the time interval.  From the above observations a computation time interval of 1.0 minute was found suitable.

3.5  Parameter Optimization

The nonlinear cascade has (n + 2) parameters:  n, x, $k_1$, $k_2$, ------$k_n$.  The parameter x reflects the degree of nonlinearity in surface

Table 3.1. Optimized model parameter values for different computation time intervals for watershed 4-H, Hastings, Nebraska.

| Watershed Name | Time Interval (min) | Optimized Model Parameters | | | Objective Function $F$ |
|---|---|---|---|---|---|
| | | $k_1$ | $k_2$ | $k_3$ | |
| Hastings, Nebraska | | | | | |
| 4-H | 0.5 | 0.3300 | 7.57 | 10.6 | 27.4382 |
| | 1.0 | 0.1800 | 3.78 | 4.3 | 13.9757 |
| | 1.5 | 0.9276 | 0.8545 | 0.5 | 44.4446 |
| | 2.0 | 0.2935 | 0.5670 | 0.6 | 59.8454 |
| | 2.5 | 0.3325 | 0.3173 | 0.3 | 60.9275 |
| | 3.0 | 0.1603 | 0.3786 | 0.4 | 51.6472 |
| | 3.5 | 0.1042 | 0.2133 | 0.4 | 145.4256 |
| | 4.0 | 0.0875 | 0.1351 | 0.3 | 197.1997 |
| | 4.5 | 0.0639 | 0.1128 | 0.2 | 326.8289 |

Table 3.2. Comparison of predicted and observed hydrograph peak and its
time for different computation time intervals

| Watershed Name | Date | Time Interval 0.5 min | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| | | Observed Hydrograph Peak (cm/hr) | Predicted Hydrograph Peak (cm/hr) | Relative Error (%) | Observed Hydrograph Peak Time (min) | Predicted Hydrograph Peak Time (min) | Relative Error (%) |
| Hastings | | | | | | | |
| 4-H | 8-11-39 | 4.5212 | 4.8725 | 7.7 | 5.0 | 5.5 | -10.0 |
| | 6-20-42 | 5.8166 | 5.7329 | 1.44 | 8.0 | 10.5 | -31.25 |
| | 9-5-46 | 3.8862 | 3.0823 | 20.69 | 12.0 | 6.5 | 45.83 |
| | 6-1-51 | 6.7564 | 6.8449 | -1.31 | 123.0 | 130.5 | -6.09 |
| | 7-3-52 | 9.1948 | 9.4789 | -3.09 | 21.0 | 18.5 | 11.90 |
| | 6-12-58 | 0.9195 | 0.9254 | -0.65 | 14.0 | 8.5 | 39.28 |
| | 6-12-65 | 9.7028 | 8.8887 | 8.39 | 19.0 | 14.5 | 23.68 |
| | 6-12-65 | 6.1468 | 5.8334 | 5.10 | 7.0 | 11.5 | -64.28 |

Table 3.2. (continued)

| | Time Interval 1.00 min | | | | Time Interval 1.5 min | | | |
|---|---|---|---|---|---|---|---|---|
| Predicted Hydrograph Peak (cm/hr) | Relative Error (%) | Predicted Hydrograph Peak Time (min) | Relative Error (%) | | Predicted Hydrograph Peak (cm/hr) | Relative Error (%) | Predicted Hydrograph Peak Time (min) | Relative Error (%) |
| 3.4254 | 24.24 | 6.0 | -20.00 | | 3.9865 | 11.83 | 7.5 | -50.00 |
| 5.0194 | 13.71 | 11.0 | -37.50 | | 5.7994 | 0.30 | 10.5 | -31.25 |
| 2.1212 | 45.42 | 8.0 | 33.33 | | 2.6170 | 32.66 | 8.5 | 70.83 |
| 6.3084 | 6.63 | 131.0 | -6.50 | | 6.8863 | -1.92 | 126.0 | -2.43 |
| 9.9094 | -7.77 | 19.0 | 19.52 | | 9.6060 | -4.47 | 19.5 | 7.14 |
| 0.5000 | 45.62 | 11.0 | 21.43 | | 0.5426 | 40.98 | 13.5 | 3.57 |
| 8.2274 | 15.21 | 15.0 | 21.05 | | 9.0837 | 6.38 | 15.0 | 5.26 |
| 4.9645 | 19.23 | 12.0 | -71.42 | | 5.9855 | 2.62 | 12.00 | -71.42 |

Table 3.2. (continued)

| | Time Interval 2.00 min | | | | Time Interval 2.5 min | | | |
|---|---|---|---|---|---|---|---|---|
| Predicted Hydrograph Peak (cm/hr) | Relative Error (%) | Predicted Hydrograph Peak Time (min) | Relative Error (%) | | Predicted Hydrograph Peak (cm/hr) | Relative Error (%) | Predicted Hydrograph Peak Time (min) | Relative Error (%) |
| 2.7216 | 39.80 | 12.0 | -140.0 | | 1.8065 | 60.04 | 15.0 | -200 |
| 5.3338 | 8.30 | 12.0 | -50.0 | | 4.1275 | 29.04 | 15.0 | -87.5 |
| 1.9207 | 50.58 | 14.0 | -16.66 | | 1.5414 | 60.34 | 17.5 | -45.83 |
| 6.5639 | 2.85 | 128.0 | -4.06 | | 5.4795 | 18.90 | 132.5 | -7.72 |
| 9.5568 | -3.94 | 20.0 | 4.76 | | 9.2300 | -0.38 | 20.0 | 4.76 |
| 0.3166 | 65.57 | 20.0 | -42.85 | | 0.2163 | 76.47 | 25.0 | -78.57 |
| 9.0079 | 7.16 | 16.0 | 15.78 | | 8.1537 | 15.97 | 17.5 | 7.89 |
| 5.5478 | 9.75 | 14.0 | -100 | | 4.1666 | 32.22 | 17.5 | -150.0 |

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Table 3.2. (continued)

| | Time Interval 3.00 min | | | | Time Interval 3.5 min | | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| | Predicted Hydrograph Peak (cm/hr) | Relative Error (%) | Predicted Hydrograph Peak Time (min) | Relative Error (%) | Predicted Hydrograph Peak (cm/hr) | Relative Error (%) | Predicted Hydrograph Peak Time (min) | Relative Error (%) |
| | 1.5557 | 65.59 | 15.00 | -200.0 | 0.8980 | 80.14 | 21.0 | -320.0 |
| | 3.7273 | 35.92 | 15.00 | -88.88 | 2.6531 | 54.39 | 21.0 | -200.0 |
| | 1.3767 | 64.57 | 21.0 | -75.00 | 0.9331 | 75.99 | 24.0 | -104.0 |
| | 6.6588 | 1.44 | 133.0 | -9.87 | 6.2974 | 6.79 | 136.5 | -10.90 |
| | 9.4169 | -2.42 | 21.0 | 0 | 8.5906 | 6.57 | 21.0 | 0 |
| | 0.1798 | 80.45 | 27.0 | -92.84 | 0.1177 | 87.20 | 38.5 | -175.0 |
| | 8.3628 | 13.81 | 18.0 | 5.26 | 6.8909 | 28.98 | 21.0 | -10.52 |
| | 4.1034 | 33.24 | 18.0 | -155.55 | 2.9706 | 51.67 | 21.0 | -200.0 |

Table 3.2. (continued)

