STUDIES ON RAINFALL-RUNOFF MODELING

5. A Uniformly Nonlinear Hydrologic Cascade Model

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ABSTRACT

Based on the concept of uniform nonlinearity (Dooge, 1967) a hydrologic cascade is formulated for prediction of surface runoff. By applying it to several natural agricultural watersheds its predictive ability is evaluated. The cascade contains three parameters. Utilizing watershed morphology equations are developed to estimate these parameters. The results of the model application to natural watersheds suggest that the model parameters can be reliably estimated. This cascade model seems to be a compromise between simple linear and complex nonlinear surface runoff models.

CHAPTER 1

INTRODUCTION

1.1 GENERAL REMARKS

The nonlinearity of watershed surface runoff has long been recognized. As a result of this recognition several nonlinear surface runoff models have appeared in the hydrological literature in the past ten years or so. The approaches, employed in the development of these models, can be grouped into two classes:

(1) Hydrodynamical

(2) Operational

The hydrodynamical approach (Chow, 1964; Singh, 1964; Kulandaiswamy, 1964; Wooding, 1965a, 1965b, 1966; Kibler and Woolhiser, 1970; Smith and Woolhiser, 1970; Eagleson, 1970, 1972; Harley, Perkins and Eagleson, 1970; Singh, 1975a, 1975b, 1975c, 1976a) requires the assumption that certain laws of physics hold and further requires a geometrical abstraction of the real-world phenomenon.

The operational approach (Amorocho and Orlob, 1961; Amorocho, 1963, 1967, 1973; Amorocho and Brandstetter, 1971; Jacoby, 1966; Harder and Zand, 1969; Bidwell, 1970; Chiu and Huang, 1970; Diskin and Boneh, 1972) develops input-output relationships by data fitting without making any explicit assumptions regarding the internal structure of the system.

This dichotomy in the modes of approaches emerges from the positions taken by the representatives of those two groups of approaches, and is well illustrated in a quote by Amorocho and Hart (1964):

> The first group espouses the pursuit of scientific research into the basic operation of each component of the hydrologic cycle in order to gain full understanding of their mechanisms and interactions. Although the immediate motivation of an individual researcher may not transcend the narrow confines of a set of special

> phenomena, it is implicit that a full synthesis of the hydrologic cycle may eventually be sought. The concept of a full synthesis is held to be the only rational approach to hydrology.
>
> The second group is motivated by the need to establish workable relationships between measurable parameters in the hydrologic cycle to be used in solving pressing practical technological problems. These people generally hold that the vast complexity of the systems involved in these studies and the inadequacy of the knowledge now available and the knowledge likely to exist in the foreseeable future, make the possibility of a full synthesis so remote in most cases that it must be discarded for practical purposes.

Because of inherent complexities of nonlinear models, both hydrodynamical and operational, they have not yet succeeded in occupying the place of operational tools in applied hydrology. What puzzles here is that the very intent of operational nonlinear models is operational, because they contribute little to the understanding of physical mechanisms governing surface runoff, and yet they have not become truly operational. The consequence is that the linear models continue to dominate, understandably, hydrologic applications even where they should justifiably be replaced by nonlinear models. This predicament can perhaps be overcome by an approach proposed by Dooge (1967). Based on the concept of uniform nonlinearity, this approach is a special case of a general nonlinear approach. This approach has the advantage that it avoids much of the complexity of the general nonlinear approach and hopefully accounts for some of the nonlinear effects which are important in runoff modeling.

1.2 OBJECTIVES

The objectives of the present investigation are:

(1) To develop a uniformly nonlinear surface runoff model.

(2) To apply the model to natural agricultural watersheds and examine its potential usefulness.

(3) To estimate the model parameters from watershed morphology.

CHAPTER 2

A UNIFORMLY NONLINEAR HYDROLOGIC CASCADE MODEL

2.1 CONCEPT OF UNIFORM NONLINEARITY

A uniformly nonlinear time-invariant system (Dooge, 1967) is one whose response can be simulated with sufficient accuracy by a model consisting of some arrangement of equal nonlinear storage elements or reservoirs. The storage elements can be arranged in series, in parallel, or a combination of both. The governing equations for a nonlinear storage element consist of a spatially lumped form of continuity equation and a nonlinear storage-discharge relationship which can be respectively written as:

$$p = q + \frac{ds}{dt} \tag{2-1}$$

$$q = ks^x \tag{2-2}$$

where

P = inflow to the element in cm/hr;

q = outflow from the element in cm/hr;

s = storage in the element in cm;

t = time in hours;

$\frac{ds}{dt}$ = rate of change of storage in the element;

k = characteristic parameter; and

x = an index of nonlinearity.

In a uniformly nonlinear model the parameters k and x do not vary from one storage element to another.

2.2 MATHEMATICAL FORMULATION OF HYDROLOGIC CASCADE

Employing the notion of uniform nonlinearity we now formulate a hydrologic cascade as a nonlinear model for surface runoff prediction. Let there be a cascade of n reservoirs with lateral inflows as shown in Fig. 2-1. By

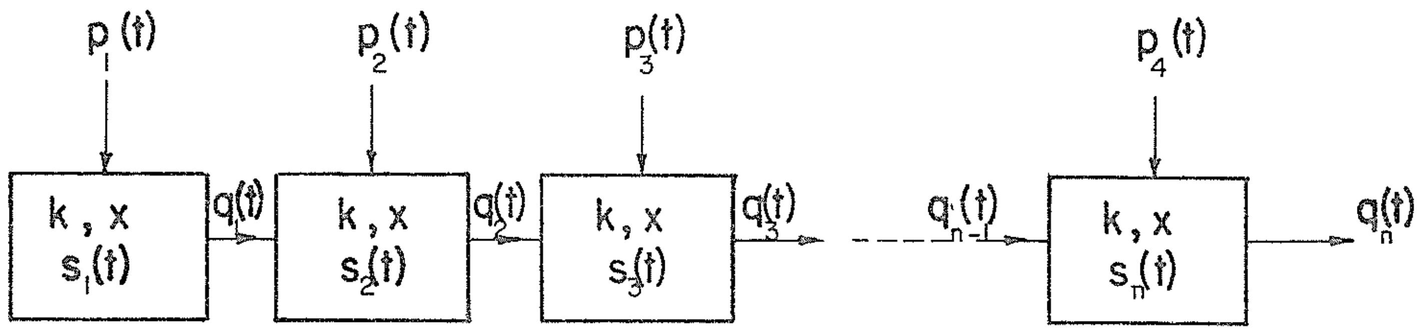


Fig. 2-1. Uniformly nonlinear hydrologic cascade with distributed input.

combining Eqs. (2-1) and (2-2) we obtain a single differential equation for a single nonlinear storage element relating storage (hence outflow) to inflow:

$$\frac{ds}{dt} = p - ks^x \tag{2-3}$$

Then for the cascade of Fig. 2-1 we get a system of equations of the type:

$$\frac{ds_1}{dt} = p_1 - k\, s_1^x$$

$$\frac{ds_2}{dt} = p_2 + k\, s_1^x - k\, s_2^x$$

$$\frac{ds_3}{dt} = p_3 + k\, s_2^x - k\, s_3^x$$

$$\vdots$$

$$\frac{ds_j}{dt} = p_j + k\, s_{j-1}^x - k\, s_j^x \tag{2-4}$$

$$\vdots$$

$$\frac{ds_{n-1}}{dt} = p_{n-1} + k\, s_{n-2}^x - k\, s_{n-1}^x$$

$$\frac{ds_n}{dt} = p_n + k\, s_{n-1}^x - k\, s_n^x$$

It is easier to write the system of equations (2-4) in a matrix form:

$$\dot{S} = P + kBS \tag{2-5}$$

where

$$\dot{S} = \begin{bmatrix} \frac{ds_1}{dt} \\ \frac{ds_2}{dt} \\ \vdots \\ \frac{ds_j}{dt} \\ \vdots \\ \frac{ds_n}{dt} \end{bmatrix} \quad ;$$

$$P = \begin{bmatrix} p_1 \\ p_2 \\ | \\ p_j \\ | \\ p_{n-1} \\ p_n \end{bmatrix} \quad ;$$

$$S = \begin{bmatrix} s_1 \\ s_2 \\ | \\ s_j \\ | \\ s_{n-1} \\ s_n \end{bmatrix} \quad ;$$

and

$$B = \begin{bmatrix} s_1^{x-1} & 0 & 0 & \text{———} & 0 & 0 \\ s_1^{x-1} & -s_2^{x-1} & 0 & \text{———} & 0 & 0 \\ 0 & s_2^{x-1} & -s_3^{x-1} & \text{———} & 0 & 0 \\ | & | & | & & | & | \\ 0 & 0 & 0 & & s_{n-1}^{x-1} & -s_n^{x-1} \end{bmatrix} .$$