| | Time Interval 4.00 min | | | | Time Interval 4.5 min | | | |
|---|---|---|---|---|---|---|---|---|
| Predicted Hydrograph Peak (cm/hr) | Relative Error (%) | Predicted Hydrograph Peak Time (min) | Relative Error (%) | | Predicted Hydrograph Peak (cm/hr) | Relative Error (%) | Predicted Hydrograph Peak Time (min) | Relative Error (%) |
| 0.5825 | 87.12 | 28.0 | -460.0 | | 0.3779 | 91.64 | 40.5 | -710.0 |
| 2.0030 | 65.56 | 24.0 | -200.0 | | 1.4477 | 75.11 | 31.5 | -293.75 |
| 0.6437 | 83.44 | 32.0 | -166.66 | | 0.4421 | 88.62 | 40.5 | -237.50 |
| 6.3478 | 6.05 | 140.0 | -13.82 | | 4.9579 | 26.62 | 144.0 | -17.07 |
| 7.3808 | 19.73 | 24.0 | -14.28 | | 6.1893 | 32.69 | 27.0 | -28.57 |
| 0.0847 | 90.79 | 48.0 | -242.85 | | 0.0611 | 93.35 | 63.0 | -350.00 |
| 5.2705 | 45.68 | 24.0 | -26.31 | | 4.1076 | 57.67 | 27.0 | -28.57 |
| 2.3216 | 62.23 | 24.0 | -242.85 | | 1.5443 | 74.88 | 40.5 | -478.57 |
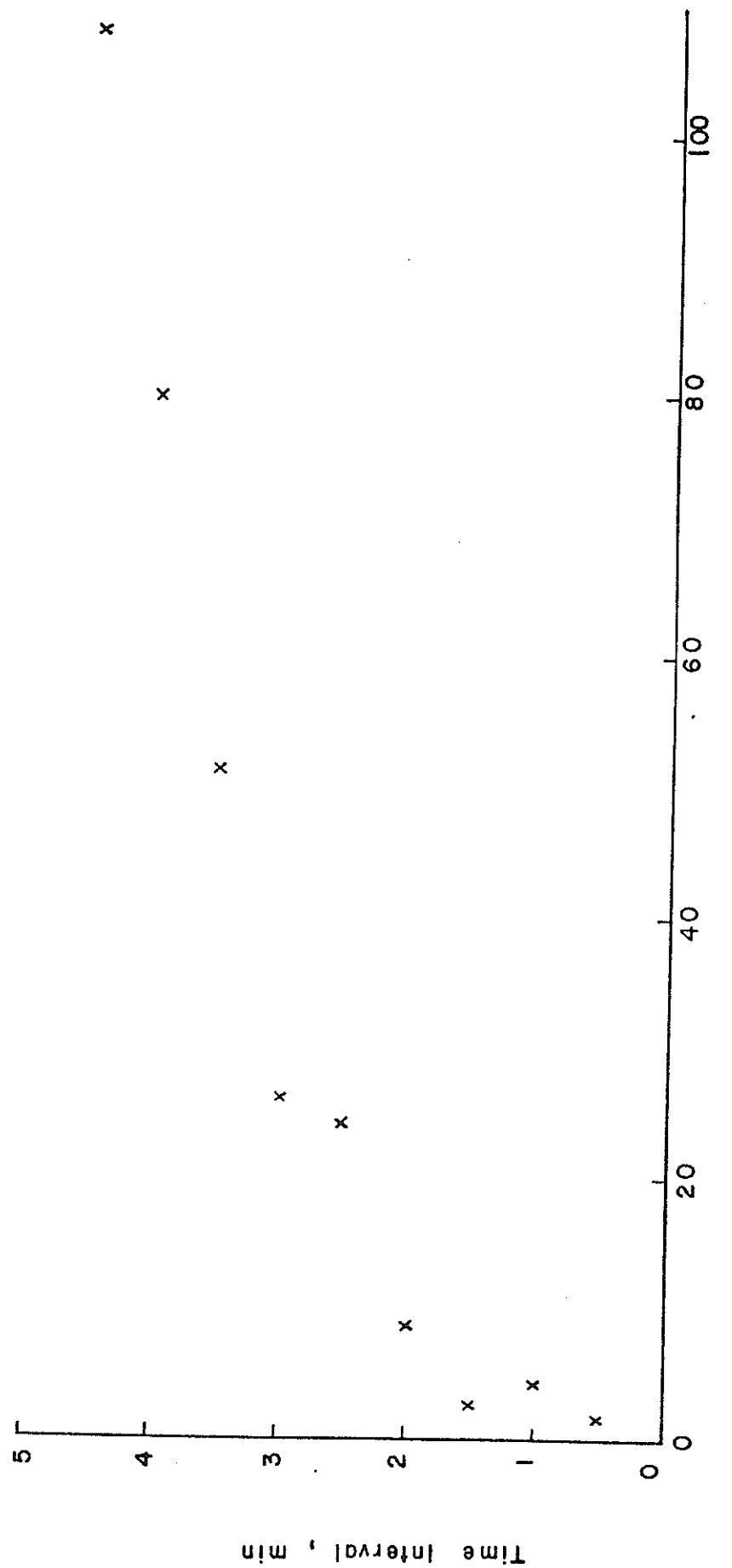
Fig. 3.2. Time interval versus error function of predicted runoff on watershed 4-H, Hastings, Nebraska.
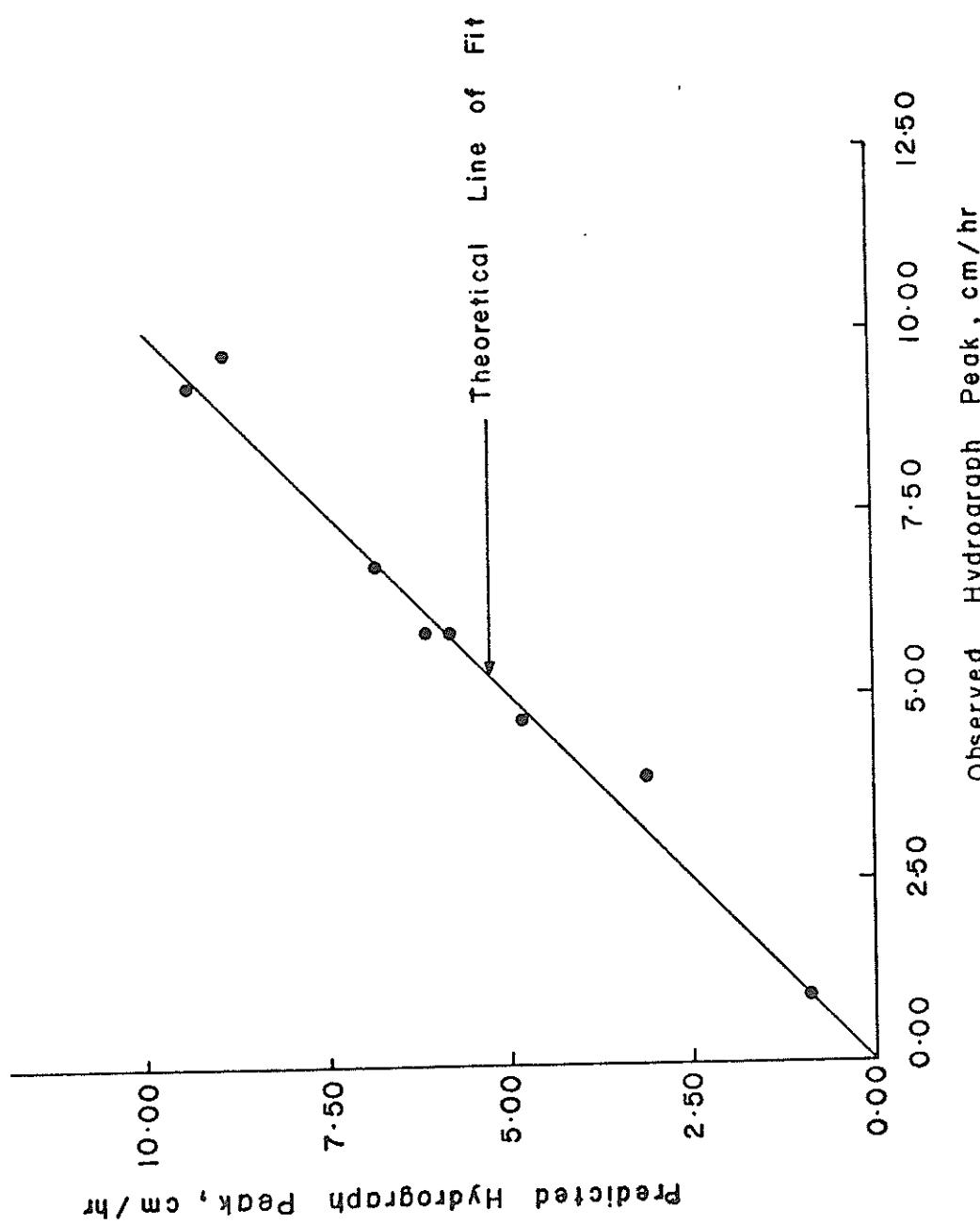
Fig. 3.3. Predicted hydrograph peak versus observed hydrograph peak for computation time interval of 0.5 min on watershed 4-H, Hastings, Nebraska.
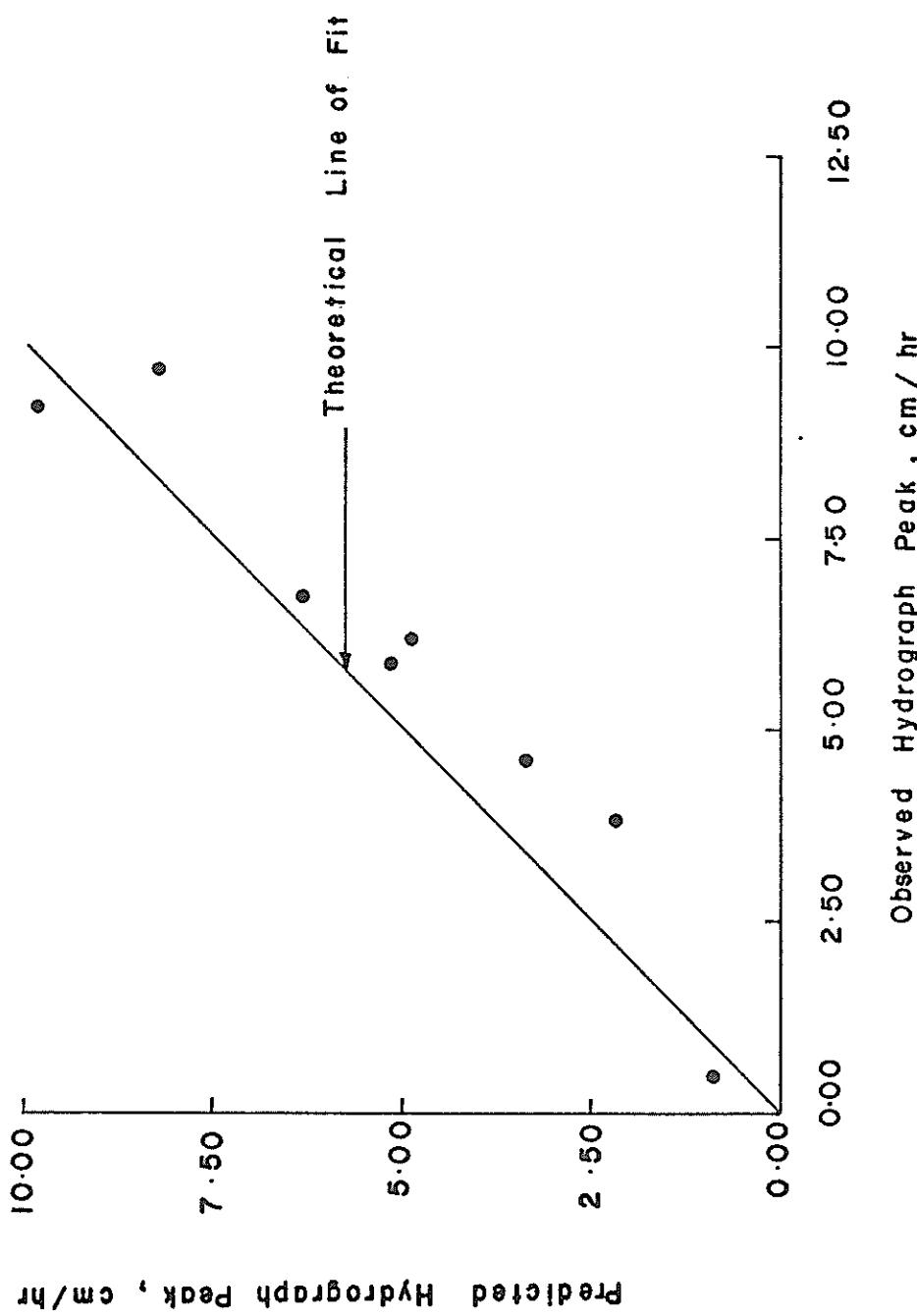
Fig. 3.4. Predicted hydrograph peak versus observed hydrograph peak for compu-
tation time interval of 1.00 min on watershed 4-H, Hastings, Nebraska.
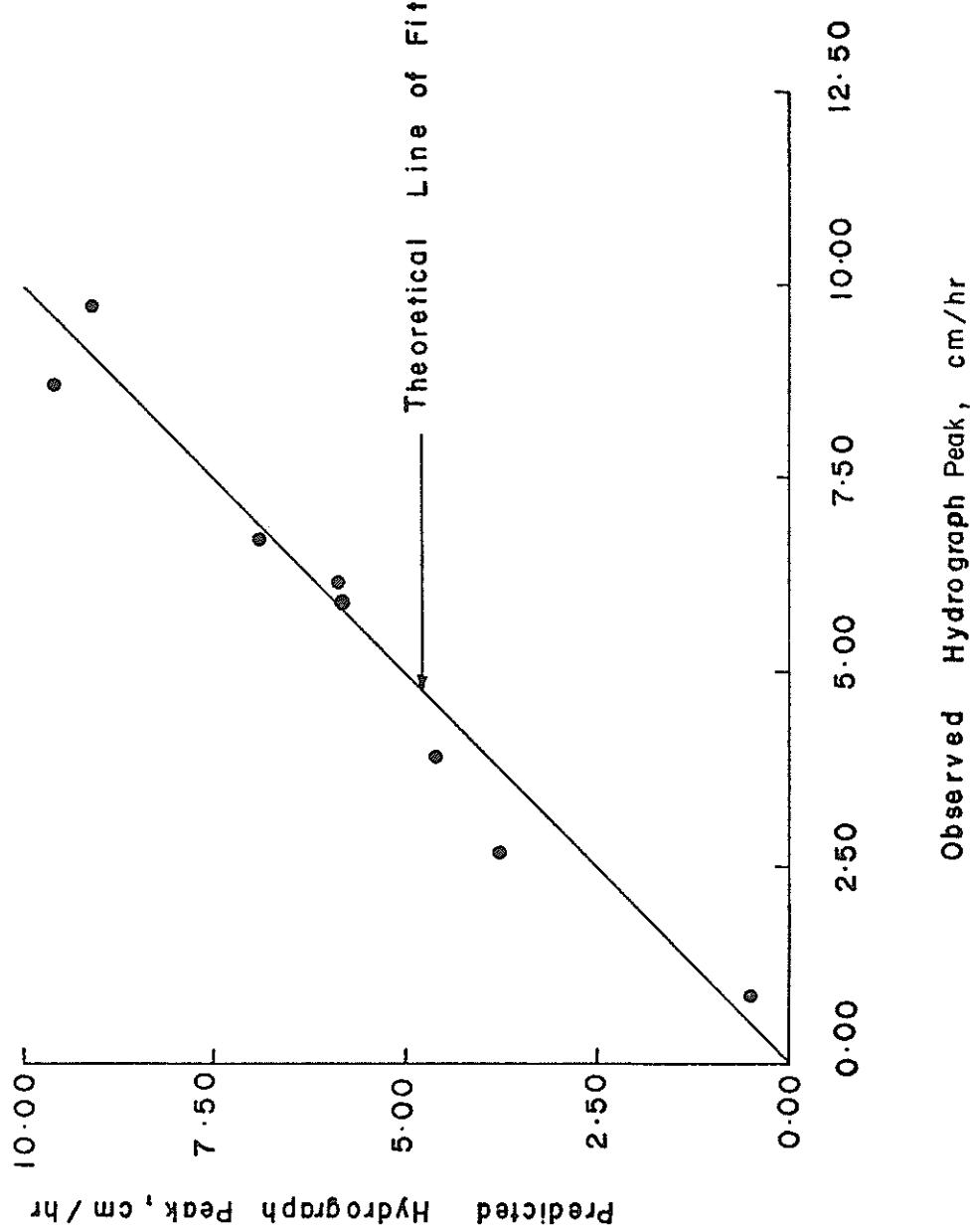
Fig. 3.5. Predicted hydrograph peak versus observed hydrograph peak for computation time interval of 1.5 min on watershed 4-H, Hastings, Nebraska.
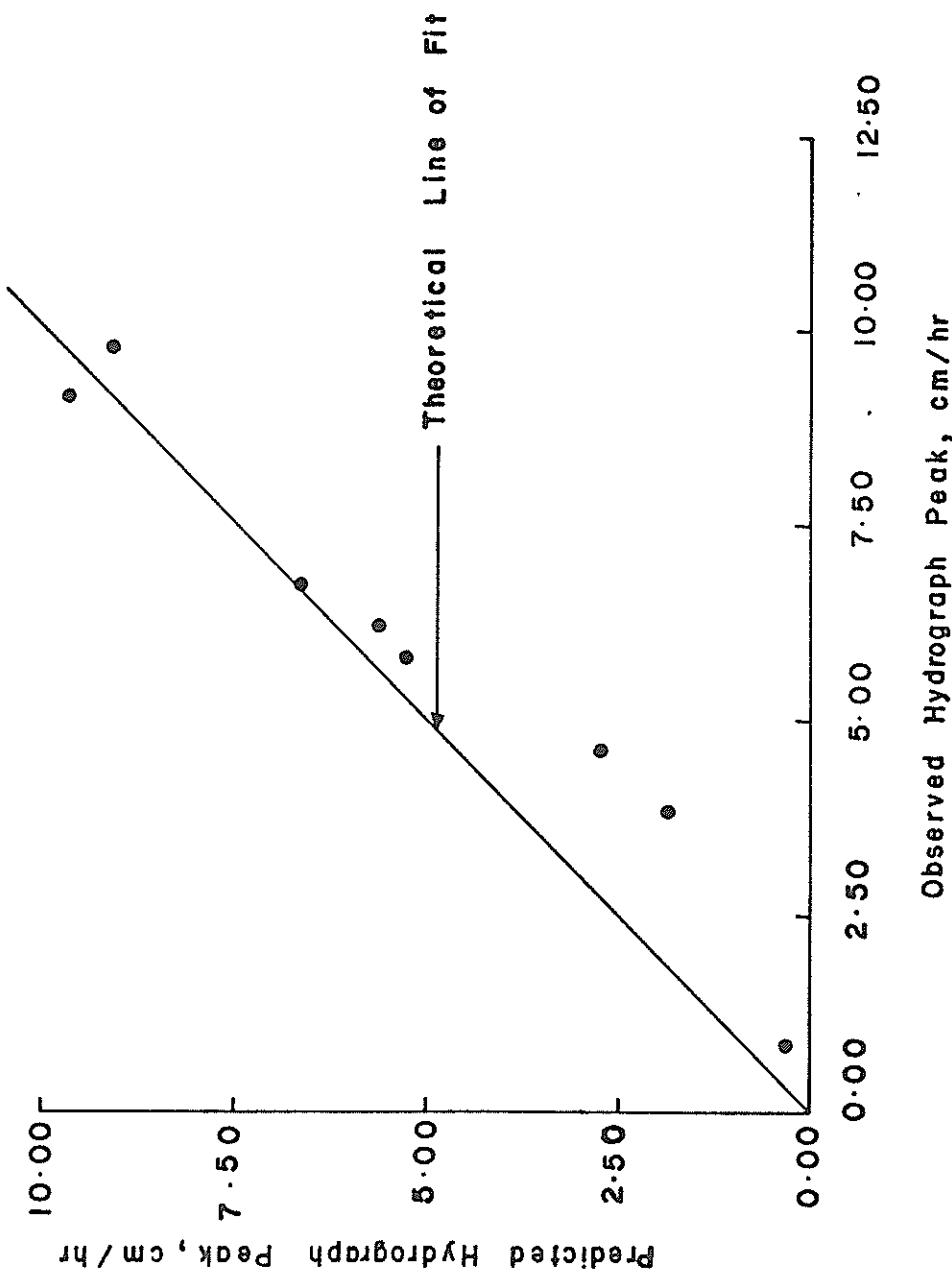
Fig. 3.6. Predicted hydrograph peak versus observed hydrograph peak for computation time interval of 2.00 min on watershed 4-H, Hastings, Nebraska.
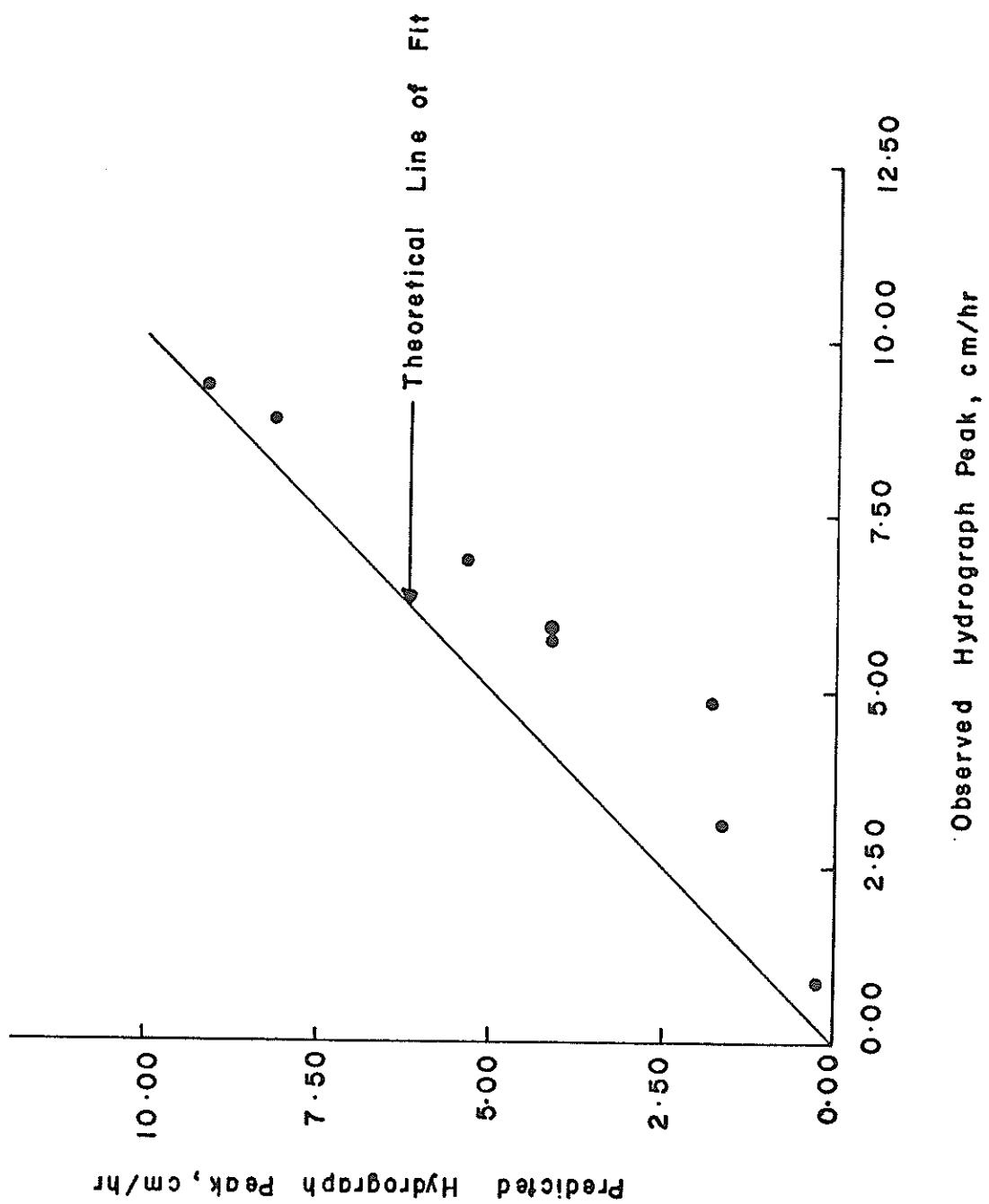
Fig. 3.7. Predicted hydrograph peak versus observed hydrograph peak for computation time interval of 2.50 min. on watershed 4-H, Hastings, Nebraska.
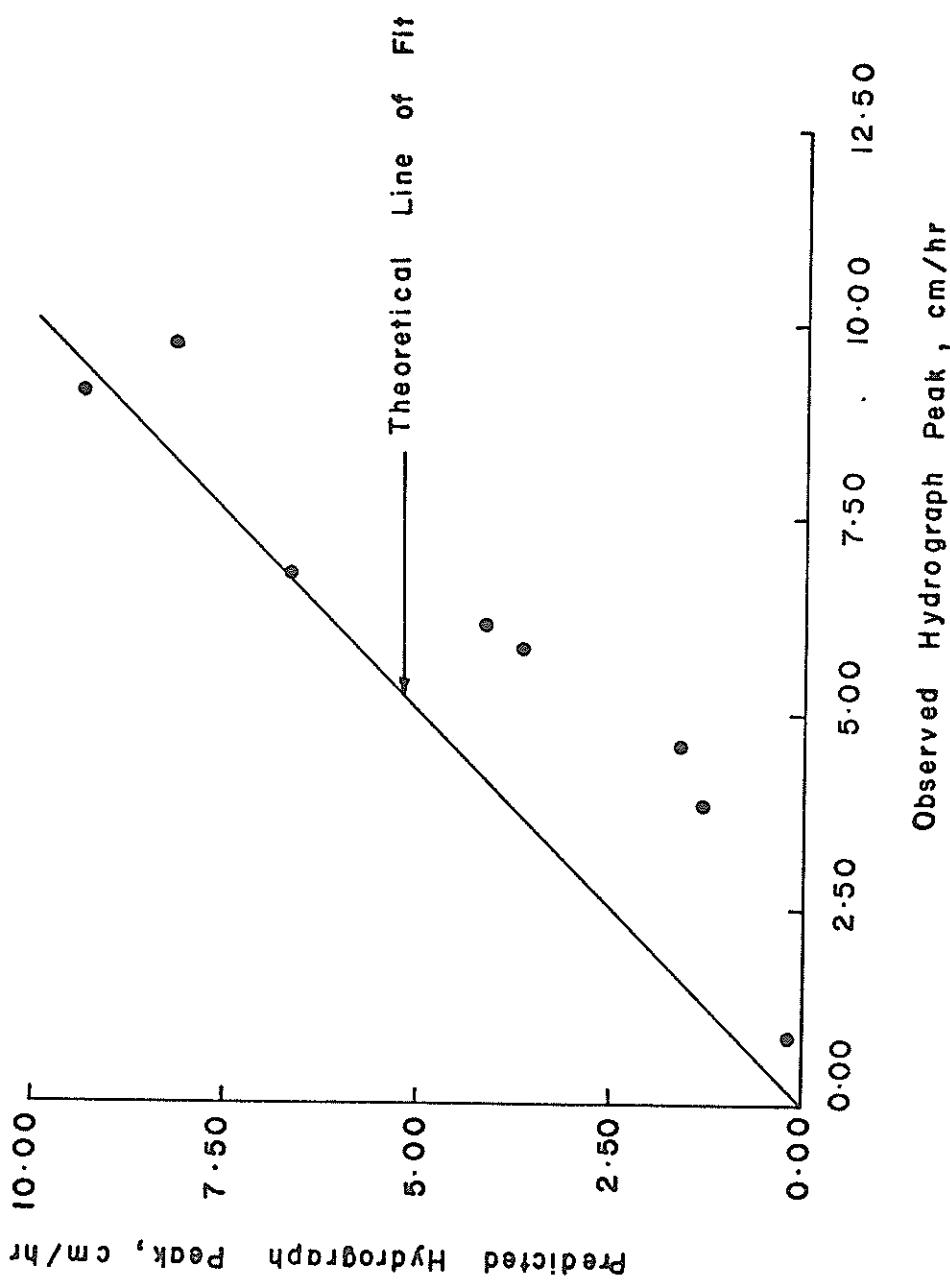
Fig. 3.8. Predicted hydrograph peak versus observed hydrograph peak for computation time interval of 3.00 min on watershed 4-H, Hastings, Nebraska.
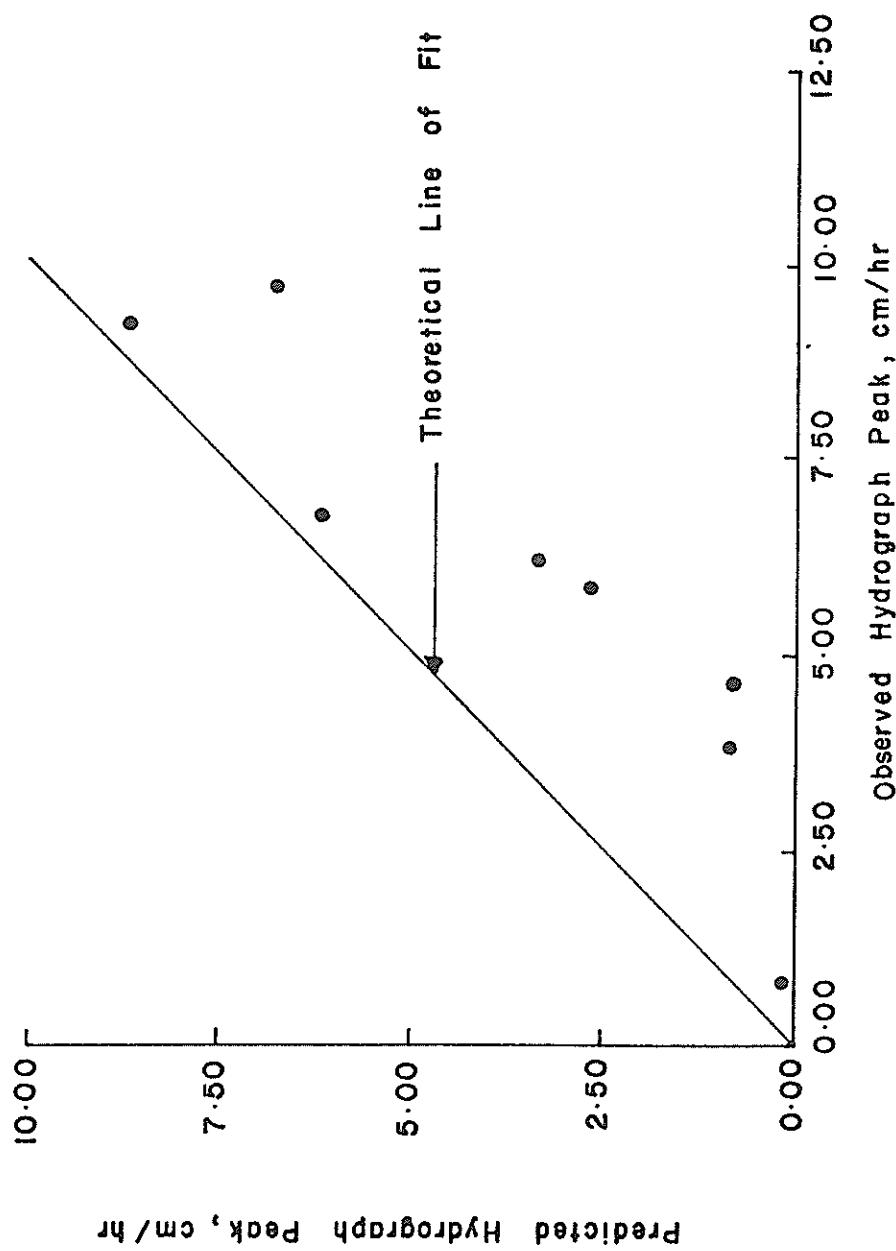
Fig. 3.9. Predicted hydrograph peak versus observed hydrograph peak for computation time interval of 3.50 min on watershed 4-H, Hastings, Nebraska.
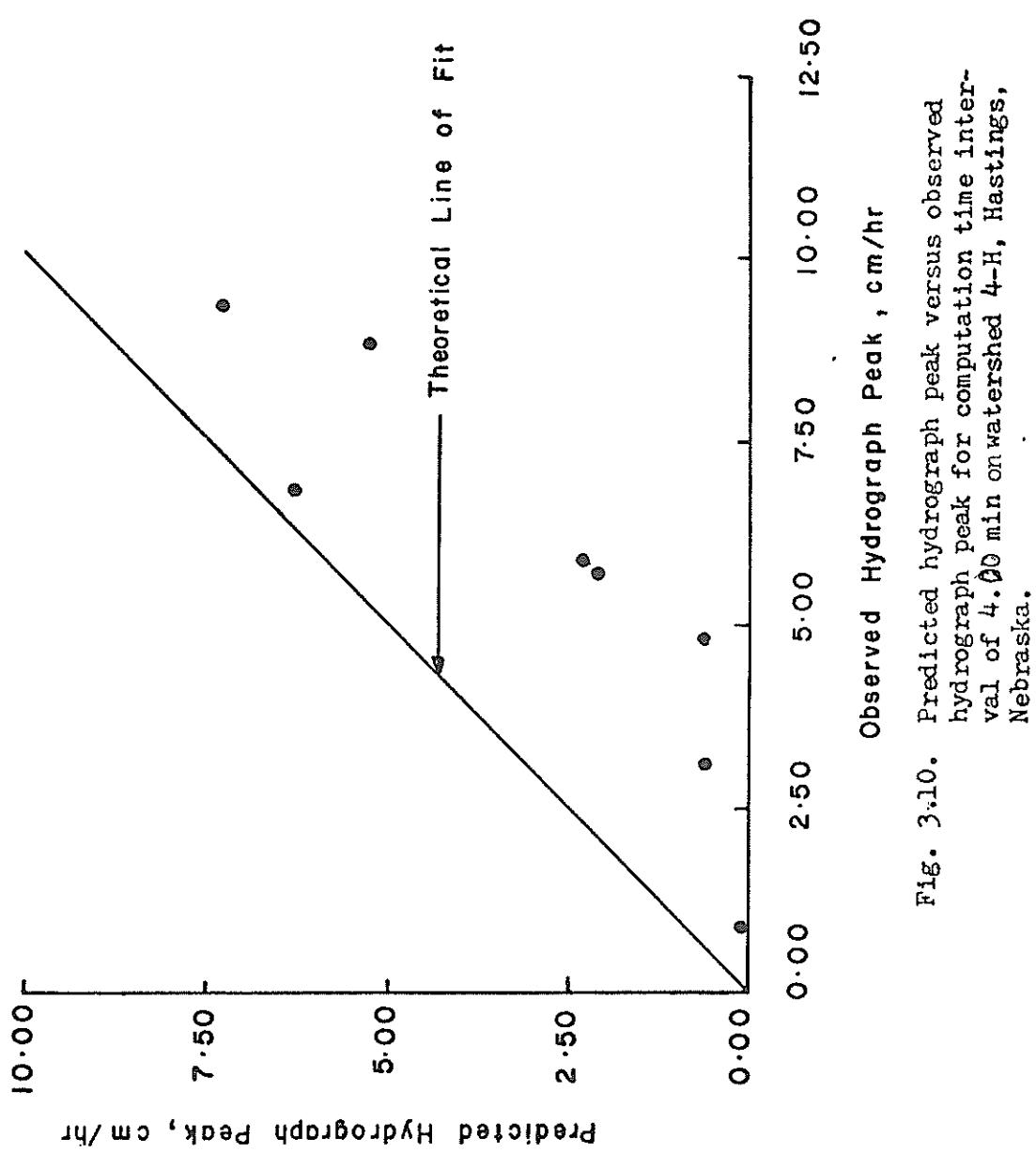
Fig. 3.10. Predicted hydrograph peak versus observed hydrograph peak for computation time interval of 4.00 min on watershed 4-H, Hastings, Nebraska.
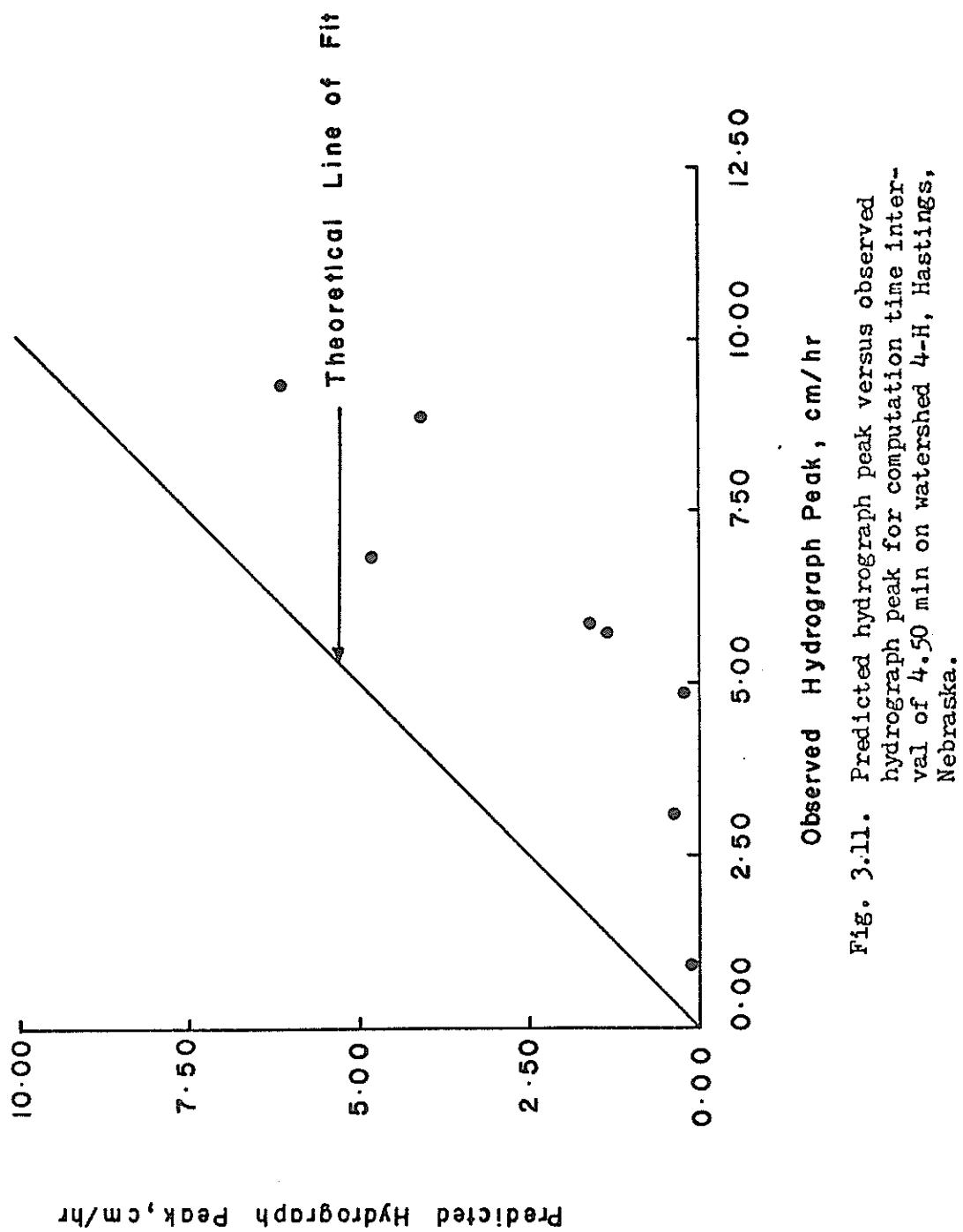
Fig. 3.11.  Predicted hydrograph peak versus observed
hydrograph peak for computation time inter-
val of 4.50 min on watershed 4-H, Hastings,
Nebraska.

runoff. Based on the findings of Singh (1976), x was fixed at 1.5. The parameter n denotes the number of storage elements in the cascade. The value of n must be greater than one to obtain proper hydrograph shape and will depend on the topographic complexity. Again, based on the study by Singh (1976) and Shelburne and Singh (1976) n was fixed at 3. Thus parameters for optimization are $k_1$, $k_2$, and $k_3$. As before, rainfall-runoff events of each of 38 watersheds were divided into an optimization set and a prediction set. Parameters $k_1$, $k_2$, and $k_3$ were then optimized for each optimized set of events by the modified Rosenbrock-Palmer algorithm (Rosenbrock, 1960; Palmer, 1969; Himmelblau, 1972) in accordance with the objective function of Eq. (3-2). The results of optimization are given in Table 3.3. It should be remarked that the parameters do not change very much from one watershed to another in a given region. This observation may be useful in specification of the parameters on a regional basis.

3.6  Hydrograph Prediction

Hydrographs were predicted for the events in the prediction sets of several watersheds, utilizing the optimized parameter values. For a few sample watersheds hydrograph peak characteristics are given in Table 3.4. It is clear that the predicted hydrograph peak characteristics are in good agreement with the observed peak characteristics. A comparison of observed and predicted hydrographs for some sample events is shown in Figs. 3.12-3.22. These figures indicate that the cascade can simulate the entire hydrograph well.

3.7  Discussion of Prediction Results

Table 3.4 and Figs. 3.12-3.22 indicate that in most cases hydrograph peak, its time and hydrograph shape are well predicted by the model. However, the prediction errors go high in some cases. The reason appears

to be the inaccurate determination of rainfall-excess and poor syn-
chronization between rainfall and runoff observations. Poor hydrograph
prediction is particularly evident for complex rainfall distribution,
indicating thereby that the model is sensitive to spatial distribution
of rainfall as illustrated in Figs. 3.20 and 3.21. The best predicted
hydrographs were found for the events which approximated a normal dis-
tribution and when the rainfall-excess was large.

Accurate determination of rainfall-excess is a major problem in
rainfall-runoff modeling (Singh, 1976; Shelburne and Singh, 1976). In
this study rainfall-excess was determined by substracting infiltration
from rainfall. Philip's equation (Philip, 1957) was used to determine
infiltration. There are two parameters in this equation. These parameters
depend on soil characteristics and initial soil moisture content. In
most cases these parameters cannot be estimated accurately.

The other sources of error might be inaccurate measurement of rain-
fall and inaccurate estimation of mean areal rainfall. Although a central
raingage might represent the mean areal rainfall for a small watershed,
this, however, is not always true. This causes errors in the optimiza-
tion. The gaged stream flow, which is used in comparison with the pre-
dicted runoff, may also contain errors. Some errors may be due to fixing
the values of parameters x and n and determination of $k_1$, $k_2$, and $k_3$ by
optimization.

Table 3.3. Optimized parameters of nonlinear hydrologic cascade.

| Watershed Name and Location | Watershed | Number of Events Used in Optimization | Optimized Parameters | | | Objective Function |
|---|---|---|---|---|---|---|
| | | | $k_1$ | $k_2$ | $k_3$ | |
| Riesel(WACO), Texas | C | 9 | 0.01 | 0.1885 | 0.1 | 0.7107 |
| | D | 8 | 0.0064 | 0.0507 | 0.1 | 2.1113 |
| | G | 8 | 0.002 | 4.6758 | 0.1 | 0.0943 |
| | SW-12 | 10 | 0.2151 | 2.8752 | 0.2 | 0.2644 |
| | SW-17 | 9 | 0.0753 | 2.9470 | 0.2 | 10.7572 |
| | W-1 | 5 | 0.1107 | 0.1881 | 0.2 | 2.4099 |
| | W-2 | 8 | 0.0427 | 0.2184 | 0.1 | 0.5318 |
| | W-6 | 8 | 0.0452 | 1.1088 | 0.1 | 8.4204 |
| | W-10 | 9 | 0.0508 | 2.4812 | 0.3 | 12.5929 |
| | Y | 8 | 0.0192 | 0.8077 | 0.6 | 6.5005 |
| | Y-2 | 9 | 0.0196 | 4.7117 | 5.4 | 0.7334 |
| | Y-4 | 9 | 0.0192 | 3.5646 | 0.4 | 0.5009 |
| | Y-6 | 9 | 0.0150 | 6.0058 | 5.3 | 8.3673 |
| | Y-7 | 9 | 0.0384 | 4.0105 | 0.9 | 5.0492 |
| | Y-8 | 5 | 0.1127 | 0.2012 | 0.3 | 5.3155 |
| | Y-10 | 5 | 0.1045 | 0.1874 | 0.2 | 2.0791 |
| Coshocton, Ohio | 5 | 7 | 3.77 | 7.14 | 0.02 | 0.2814 |
| | 92 | 8 | 5.6 | 0.19 | 0.02 | 0.1779 |
| | 95 | 7 | 0.0271 | 0.3119 | 0.03 | 0.2789 |
| | 97 | 9 | 1.7627 | 0.0551 | 0.01 | 0.4541 |
| | 177 | 9 | 8.7690 | 0.4205 | 0.04 | 0.1009 |
| McCredie, Missouri | W-1 | 9 | 0.0248 | 2.9309 | 0.1 | 1.5632 |
| Watkinsville, Georgia | W-1 | 9 | 0.0777 | 3.5816 | 0.2 | 18.6779 |
| Fennimore, Wisconsin | W-1 | 10 | 0.0772 | 2.9436 | 0.2 | 0.8152 |
| Ralston Creek, Iowa | | 10 | 0.0088 | 0.5955 | 0.5 | 2.1323 |
| Hastings, Nebraska | 2-H | 10 | 0.1207 | 1.7874 | 2.2 | 7.7348 |
| | 4-H | 10 | 0.1800 | 3.78 | 4.3 | 13.9757 |
| | W-3 | 10 | 0.0242 | 0.5955 | 0.5 | 2.1323 |
| | W-8 | 7 | 0.0040 | 0.7471 | 0.7 | 0.1437 |
| | W-11 | 10 | 0.0029 | 0.5687 | 0.02 | 1.7775 |

Table 3.3.(continued)

| Watershed Name and Location | Number of Events Used in Optimization | Optimized Parameters | | | Objective Function |
|---|---|---|---|---|---|
| | | $k_1$ | $k_2$ | $k_3$ | |
| Oxford, Mississippi | | | | | |
| W-5 | 6 | 0.0148 | 0.6162 | 0.1 | 0.1603 |
| W-35 | 4 | 0.0051 | 0.7313 | 1.1 | 0.1005 |
| W-10 | 5 | 0.0141 | 0.5850 | 0.4 | 0.0812 |
| W-24 | 5 | 0.0068 | 1.0636 | 0.9 | 0.3866 |
| WC-1 | 7 | 0.1510 | 2.0277 | 2.4 | 8.1092 |
| WC-2 | 9 | 0.2460 | 3.25 | 0.2 | 13.4011 |
| WC-3 | 9 | 0.1665 | 2.0692 | 2.3 | 20.3453 |
| WP-4 | 9 | 0.1480 | 2.6946 | 1.2 | 3.7453 |

Table 3.4. Comparison of observed and predicted hydrograph peak and its time.

| Watershed Name | Date | Observed Hydrograph Peak (cm/hr) | Predicted Hydrograph Peak (cm/hr) | Relative Error (%) | Observed Hydrograph Peak Time (min) | Predicted Hydrograph Peak Time (min) | Relative Error (%) |
|---|---|---|---|---|---|---|---|
| Hastings 2-H | 8-11-39 | 2.8194 | 0.9774 | 65.33 | 10.0 | 10.0 | 0 |
| | 5-20-49 | 0.7036 | 0.1351 | 80.79 | 3.0 | 20.0 | -566.66 |
| | 6-12-65 | 8.8138 | 7.9924 | 9.32 | 9.0 | 24.0 | -166.66 |
| | 6-12-65 | 2.1565 | 2.1880 | -1.46 | 13.0 | 15.0 | -23.08 |
| | 6-29-65 | 2.0676 | 1.5737 | 23.89 | 9.0 | 9.0 | 0 |
| | 8-7-46 | 3.7592 | 1.5174 | 59.64 | 7.0 | 10.0 | -42.86 |
| | 8-7-42 | 2.5197 | 1.5120 | 39.99 | 10 | 23.0 | -130.00 |
| | 6-16-50 | 0.1735 | 0.0040 | 97.69 | 2.0 | 43.0 | -2050.0 |
| | 9-7-42 | 3.5306 | 1.8000 | 49.02 | 15.0 | 12.0 | 20.0 |
| | 9-5-46 | 2.2885 | 1.3593 | 40.60 | 9.0 | 12.0 | -33.33 |
| Hastings 4-H | 8-11-39 | 4.5212 | 3.4254 | 24.24 | 5.0 | 6.0 | -20.00 |
| | 6-20-42 | 5.8166 | 5.0194 | 13.71 | 8.0 | 11.0 | -37.50 |
| | 9-5-46 | 3.8862 | 2.1212 | 45.42 | 12.0 | 8.0 | 33.33 |
| | 6-1-51 | 6.7564 | 6.3084 | 6.63 | 123.0 | 131.0 | -6.50 |
| | 7-13-52 | 9.1948 | 9.9094 | -7.77 | 21.0 | 19.0 | 9.52 |
| | 6-12-58 | 0.9195 | 0.5000 | 45.62 | 14.0 | 11.0 | 21.43 |
| | 6-12-65 | 9.7028 | 8.2274 | 15.21 | 19.0 | 15.0 | 21.05 |
| | 6-12-65 | 6.1468 | 4.9645 | 19.23 | 7.0 | 12.0 | -71.42 |
| Riesel (Waco) | | | | | | | |
| W-1 | 3-12-53 | 2.8702 | 3.1407 | -9.42 | 59.0 | 26.0 | 55.93 |
| | 6-23-59 | 4.8006 | 4.6877 | 2.35 | 87.0 | 111.0 | -27.58 |
| | 3-29-65 | 5.8750 | 7.9949 | -36.08 | 101.0 | 71.0 | 29.70 |
| Y-8 | 5-13-57 | 5.6642 | 5.5050 | 2.81 | 19.0 | 28.0 | -47.36 |
| | 3-23-65 | 5.7122 | 6.9353 | -21.41 | 61.0 | 73.0 | -19.67 |
| | 6-18-61 | 0.1986 | 0.4147 | -108.80 | 100 | 72.0 | 28.00 |
| Y-10 | 4-24-57 | 6.8580 | 6.7709 | 1.27 | 35.0 | 36.0 | -2.85 |
| | 5-13-57 | 4.8514 | 4.4727 | 7.81 | 26.0 | 33.0 | -26.92 |
| | 6-9-62 | 1.0008 | 1.3655 | -36.45 | 38.0 | 38.0 | 0 |
| | 3-23-65 | 6.9248 | 7.8860 | -13.88 | 69.0 | 81.0 | -17.39 |