The bold-faced letters will henceforth symbolize either matrices or vectors. Our main interest is in the relationship between the final outflow q_n and the set of lateral inflows $p_1, p_2, p_3, \ldots p_j \ldots p_n$. The relationship for discharge q can then be expressed as:

$$Q = kCS \tag{2-6}$$

where

$$Q = \begin{bmatrix} q_1 \\ q_2 \\ | \\ q_j \\ | \\ q_{n-1} \\ q_n \end{bmatrix}$$

and

$$C = \begin{bmatrix} s_1^{x-1} & 0 & \text{———} & 0 & 0 \\ 0 & s_2^{x-1} & \text{———} & 0 & 0 \\ | & | & & | & | \\ 0 & 0 & & s_{n-1}^{x-1} & 0 \\ 0 & 0 & & 0 & s_n^{x-1} \end{bmatrix}$$

Equations (2-5) and (2-6) constitute what may be termed as a state variable representation of a uniformly nonlinear hydrologic cascade for simulation of watershed runoff response. Equation (2-5) represents the case where a number of cascades of equal storage elements are combined in parallel.

2.3 SOME MATHEMATICAL PROPERTIES

The principle of proportionality and the principle of superposition are the two fundamental principles that make the linear time-invariant systems so attractive. These principles do not hold for nonlinear systems in general and that is why the mathematics of nonlinear systems becomes so complex. For convenience, let us define these principles before examining them with particular regard to uniformly nonlinear systems.

Proportionality implies that input and output have the same scale ratio. It ensures that for a given pattern of inflow a change in the average inflow will not affect the shape of the outflow but merely its scale.

Superposition means that the output due to the combined effect of a number of separate inputs is equal to the sum of the separate outputs due to each individual input. Thus it allows a complex input to be broken down into simple elements and the output obtained by summing the outputs due to these simple elements. This is the reason that the output from one complex input can be used as a basis for computing the output from another complex input of totally different pattern.

For uniformly nonlinear systems the principle of superposition does not hold. However, it can be shown that for such systems the principle of proportionality will hold provided that the time scale has been previously transformed in accordance with the input intensity. To prove it, it will be convenient if we define Eq. (2-5) in a dimensionless form by the use of the following normalizing quantities:

P_o = normalizing rate of inflow. This can be taken as average input rate, or any other input rate that may be suitable.

S_o = normalizing storage. This can be expressed in terms of P_o as:

$$S_o = \left(\frac{P_o}{k}\right)^{\frac{1}{x}}$$

T_o = normalizing time. This can be expressed in terms of P_o and S_o as:

$$T_o = \frac{S_o}{P_o} = \frac{1}{(k)^{\frac{1}{x}} \, (P_o)^{\frac{x^2-1}{x}}}$$

It must be remarked that once the normalizing rate P_o has been defined, the normalizing storage and the normalizing time can be calculated from it using the parameters k and x of the nonlinear storage element. Thus the normaliz-ed quantities, denoted by asterisks, are:

$$p_{*j} = \frac{p_j(t/T_o)}{P_o}$$

$$t_* = \frac{t}{T_o}$$

$$s_{*j} = \frac{s}{S_o}$$

$$\frac{ds_{*j}}{dt_*} = \frac{d(s_j/S_o)}{d(t/T_o)}$$

$$q_{*j} = \frac{q_j}{P_o}$$

Then Eqs. (2-5) and (2-6) can be written as:

$$\dot{S}_* - kB_*S_* = P_* \qquad (2\text{-}7)$$

$$Q_* = kC_*S_* \qquad (2\text{-}8)$$

Let us now suppose that two distinctly different input patterns, as shown in Fig. 2-2, are given for which runoff hydrograph is desired. If these inputs are such that they both lead to one and the same p_*, then q_* will also be the same in both cases as is evident from Eqs. (2-7) and (2-8). Such inputs were defined by Dooge (1967) as similar inputs, and for these inputs the principle of proportionality holds for uniformly nonlinear hydrologic systems. If, however, given input patterns do not lead to the same p_* as shown in Fig. 2-3, then the principle of proportionality will no longer hold even though the inputs may be proportional in an absolute sense.