Table 3.4. (continued)

| Watershed Name | Date | Observed Hydrograph Peak (cm/hr) | Predicted Hydrograph Peak (cm/hr) | Relative Error (%) | Observed Hydrograph Peak Time (min) | Predicted Hydrograph Peak Time (min) | Relative Error (%) |
|---|---|---|---|---|---|---|---|
| Hastings W-8 | 6-7-42 | 0.2718 | 0.1693 | 37.69 | 64.0 | 48.0 | 25.00 |
| | 5-11-44 | 0.4521 | 0.0819 | 81.89 | 75.0 | 112.0 | -49.33 |
| | 7-6-45 | 0.4267 | 0.2626 | 38.46 | 71.0 | 55.0 | 22.54 |
| | 6-1-51 | 1.1328 | 1.2051 | -6.38 | 154.0 | 168.0 | -9.09 |
| | 6-26-52 | 0.4491 | 0.4790 | -6.65 | 106.0 | 88.0 | 16.98 |
| | 7-6-52 | 0.2195 | 0.0720 | 67.19 | 62.0 | 85.0 | -37.10 |
| | 6-16-57 | 0.6579 | 0.9788 | -48.78 | 212.0 | 149.0 | 29.72 |
| Coshocton 97 | 4-25-61 | 1.3919 | 1.6009 | -15.01 | 130.0 | 108.0 | 16.92 |
| | 6-18-40 | 0.2946 | 0.3684 | -25.05 | 122.0 | 57.0 | 53.28 |
| | 3-9/10-64 | 0.4729 | 0.6060 | -28.12 | 394.0 | 393.0 | 0.25 |
| | 6-12-57 | 0.6604 | 0.5747 | 12.97 | 150.0 | 80.0 | 46.67 |
| | 6-28/29-40 | 0.5182 | 0.6941 | -33.96 | 150.0 | 113.0 | 24.67 |
| | 6-4-41 | 0.9144 | 0.6623 | 27.57 | 156.0 | 62.0 | 60.26 |
| | 9-23-45 | 0.8204 | 0.8292 | -1.07 | 120.0 | 139.0 | -15.83 |
| | 7-11-46 | 0.5359 | 0.3902 | 27.19 | 212.0 | 100.0 | 52.83 |
| | 6-28-57 | 1.8390 | 1.5310 | 16.75 | 416.0 | 379.0 | 8.89 |
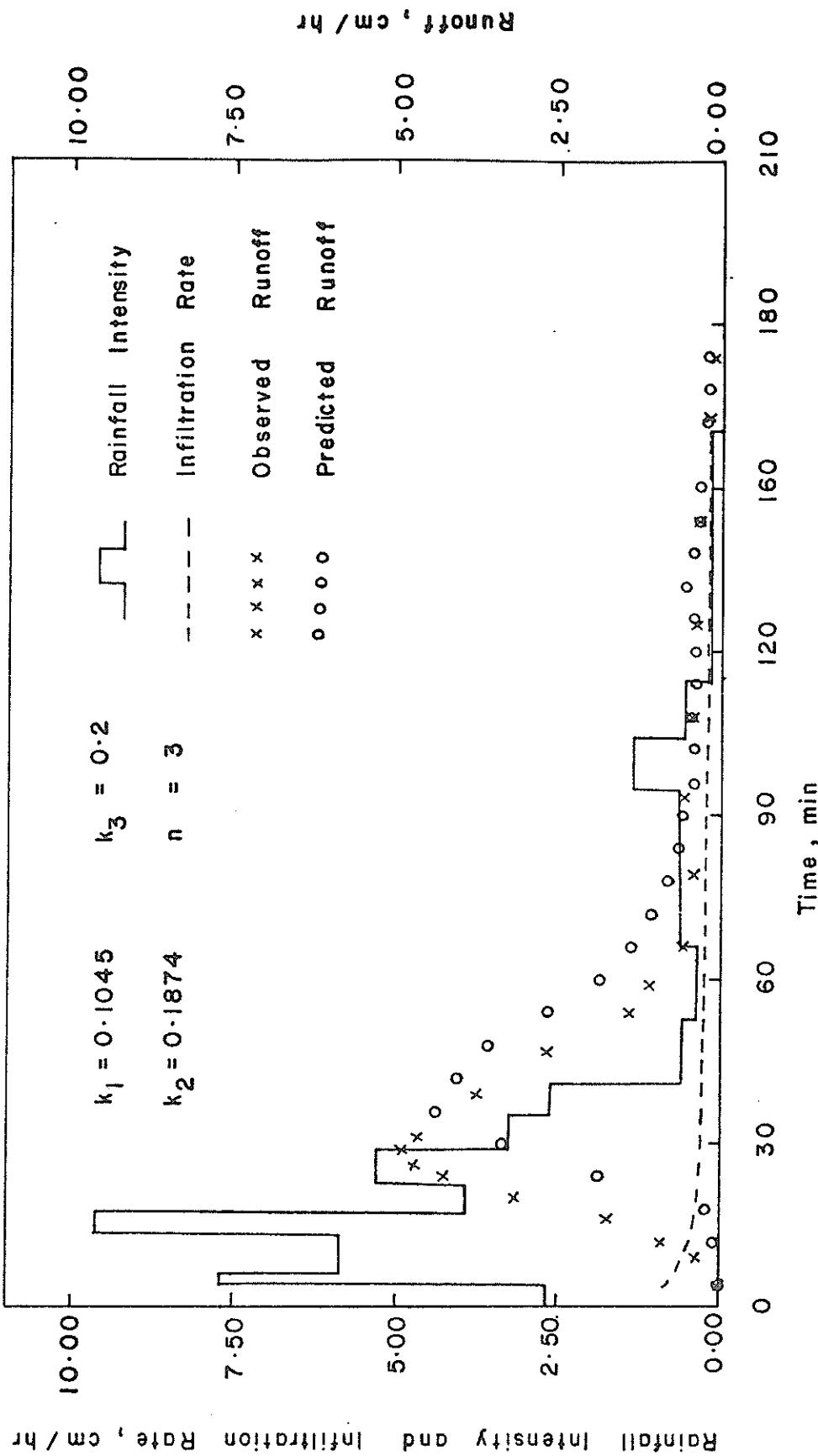
Fig. 3.12. Hydrograph prediction for rainfall event of 5-13-57 on water-
shed Y-10, Riesel (Waco), Texas.
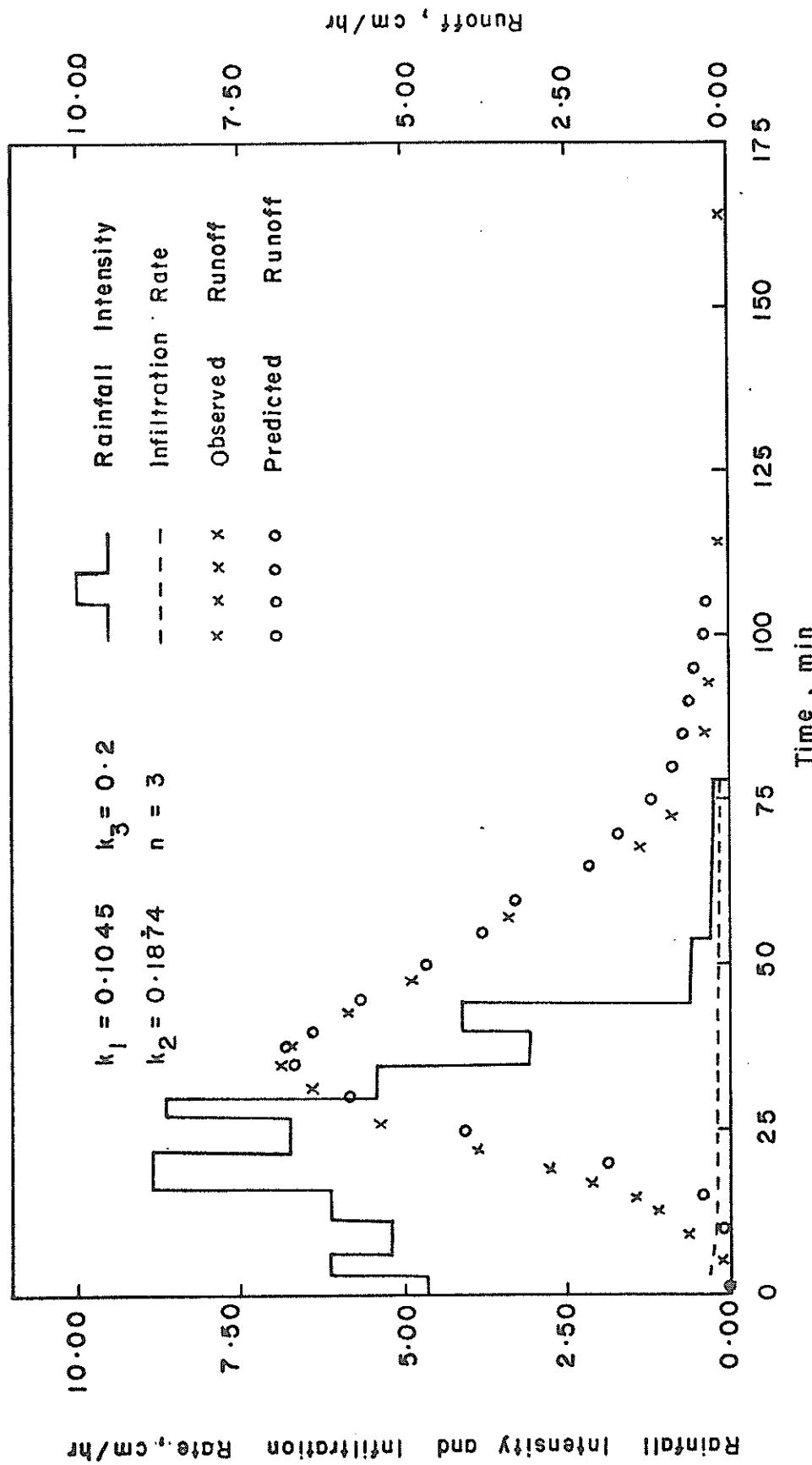
Fig. 3.13. Hydrograph prediction for rainfall event of 4-24-57 on watershed Y-10, Riesel (Waco), Texas.
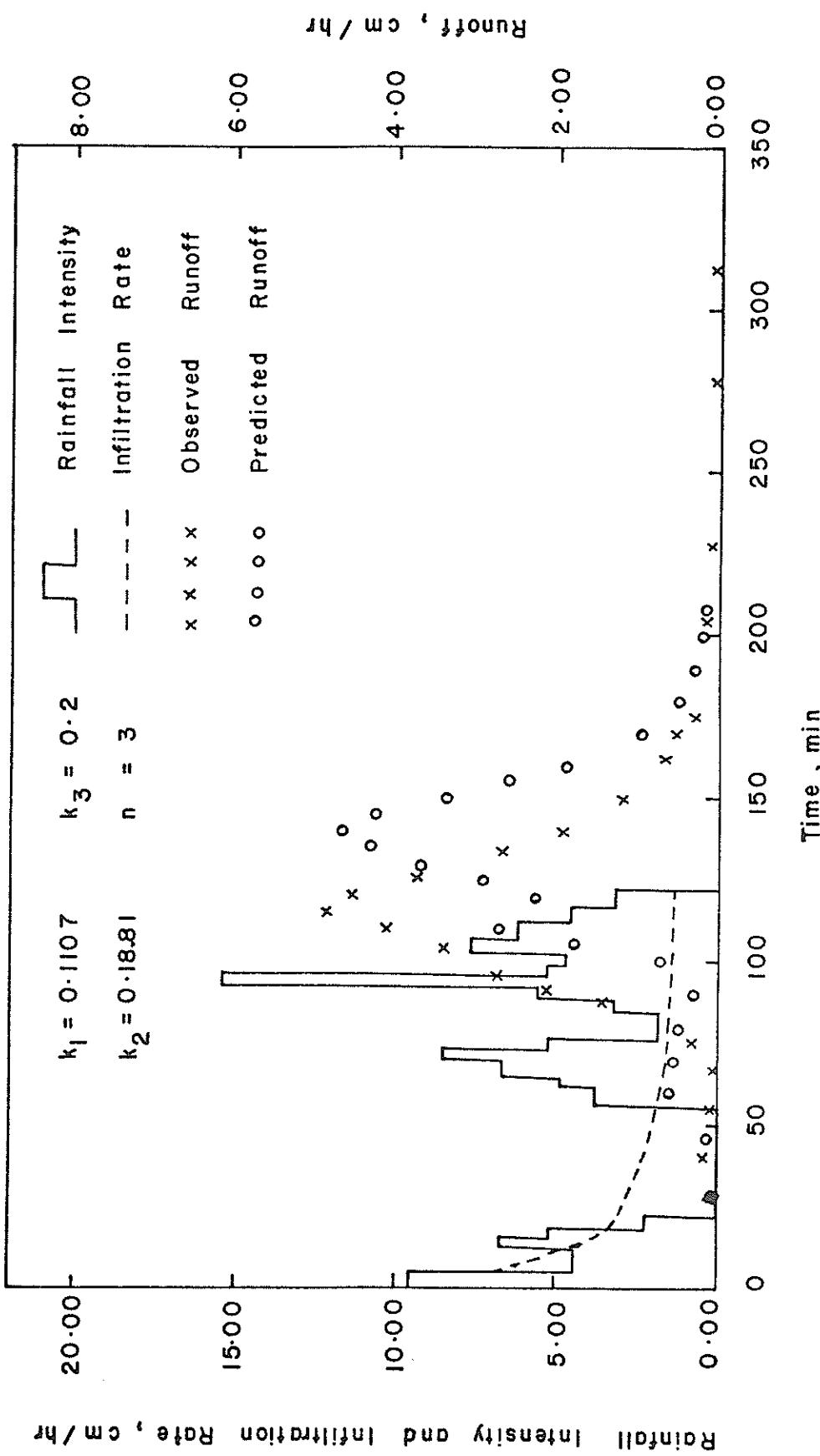
Fig. 3.14. Hydrograph prediction for rainfall event of 6-23-59 on watershed W-1, Riesel (Waco), Texas.
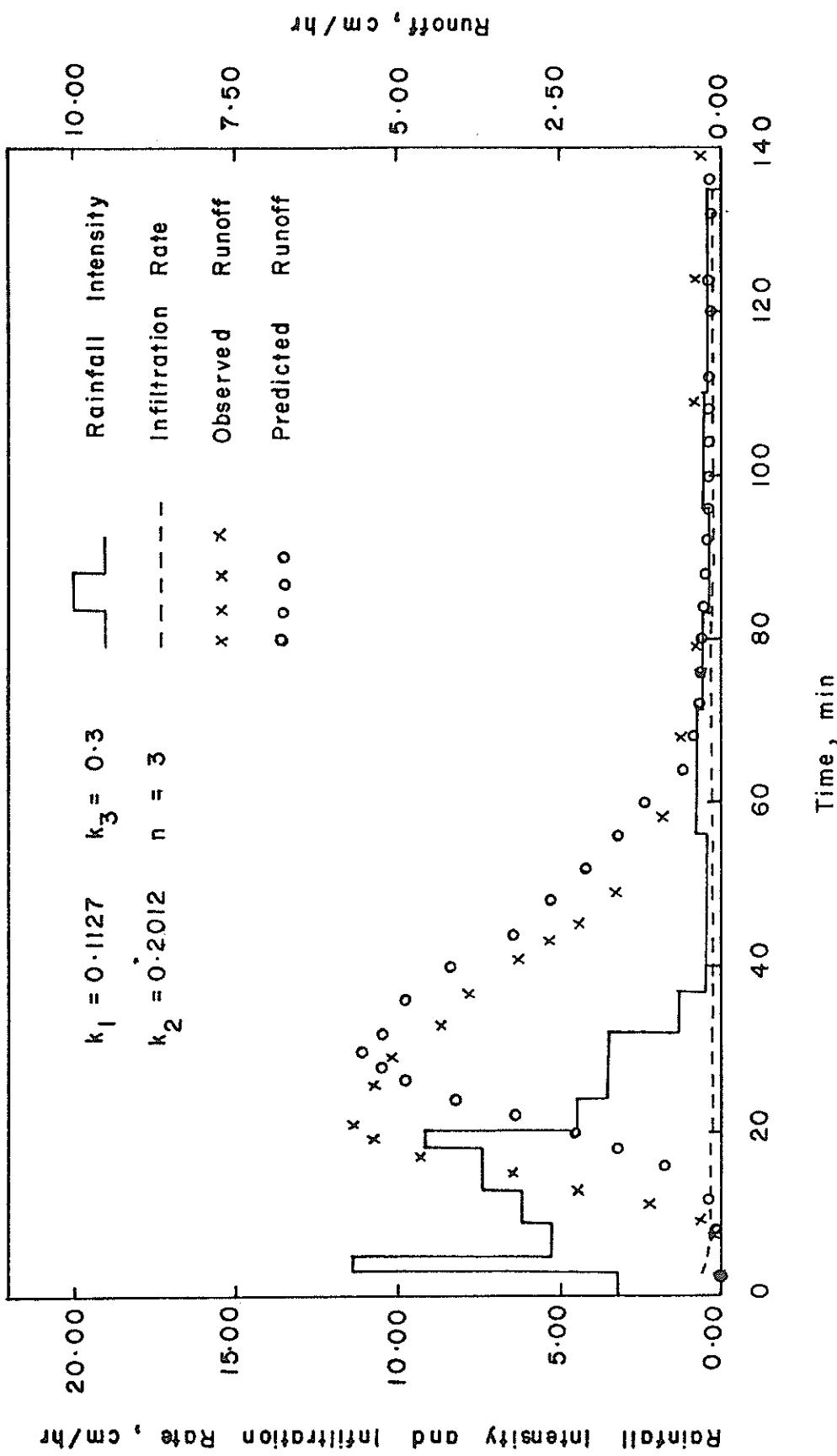
Fig. 3.15. Hydrograph prediction for rainfall event of 5-13-57 on watershed Y-8, Riesel (Waco), Texas.
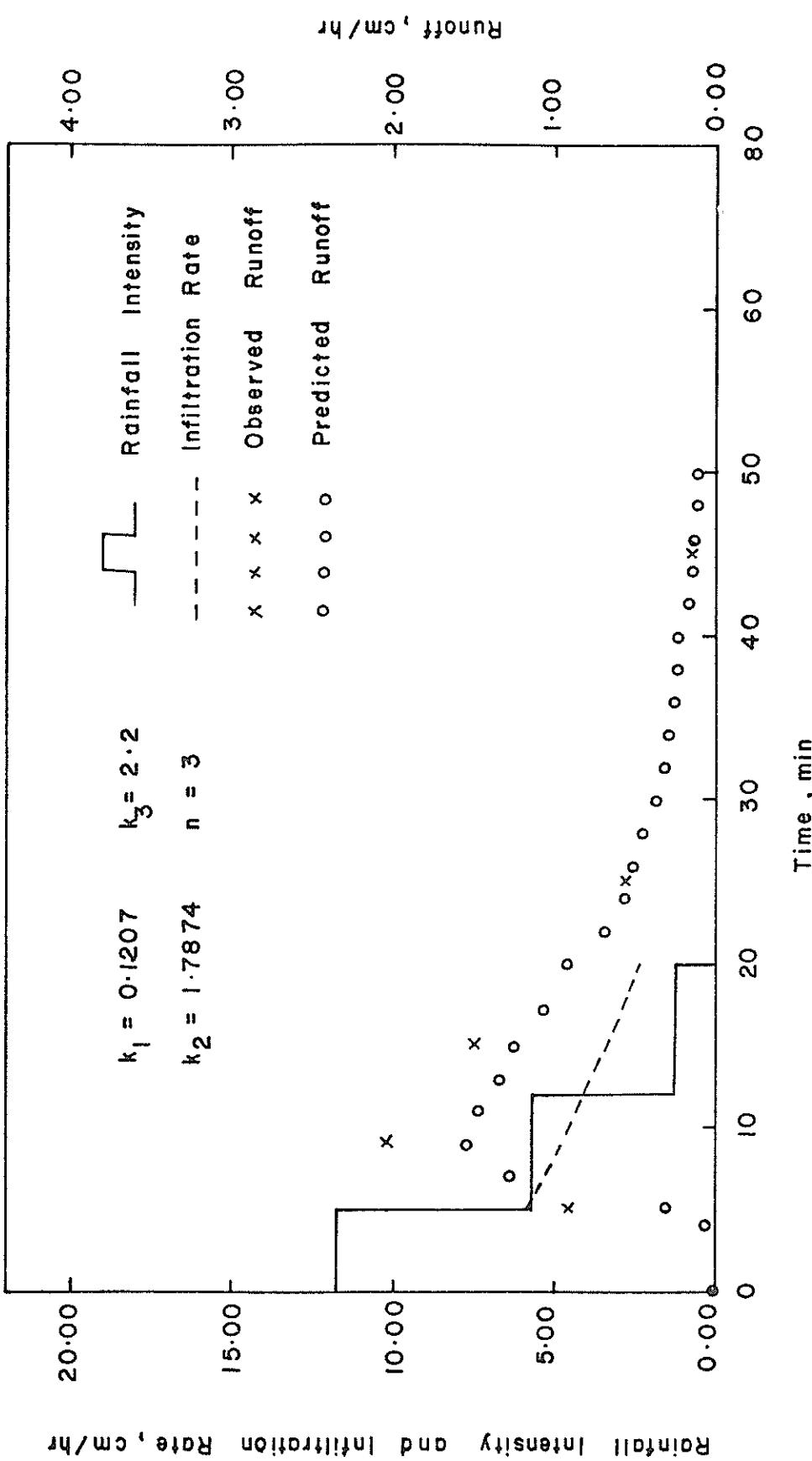
Fig. 3.16. Hydrograph prediction for rainfall event of 6-29-65 on watershed 2-H, Hastings, Nebraska.
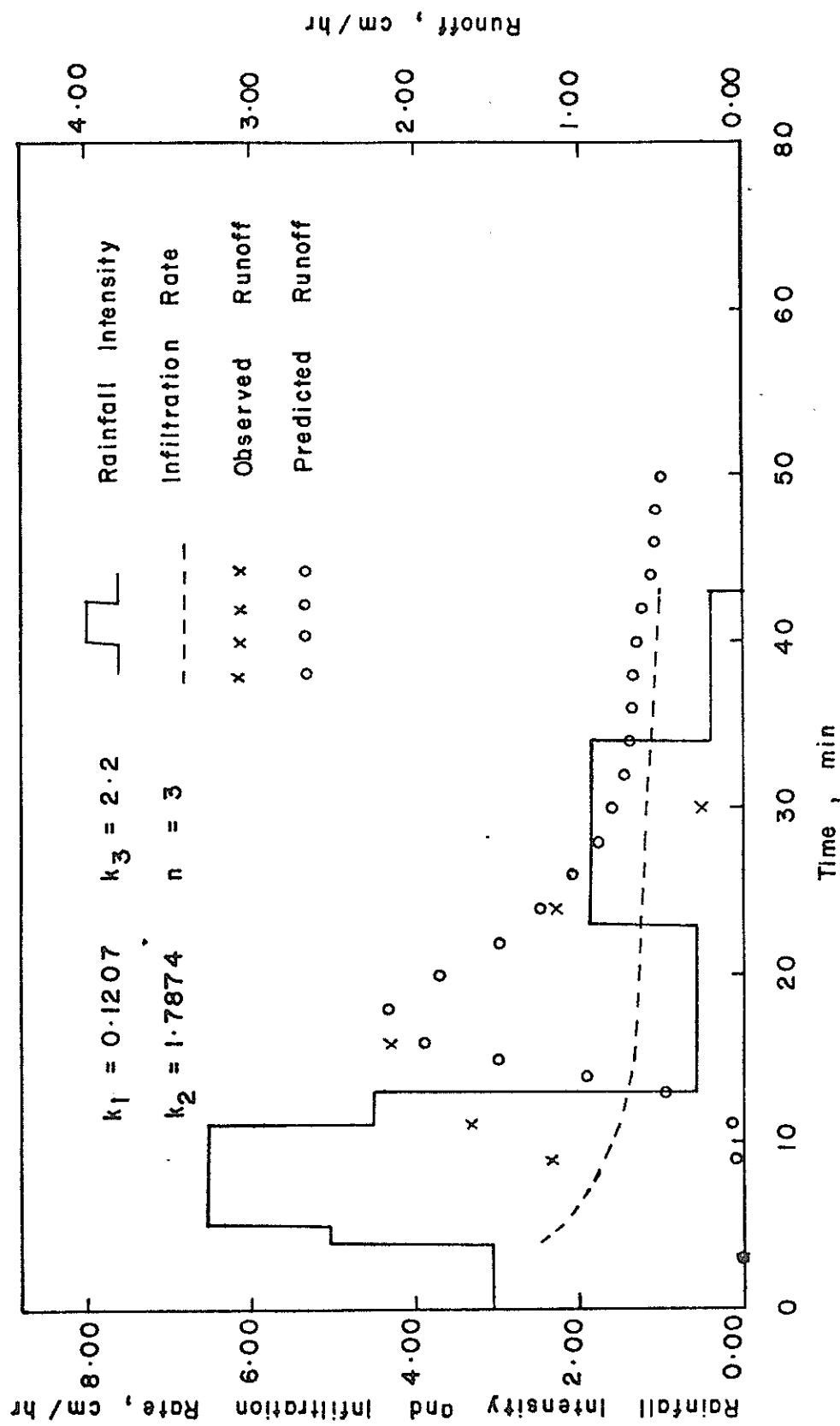
Fig. 3.17. Hydrograph prediction for rainfall event of 6-12-65 on watershed 2-H, Hastings, Nebraska.
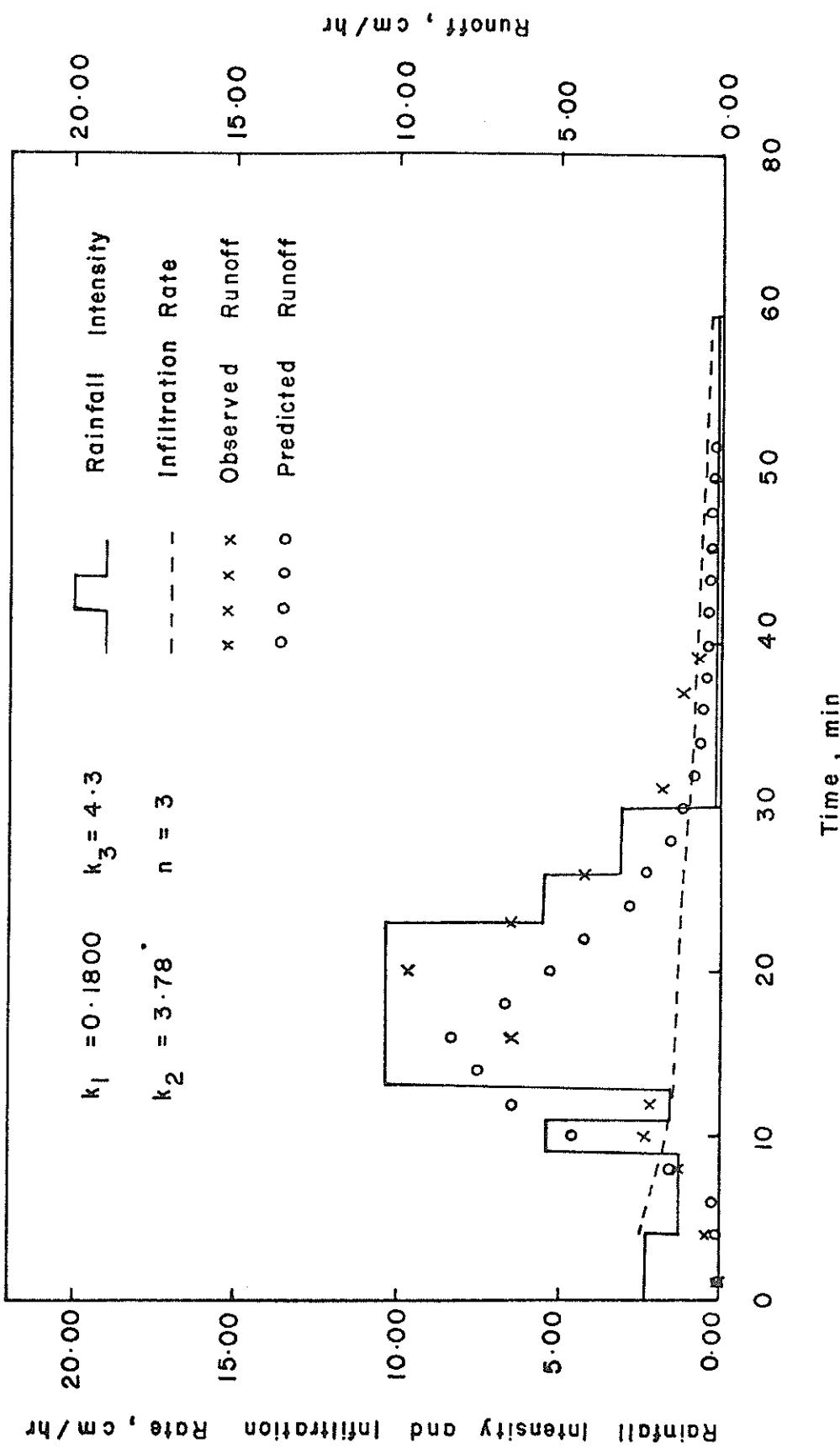
Fig. 3.18. Hydrograph prediction for rainfall event of 6-12-65 on
watershed 4-H, Hastings, Nebraska.
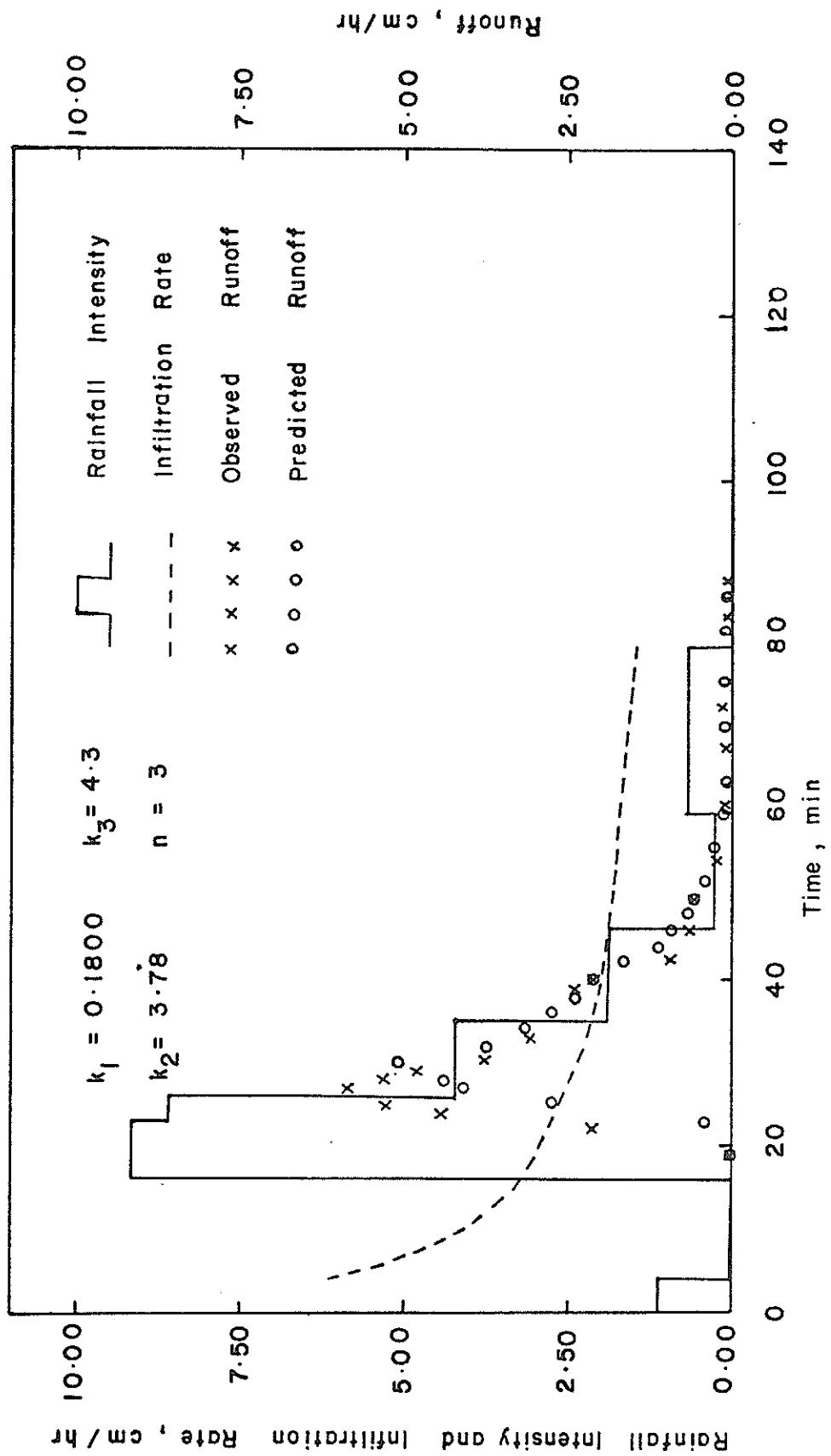
Fig. 3.19. Hydrograph prediction for rainfall event of 6-20-42 on watershed 4-H, Hastings, Nebraska.
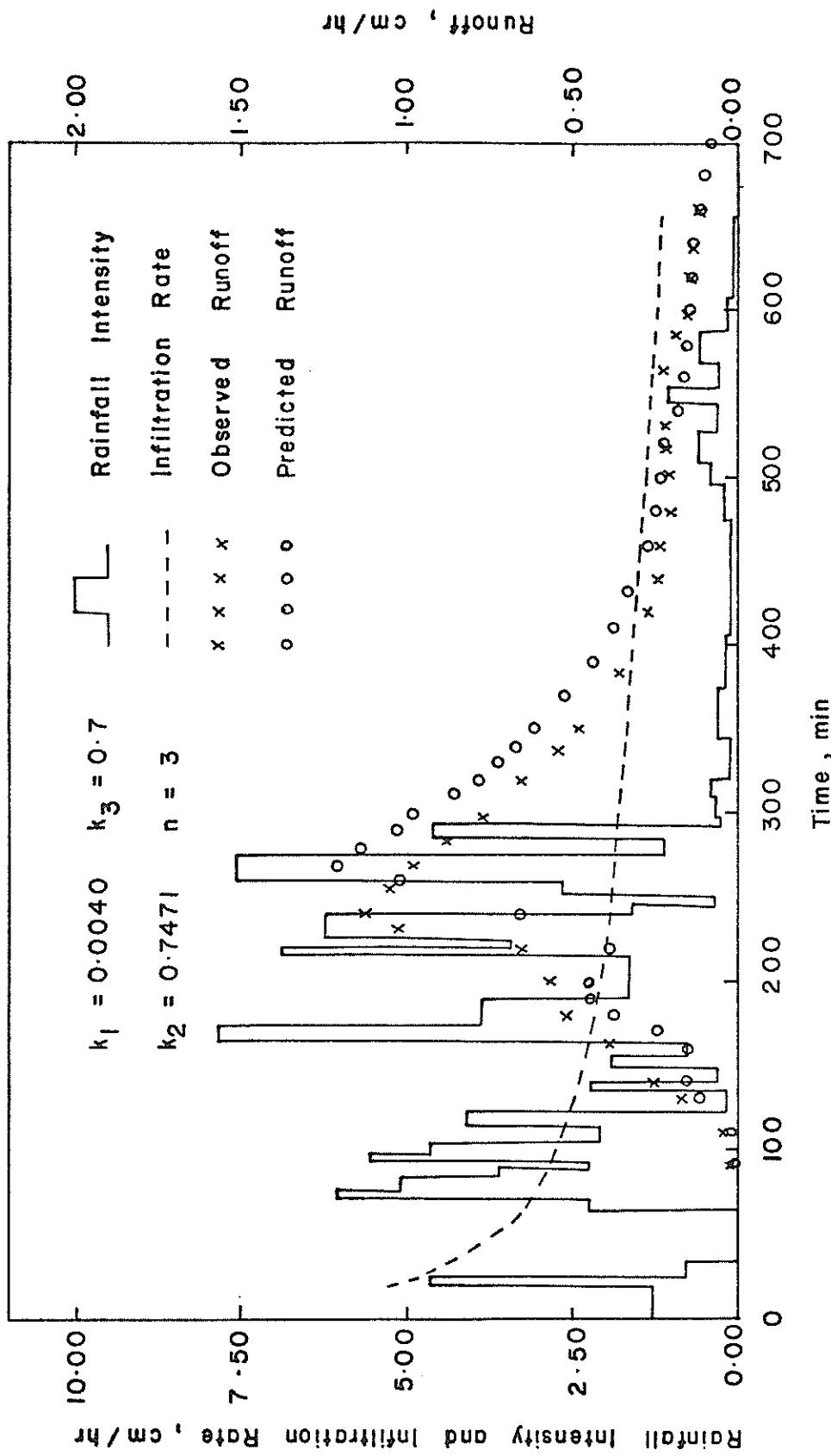
Fig. 3.20. Hydrograph prediction for rainfall event of 6-1-51 on water-shed W-8, Hastings, Nebraska.
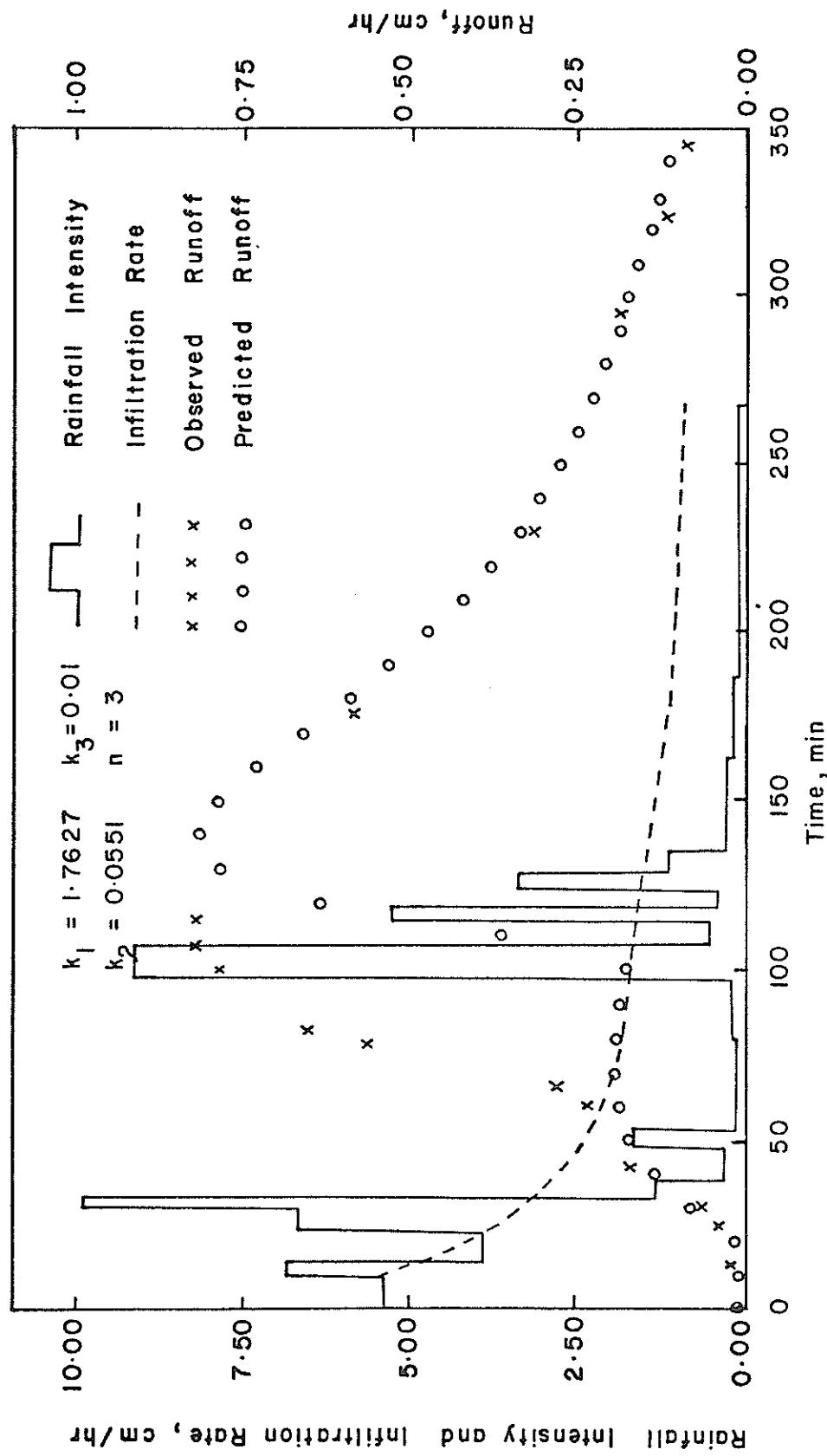
Fig. 3.21. Hydrograph prediction for rainfall event of 9-23-45 on watershed 97, Coshocton, Ohio.
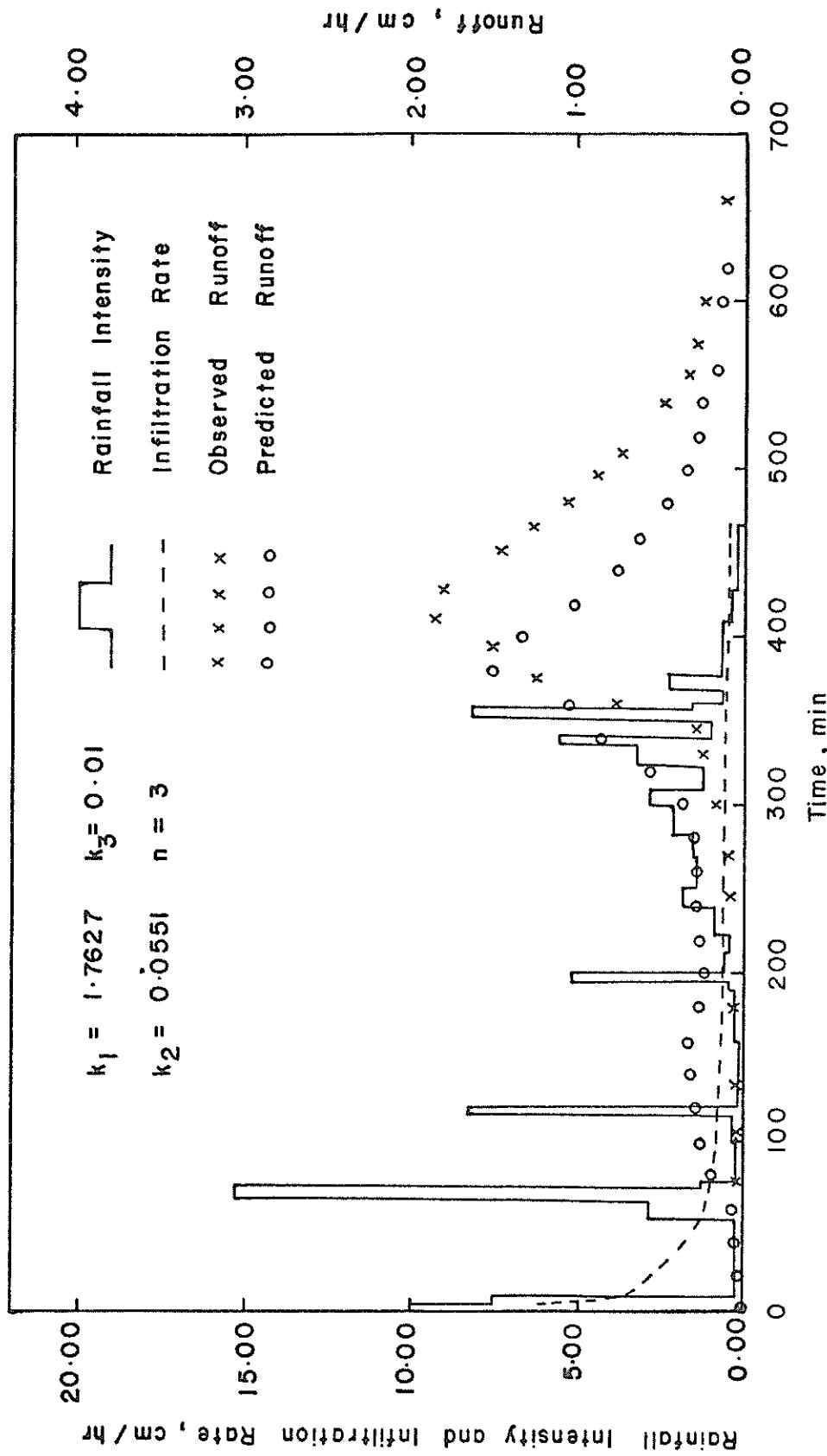
Fig. 3.22. Hydrograph prediction for rainfall event of 6-28-57 on watershed 97, Coshocton, Ohio.

CHAPTER 4

PARAMETER ESTIMATION

This chapter explores the possibility of estimating the model
parameters from physically measurable watershed characteristics. A
regression and correlation analysis is performed to examine this pos-
sibility.

4.1 Watershed Characteristics

The topographic characteristics of the watersheds, used in this
study, included average watershed slope, slope of mainstream, watershed
area, width of watershed, length of mainstream, drainage density, shape
factor and stream order. These watershed characteristics were consid-
ered to affect runoff hydrograph (Sherman, 1932; Gray, 1961; Black, 1975).
The topographic characteristics of all 38 watersheds and the optimized
parameters are given in Table 4.1.

4.1.1 Average Watershed Slope (SLOPE)

An average watershed slope was obtained by weighting each slope
with respect to the proportion of the watershed area comprising this
slope. Average watershed slope for each watershed was obtained directly
from USDA hydrologic data publications.

4.1.2 Average Mainstream Slope (CSLOPE)

The slope of main stream was taken as the slope of the principal
drainage channel, and was obtained by dividing the difference in eleva-
tions of its upstream and downstream points by its length.

4.1.3 Watershed Area (AREA), Width (WIDTH) and Length of Mainstream (XLR)

Watershed area and its width were obtained directly from the USDA
hydrologic data publications. The length of mainstream was defined as
the distance from the most remote point on the watershed to the stream
gauging station.

Table 4.1. Watershed characteristics and optimized parameters.

| Location of Watershed | Area (Hectares) | Width (Km) | Length of main-stream (Km) | Slope (%) | Shape | Drainage Density ($m^{-1}$) | Stream Order | Slope of main-stream (%) | $k_1$ | $k_2$ | $k_3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Coshocton, Ohio** | | | | | | | | | | | |
| 5 | 141.24 | 0.671 | 2.415 | 15.505 | 3.2431 | 0.0063 | 3 | 1.770 | 3.77 | 7.14 | 0.02 |
| 92 | 372.32 | 0.518 | 2.898 | 15.400 | 1.7716 | 0.0007 | 3 | 2.368 | 5.6 | 0.19 | 0.02 |
| 95 | 104.01 | 3.460 | 5.635 | 16.890 | 2.3973 | 0.0018 | 3 | 1.353 | 0.0271 | 0.3119 | 0.03 |
| 97 | 1,853.53 | 3.892 | 9.016 | 17.210 | 3.4444 | 0.0010 | 3 | 1.014 | 1.7627 | 0.0551 | 0.01 |
| 177 | 30.59 | 0.465 | 0.854 | 15.325 | 1.8707 | 0.0031 | 2 | 5.357 | 8.7690 | 0.1205 | 0.04 |
| **Fennimore, Wisconsin** | | | | | | | | | | | |
| W-1 | 133.55 | 0.914 | 1.768 | 5.975 | 1.8389 | 0.0022 | 2 | 1.724 | 0.0772 | 2.9436 | 0.2 |
| **Hastings, Nebraska** | | | | | | | | | | | |
| 2-H | 1.38 | 0.076 | 0.189 | 6.135 | 2.0395 | 0.0078 | 2 | 2.581 | 0.1207 | 1.7874 | 2.2 |
| 4-H | 1.47 | 0.107 | 0.162 | 5.960 | 1.3921 | 0.0078 | 1 | 4.528 | 0.18 | 3.78 | 4.3 |
| W-3 | 194.66 | 1.207 | 2.720 | 5.305 | 2.9861 | 0.0039 | 3 | 0.430 | 0.0242 | 0.5955 | 0.5 |
| W-8 | 844.20 | 1.811 | 7.953 | 5.500 | 5.8850 | 0.0032 | 4 | 0.430 | 0.004 | 0.7471 | 0.7 |
| W-11 | 1,412.40 | 2.012 | 11.673 | 5.576 | 7.5793 | 0.0032 | 4 | 0.430 | 0.0029 | 0.5687 | 0.02 |
| **McCredie, Missouri** | | | | | | | | | | | |
| W-1 | 62.32 | 0.415 | 1.036 | 3.420 | 1.3541 | 0.0028 | 3 | 0.176 | 0.0248 | 2.9309 | 0.1 |
| **Oxford, Mississippi** | | | | | | | | | | | |
| W-5 | 457.31 | 2.347 | 2.415 | 7.720 | 1.0013 | 0.0009 | 1 | 0.630 | 0.0148 | 0.6162 | 0.1 |
| W-10 | 2,238.00 | 4.829 | 6.440 | 10.140 | 1.4555 | 0.0010 | 3 | 0.700 | 0.0141 | 0.5850 | 0.4 |
| W-24 | 206.80 | 1.207 | 2.012 | 12.420 | 1.5382 | 0.0038 | 3 | 3.030 | 0.0068 | 1.0636 | 0.9 |
| W-35 | 3,055.48 | 3.621 | 8.855 | 7.550 | 2.0155 | 0.0014 | 4 | 0.689 | 0.0051 | 0.7313 | 1.1 |
| WC-1 | 1.57 | 0.114 | 0.149 | 7.760 | 1.1163 | 0.0248 | 3 | 6.222 | 0.1510 | 2.0277 | 2.4 |
| WC-2 | 0.59 | 0.084 | 0.092 | 7.130 | 1.1197 | 0.0146 | 1 | 6.667 | 0.2460 | 3.25 | 0.2 |
| WC-3 | 0.65 | 0.061 | 0.107 | 5.930 | 1.3725 | 0.0122 | 1 | 4.286 | 0.1665 | 2.0692 | 2.3 |
| WP-4 | 1.22 | 0.076 | 0.134 | 9.380 | 1.1602 | 0.0138 | 1 | 5.682 | 0.1480 | 2.6946 | 1.2 |

Table 4.1. (continued)

| Location of Watershed | Area (Hectares) | Width (Km) | Length of main-stream (Km) | Slope (%) | Shape | Drainage Density ($m^{-1}$) | Stream Order | Slope of main-stream (%) | Model Parameters | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | $k_1$ | $k_2$ | $k_3$ |
| Riesel(WACO), Texas | | | | | | | | | | | |
| C | 234.32 | 1.402 | 2.366 | 2.040 | 1.8761 | 0.0023 | 3 | 0.570 | 0.01 | 0.1835 | 0.1 |
| D | 449.22 | 1.982 | 3.567 | 2.100 | 2.2246 | 0.0022 | 3 | 0.510 | 0.0064 | 0.0507 | 0.1 |
| G | 1,772.59 | 2.592 | 7.829 | 2.055 | 2.7160 | 0.0020 | 4 | 0.389 | 0.002 | 4.6758 | 0.1 |
| W-1 | 71.23 | 0.610 | 1.646 | 2.180 | 2.9887 | 0.0029 | 2 | 0.833 | 0.1107 | 0.1881 | 0.2 |
| W-2 | 52.61 | 0.823 | 0.945 | 2.550 | 1.3335 | 0.0034 | 2 | 1.290 | 0.0427 | 0.2184 | 0.1 |
| W-6 | 17.12 | 0.457 | 0.445 | 2.020 | 0.9090 | 0.0129 | 2 | 1.370 | 0.0452 | 1.1088 | 0.1 |
| W-10 | 7.97 | 0.305 | 0.323 | 1.620 | 1.0289 | 0.0041 | 1 | 1.500 | 0.0508 | 2.4812 | 0.3 |
| Y | 125.05 | 0.915 | 1.537 | 2.405 | 1.4830 | 0.0026 | 2 | 0.990 | 0.0192 | 0.8077 | 0.6 |
| Y-2 | 53.42 | 0.854 | 1.000 | 2.585 | 1.4702 | 0.0027 | 2 | 1.220 | 0.0196 | 4.7117 | 5.4 |
| Y-4 | 32.34 | 0.595 | 0.610 | 2.850 | 0.9031 | 0.0011 | 2 | 1.750 | 0.0192 | 3.5646 | 0.4 |
| Y-6 | 8.46 | 0.259 | 0.338 | 3.225 | 1.0634 | 0.0015 | 1 | 1.804 | 0.0150 | 6.0053 | 5.3 |
| Y-7 | 16.19 | 0.381 | 0.543 | 1.865 | 1.4289 | 0.0017 | 1 | 0.840 | 0.0384 | 4.0105 | 0.9 |
| Y-8 | 8.42 | 0.183 | 0.244 | 1.945 | 0.5550 | 0.0016 | 1 | 1.100 | 0.1127 | 0.2012 | 0.3 |
| Y-10 | 8.50 | 0.381 | 0.338 | 2.375 | 1.0584 | 0.0038 | 1 | 1.300 | 0.1045 | 0.1874 | 0.2 |
| SW-12 | 1.20 | 0.119 | 0.116 | 3.950 | 0.8770 | 0.0171 | 1 | 2.894 | 0.2151 | 2.8752 | 0.2 |
| SW-17 | 1.21 | 0.122 | 0.116 | 1.830 | 0.8712 | 0.0066 | 1 | 1.579 | 0.0753 | 2.9470 | 0.2 |
| Ralston Creek, Iowa | 781.07 | 1.463 | 5.796 | 10.250 | 3.3780 | 0.0085 | 4 | 0.550 | 0.0088 | 0.3365 | 0.1 |
| Watkinsville, Georgia | 7.77 | 0.274 | 0.381 | 10.625 | 1.4763 | 0.0114 | 2 | 4.161 | 0.0777 | 3.5816 | 0.2 |

Watershed Characteristics

4.1.4 Drainage Density (DD)

Drainage density was defined by the accumulative length of streams divided by the total drainage area:

$$DD = \frac{\Sigma L}{A}$$ (4-1)

where DD = drainage density;

L = length of a stream; and

A = area of drainage basin.

4.1.5 Shape Factor (SHAPE)

The shape factor of Chorley, Malm and Pagorzelski (1957) was used in this study. It can be defined as:

$$SHAPE = \frac{\pi L^2}{4A}$$ (4-2)

where L = length of the mainstream; and

A = area of the watershed.

Shape factor is a dimensionless parameter.

4.1.6 Stream Order (SO)

The stream order developed by Strahler (1957) was applied in this study. Strahler suggested that the channel network map includes all intermittent and permanent flow lines located in clearly defined valleys. The smallest finger-tip tributaries are designated order 1. Where two channels of order 1 join, a channel segment of order 2 is formed; and so on. The mainstream, through which all discharge passes, has therefore the highest order.

4.2 Correlation of Model Parameters

To correlate parameters $k_1$, $k_2$, and $k_3$ with topographic characteristics of the watersheds, the UCLA Biomedical Statistical Package (BMD 02R) was used to perform regression and correlation analysis. First, a multiple linear regression was performed to correlate optimized

$k_1$, a dependent variable, with topographic characteristics, independent variables. The correlation coefficient was 0.7742 and standard error of estimate 1.2496. The regression equation for $k_1$ can be represented as:

$$k_1 = -0.30127 + 0.00113*AREA - 0.98367*WIDTH - 0.2448*XLR + 0.26045*$$
$$SLOPE + 0.17566*SHAPE - 171.4197*DD + 0.28804*SO + 0.19733*$$
$$CSLOPE \hspace{4cm} (4-3)$$

To check further the reliability of this equation the optimized $k_1$ was plotted against $k_1$ estimated from Eq. (4-3) as shown in Fig. 4.1. It shows wide scattering of points around the theoretical line of fit and thus a poor relationship between parameter $k_1$ and watershed characteristics.

Then the multiple linear regression was performed to correlate the parameters $k_2$ and $k_3$ with watershed characteristics. The correlation coefficient for $k_2$ was 0.4075 and standard error of estimate 1.8641. The correlation coefficient for $k_3$ was 0.4065 and standard error of estimate 1.4123. The linear regression equations for $k_2$ and $k_3$ can be respectively written as:

$$k_2 = 2.75611 + 0.00111*AREA - 0.73322*WIDTH - 0.19508*XLR + 0.02899*SLOPE$$
$$+ 0.11413*SHAPE - 0.14540*SO - 0.03928*CSLOPE \hspace{1cm} (4-4)$$

$$k_3 = 0.65929 + 0.00063*AREA + 0.17000*WIDTH - 0.30879*XLR$$
$$- 0.08221*SLOPE + 0.32046*SHAPE - 52.04816*DD + 0.35307*CSLOPE \hspace{0.3cm} (4-5)$$

To check further the suitability of the above equations $k_2$ computed from Eq. (4-4) was plotted against optimized $k_2$ as shown in Fig. 4.2. The plot shows considerable scattering of points around the theoretical line of fit and therefore a poor relationship for $k_2$.

Figure 4.3 shows optimized $k_3$ versus $k_3$ computed from Eq. (4-5). It shows considerable scattering of points around the theoretical line of
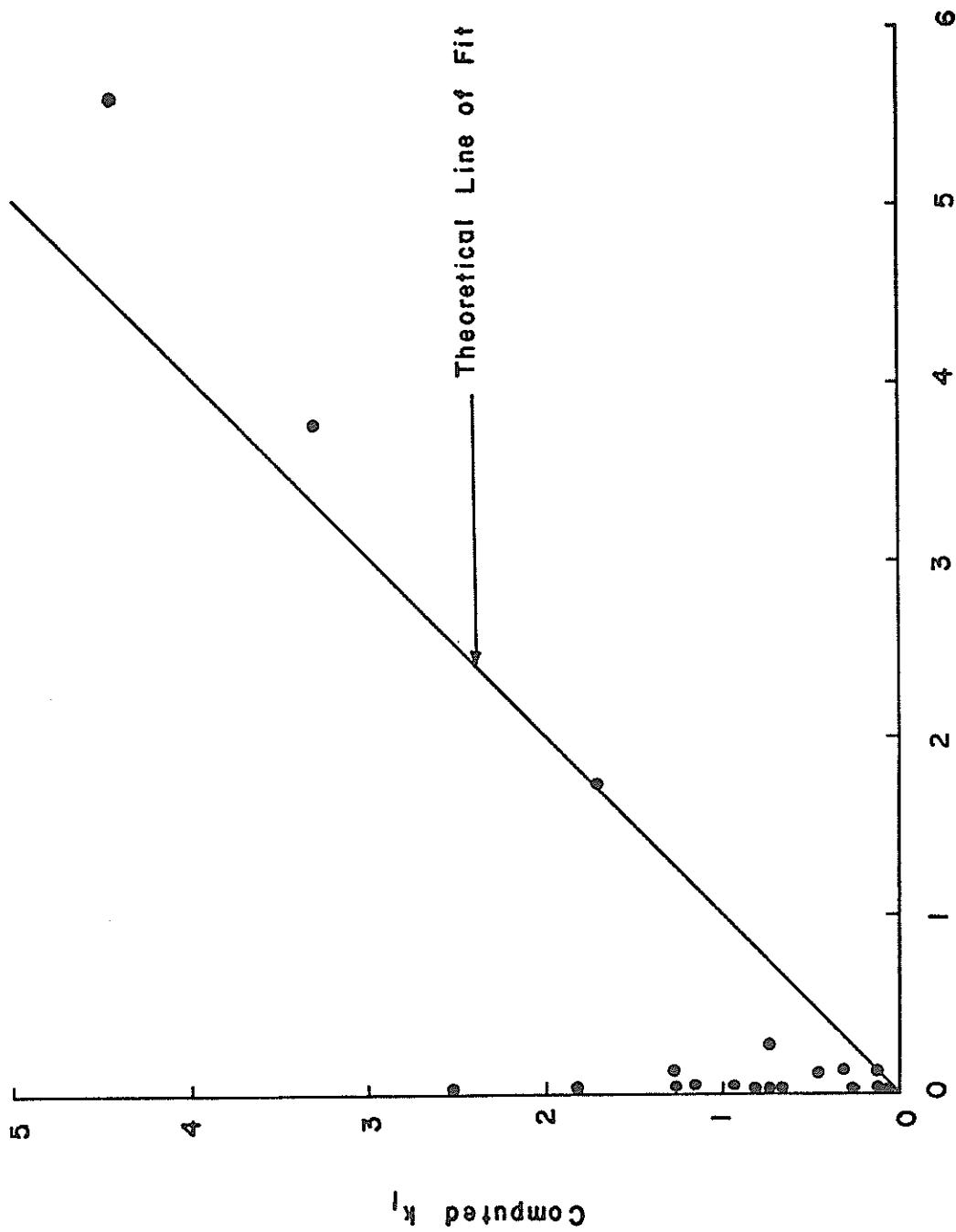
Fig. 4.1. Optimized $k_1$ versus computed $k_1$ from linear regression Eq. (4-3).
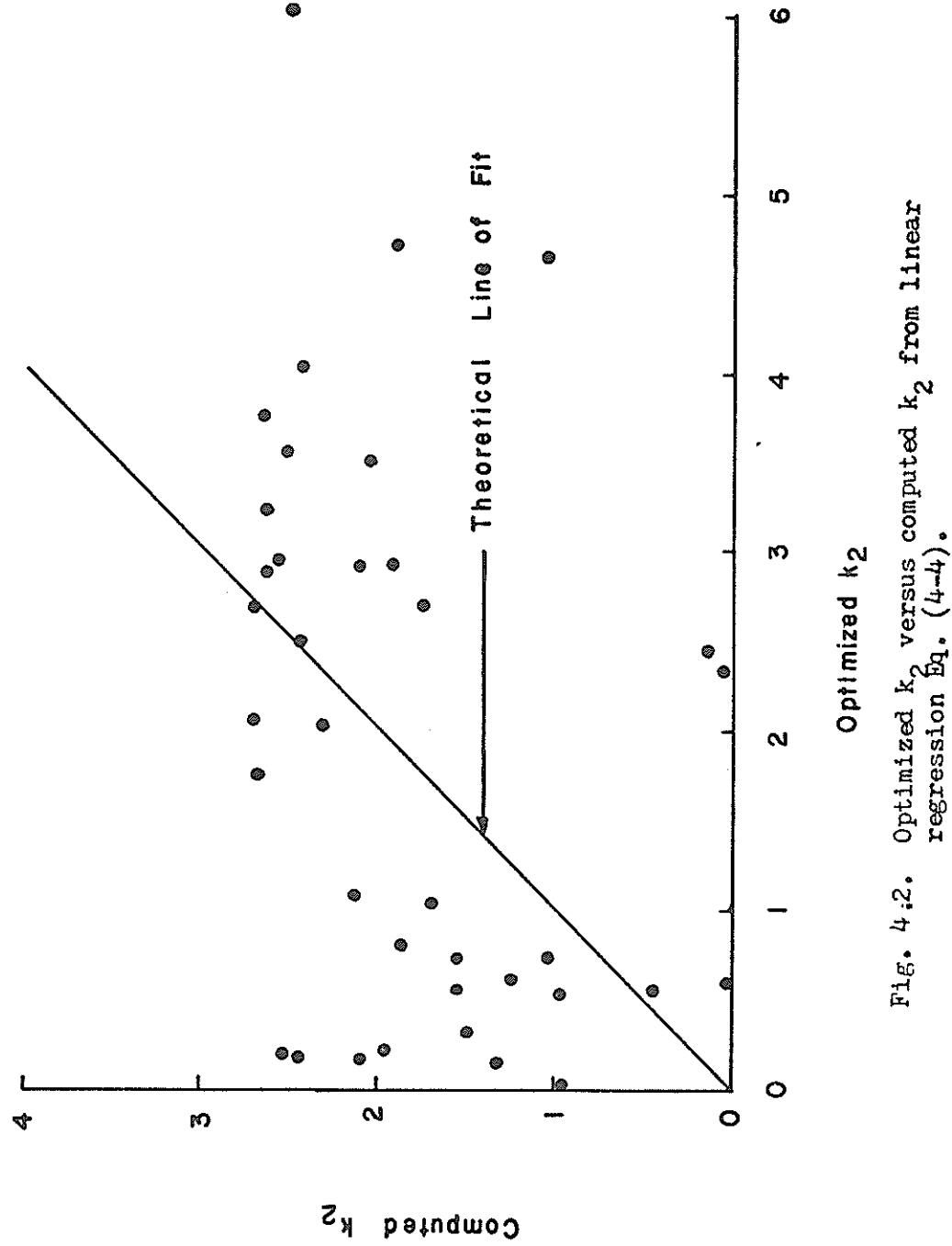
Fig. 4.2.  Optimized $k_2$ versus computed $k_2$ from linear regression Eq. (4-4).
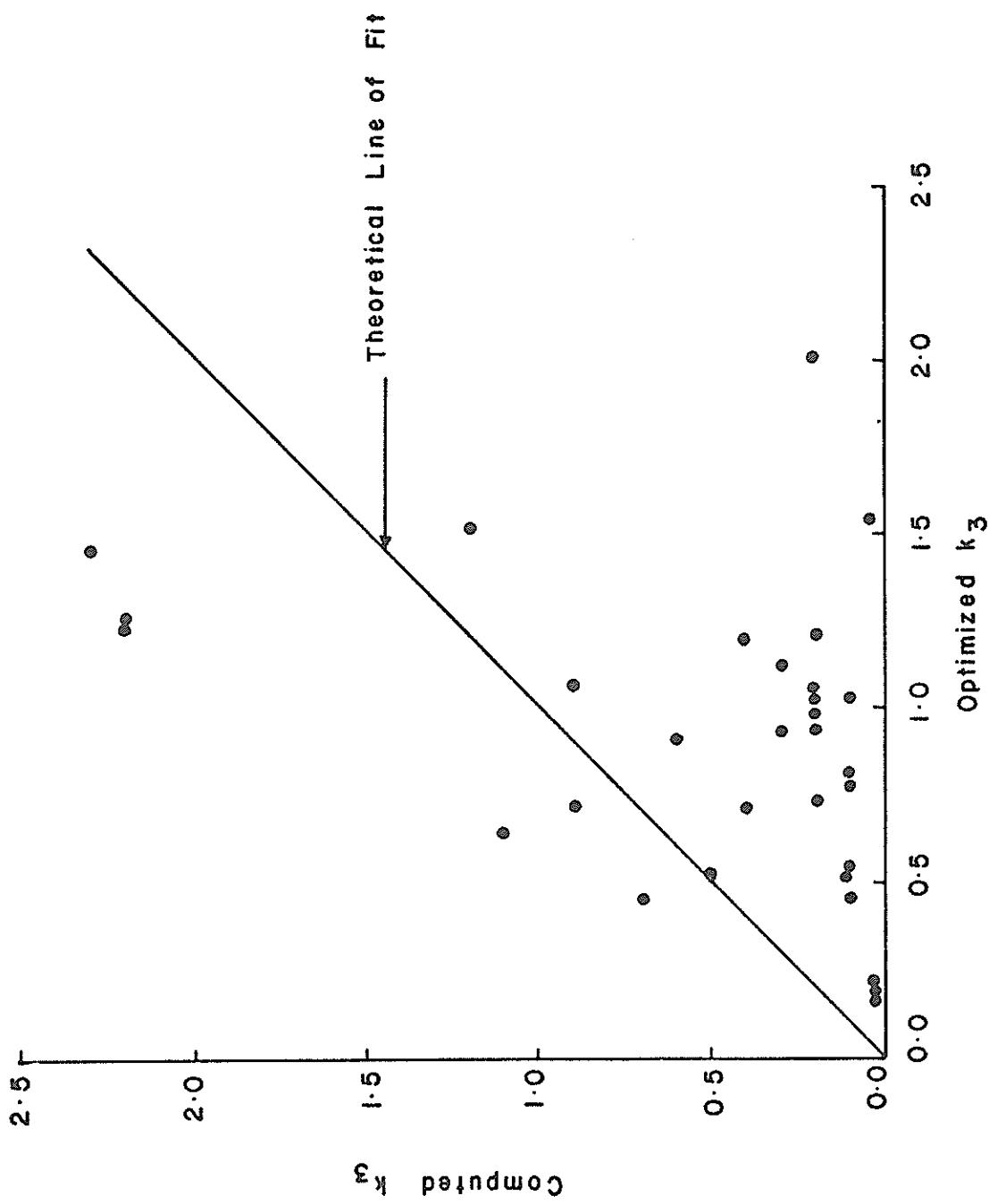
Fig. 4.3. Optimized $k_3$ versus computed $k_3$ from linear regression Eq. (4-5).

fit and therefore a poor relationship between $k_3$ and watershed characteristics.

In order to improve the correlation of model parameters all variables, independent as well as dependent, were transformed logarithmically to the base 10. Then the regression analyses were performed. The correlation coefficient for $k_1$ was 0.7264 and standard error of estimate 0.6699 where the highest correlation coefficient was contributed by stream order, then by length of mainstream, shape factor, area, drainage density, width, slope and channel slope respectively. The regression equation for $k_1$ can be written as:

$$\log k_1 = - 5.14689 + 0.4502 \log AREA - 0.95677 \log WIDTH - 0.80706 \log$$
$$LENGTH + 1.0594 \log SLOPE + 0.77707 \log SHAPE - 0.80188$$
$$\log DD - 0.19171 \log SO + 0.6892 \log CSLOPE \qquad (4-6)$$

To check the improvement in the correlation log of optimized $k_1$ was plotted against log of computed $k_1$ from Eq. (4-6) as shown in Fig. 4.4. This figure shows considerable scattering of points around the theoretical line of fit. Although improved from before, the relationship is still poor.

The correlation coefficient for $k_2$ was 0.4802 and standard error of estimate 0.5533 where the highest correlation was with width and then with drainage density, average channel slope and shape factor respectively. Area, average length of mainstream, average slope of watershed and stream order were deleted in the analysis. The regression equation for $k_2$ can be written as:

$$\log k_2 = 0.65164 - 0.3860 \log WIDTH - 0.23575 \log SHAPE + 0.28078$$
$$\log DD - 0.15567 \log CSLOPE \qquad (4-7)$$

Fig. 4.4: Optimized log $k_1$ versus computed log $k_1$ from log regression Eq. (4-6).

Figure 4.5 shows the plot of optimized log $k_2$ versus log $k_2$ computed from Eq. (4-7) and indicates that the correlation is better but the scattering of points is still wide and therefore the relationship for $k_2$ poor.

For $k_3$ the correlation coefficient of 0.5371 and standard error of estimate of 0.6691 were obtained where the highest correlation was with average channel slope and then with drainage density, width, shape factor, area, stream order, average slope of watershed and length of mainstream respectively. The regression equation for $k_3$ can be written as:

$$\log k_3 = - 2.81893 + 0.61483 \log AREA + 0.83323 \log WIDTH - 2.9685$$
$$\log XLR - 0.2086 \log SLOPE + 1.5370 \log SHAPE - 0.39749$$
$$\log DD + 0.85542 \log SO + 0.06763 \log CSLOPE \qquad . \qquad (4-8)$$

The plot of optimized log $k_3$ versus log $k_3$ computed from Eq. (4-8) is shown in Fig. 4.6. The correlation for $k_3$ now has a small improvement over linear correlation.

After the regression analyses with watershed characteristics correlation between the parameters was considered. The optimized $k_1$ was plotted versus optimized $k_2$ as shown in Fig. 4.7. It shows considerable scattering of points. The linear regression analysis was performed to find correlation coefficient between $k_1$ and $k_2$ where $k_1$ was independent variable and $k_2$ dependent variable. The correlation coefficient was 0.0674 and standard error of estimate 1.8592. This shows that there is practically no correlation between parameters $k_1$ and $k_2$.

The plots of optimized $k_2$ and $k_3$ and optimized $k_3$ and $k_1$ are shown in Fig. 4.8 and 4.9 respectively. Linear regression analysis was then performed between $k_2$ and $k_3$ where $k_2$ was independent variable and $k_3$ dependent variable. The correlation coefficient was 0.4626 and standard
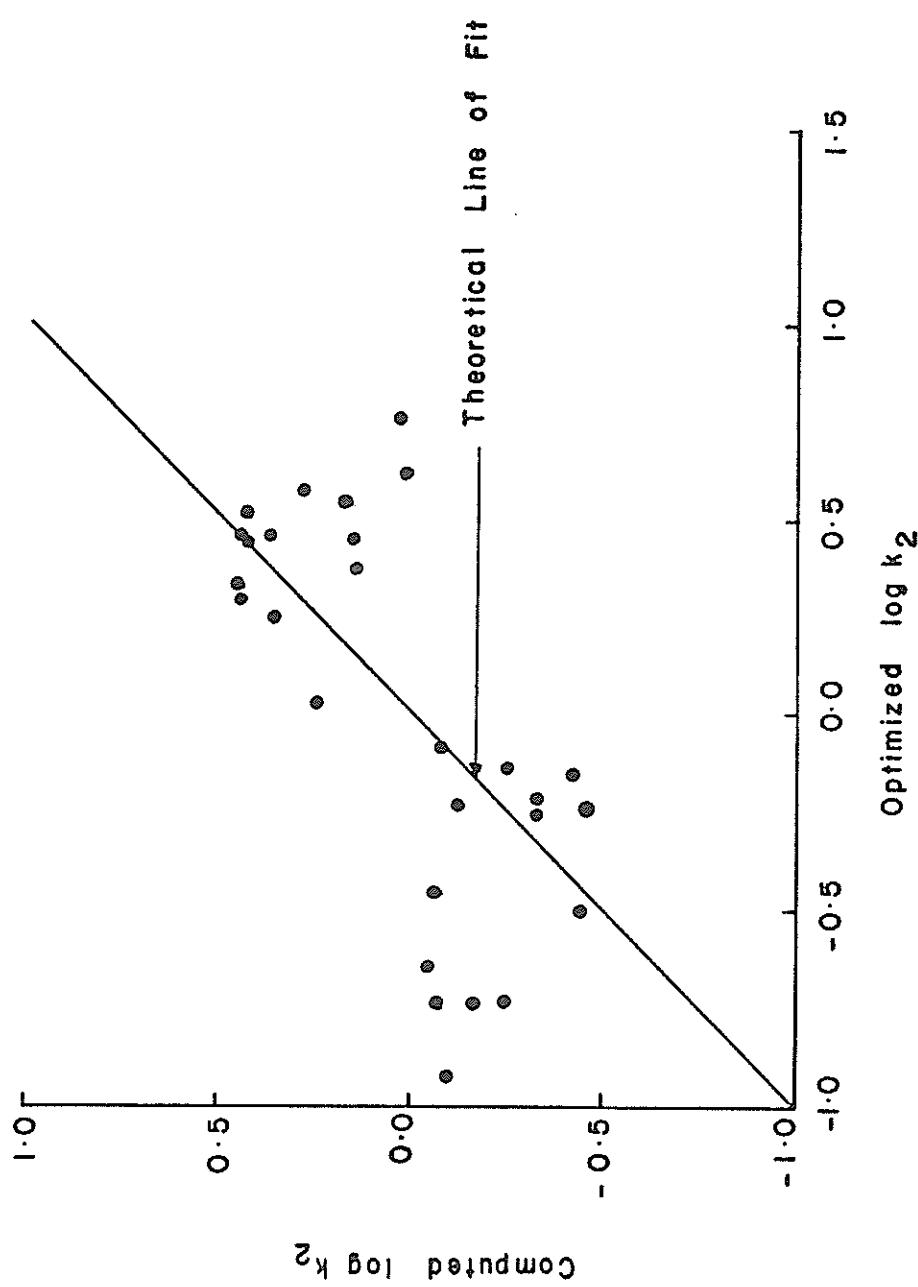
Fig. 4.5. Optimized log $k_2$ versus computed log $k_2$ from log regression Eq. (4-7).
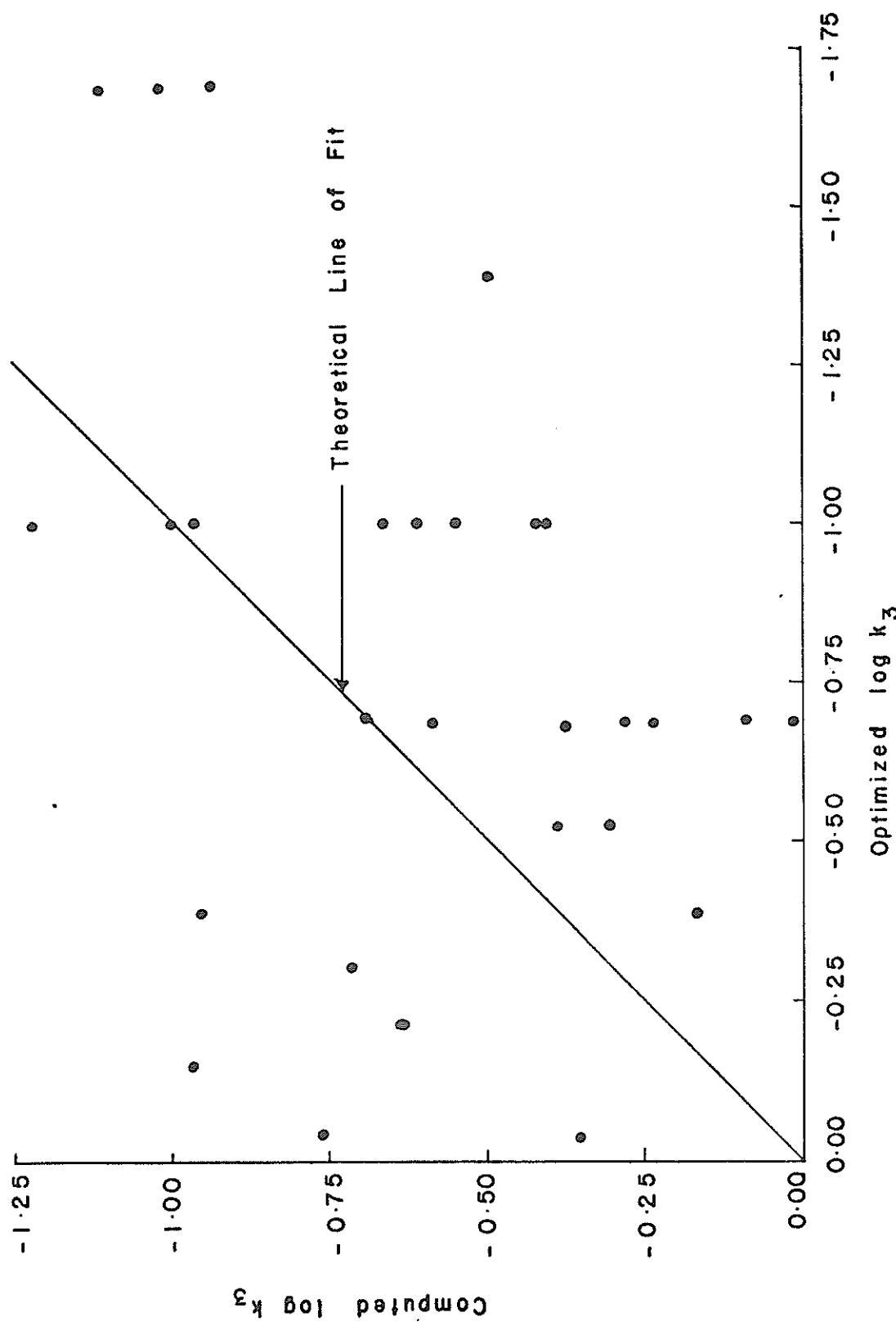
Fig. 4.6. Optimized log $k_3$ versus computed log $k_3$ from log regression Eq. (4-8).
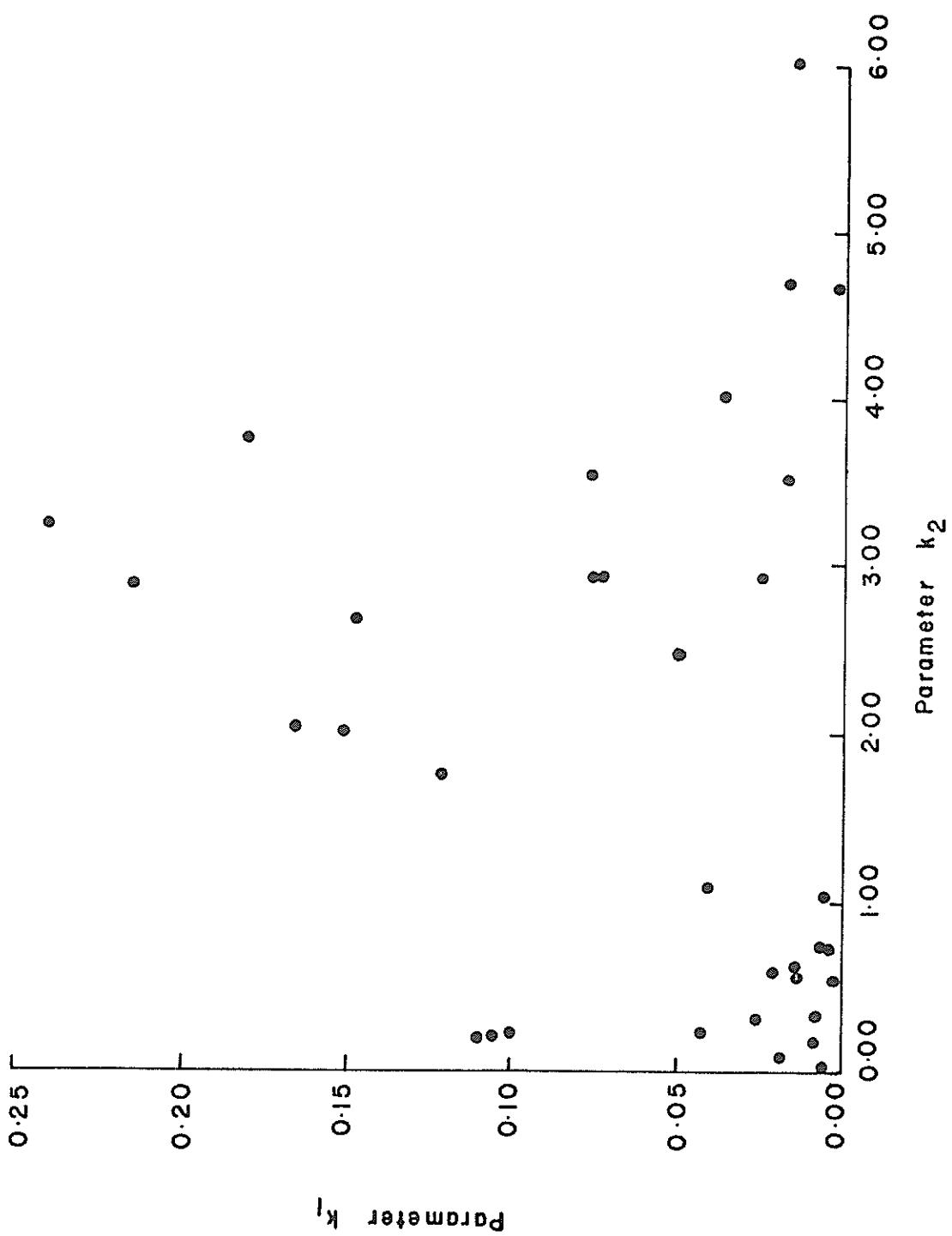
Fig. 4.7. Parameter $k_1$ versus parameter $k_2$.
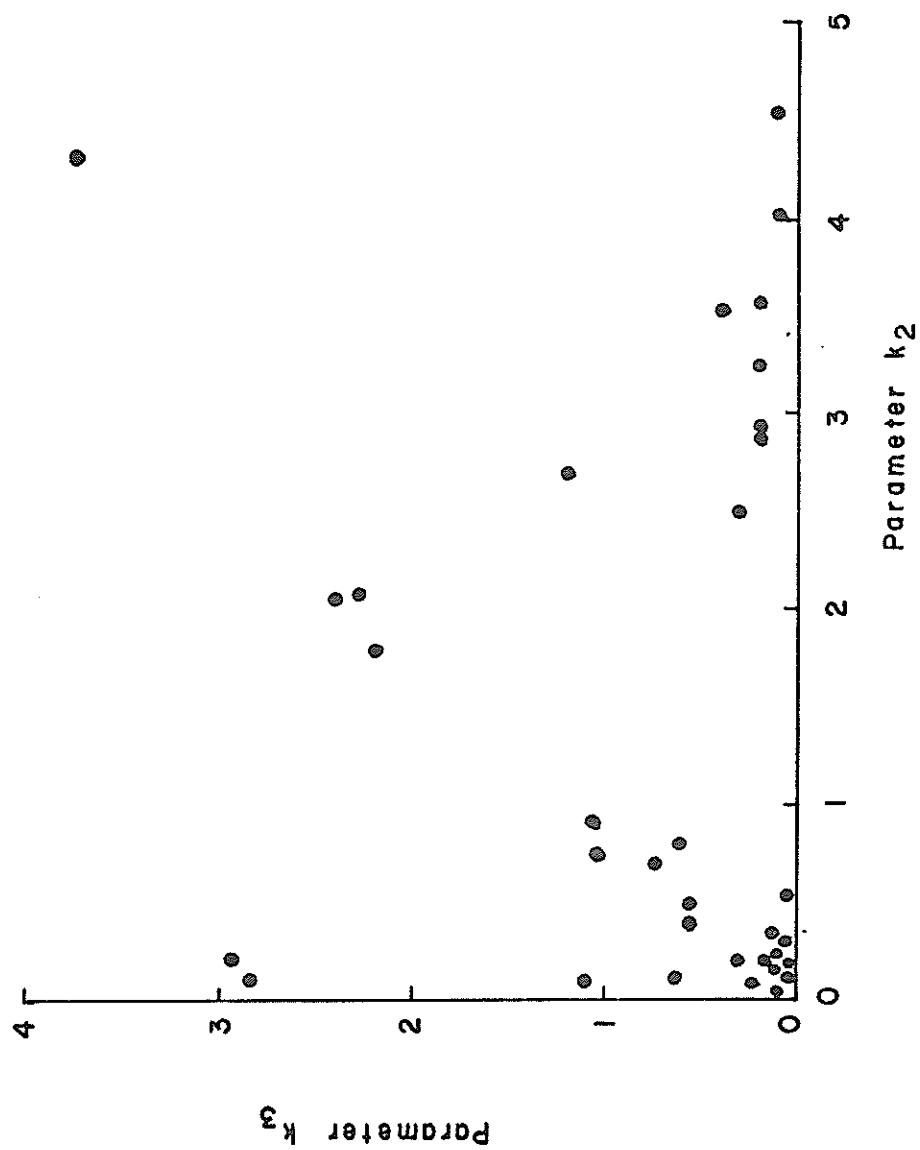
Fig. 4.8. Parameter $k_2$ versus parameter $k_3$.
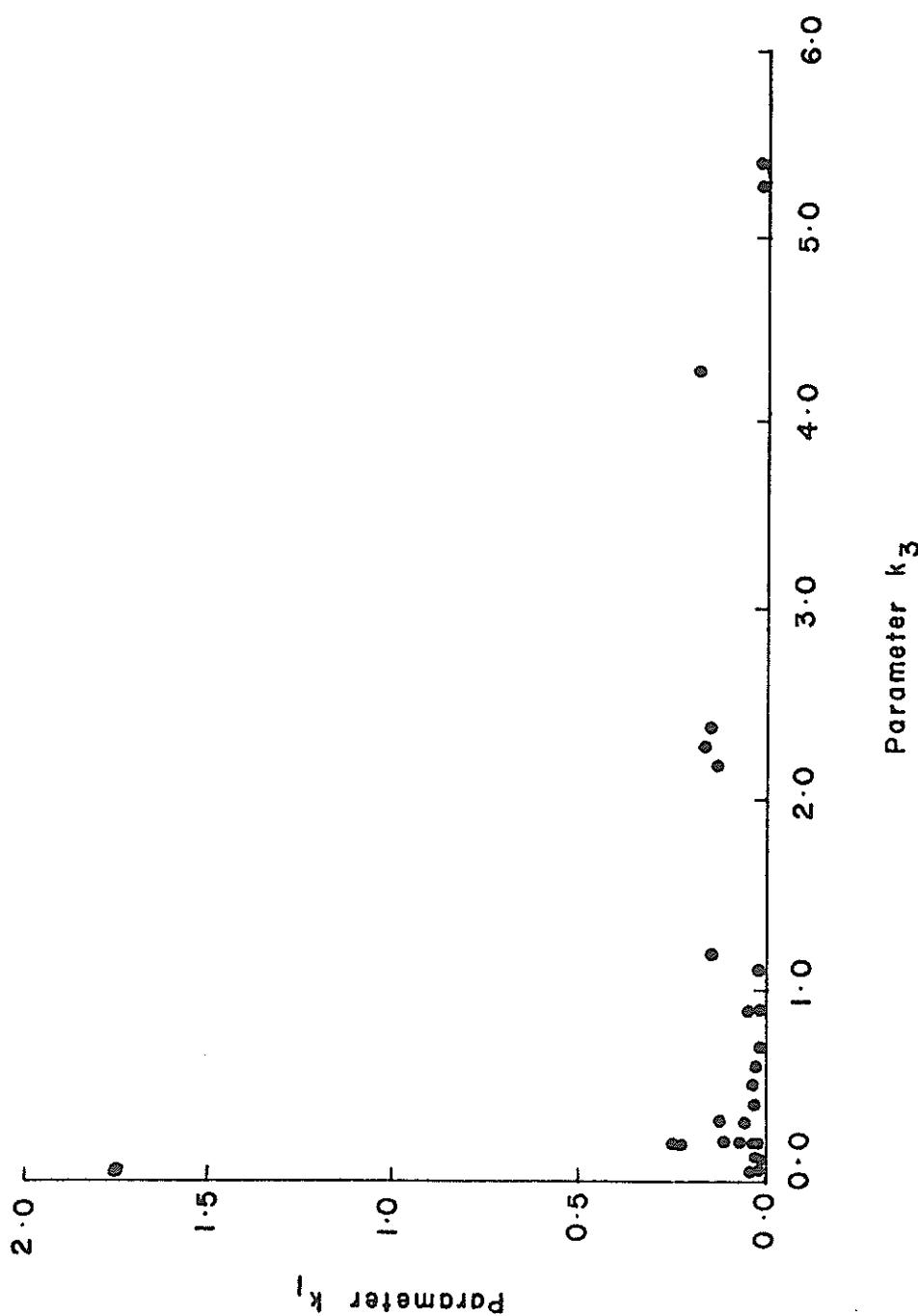
Fig. 4.9. Parameter $k_3$ versus parameter $k_1$.

error of estimate 1.2511. It indicates that $k_2$ and $k_3$ are correlated but not very strongly. Linear regression analysis was also performed between $k_3$ and $k_1$ where $k_1$ was dependent variable and $k_3$ independent variable. The correlation coefficient of 0.1699 and standard error of estimate 1.7461 were obtained. Again no significant correlation between parameters $k_1$ and $k_3$ was noticed.

## 4.3 Results and Discussions

Figures 4.1-4.3 show the plots of optimized $k_1$, $k_2$, and $k_3$ versus estimated $k_1$, $k_2$, and $k_3$ from linear Eqs. (4-3), (4-4) and (4-5) respectively. These figures show considerable scattering of points around the theoretical line of fit and indicate that the model parameters cannot be estimated reliably from the watershed characteristics in a linear fashion.

Figures 4.4-4.6 show the plot of the log values of optimized $k_1$, $k_2$, and $k_3$ versus estimated $k_1$, $k_2$, and $k_3$ from log regression Eqs. (4-6), (4-7) and (4-8) respectively. These figures show some improvement in correlation of parameters with watershed characteristics. The scattering of points is reduced. However, the correlations are still too poor to be used in the estimation of parameters from watershed characteristics. It appears that the model parameters not only depend on watershed characteristics but also equally depend on rainfall-excess characteristics and perhaps the interaction between them.

# CHAPTER 5

## CONCLUSIONS

The following conclusions can be drawn from this study.

1.  The nonlinear hydrologic cascade can predict watershed runoff satisfactorily.

2.  Computation time interval is important for model performance and can be taken as 1 minute.

3.  Reliable correlations could not be established between the model parameters and watershed characteristics.

4.  The model parameters have no correlation between themselves.

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