2.4 RELATIONSHIP WITH KINEMATIC CASCADE

The governing equations for a kinematic plane consist of an equation of continuity and an approximation to momentum equation, which can be written respectively as:

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = p(x,t) \tag{2-9}$$

$$q = \alpha\, h^c \tag{2-10}$$

where

h = mean local depth of flow in cm;

$p(x,t)$ = lateral inflow in cm/hr varying in time and space;

x = space coordinate;

t = time coordinate;

c = an index of nonlinearity; and

α = kinematic wave friction relationship parameter.

On comparing Eqs. (2-9) and (2-10) with Eqs. (2-1) and (2-2) we notice that the former account for space-time variability of rainfall p and runoff q while the latter ignore their spatial variability and account for their temporal variability only. Once this distinction is realized, it is easy to

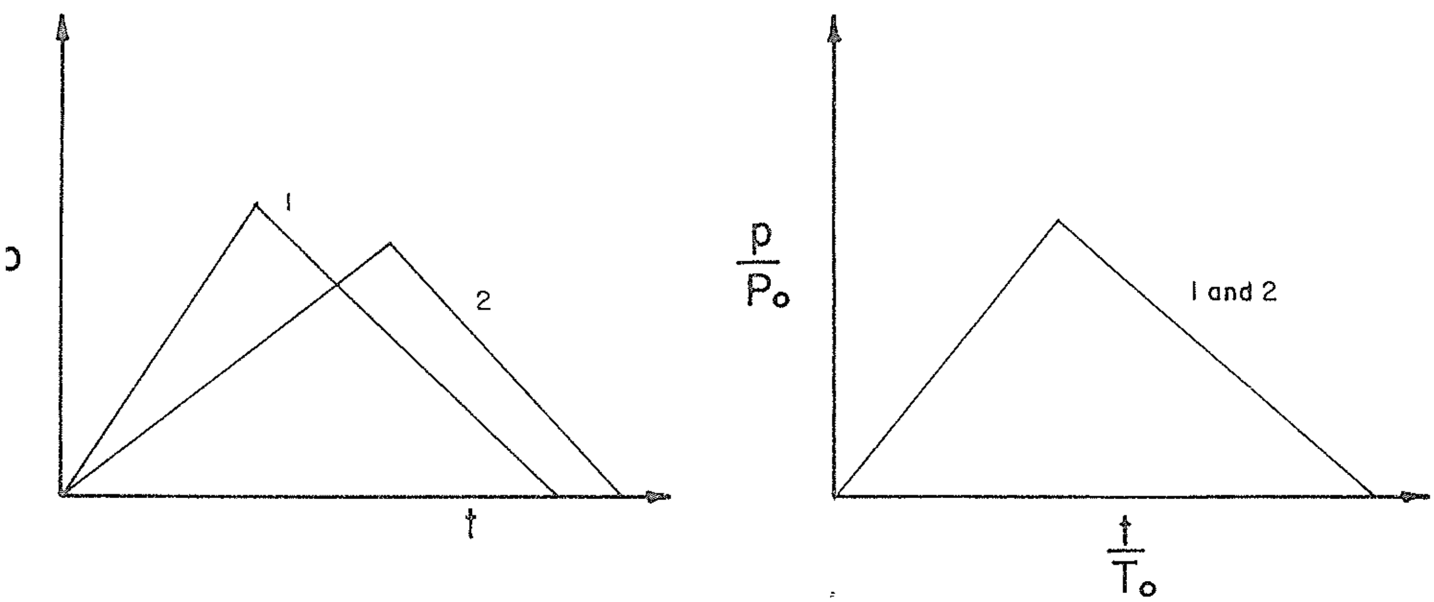


Fig. 2-2. Similar inputs.

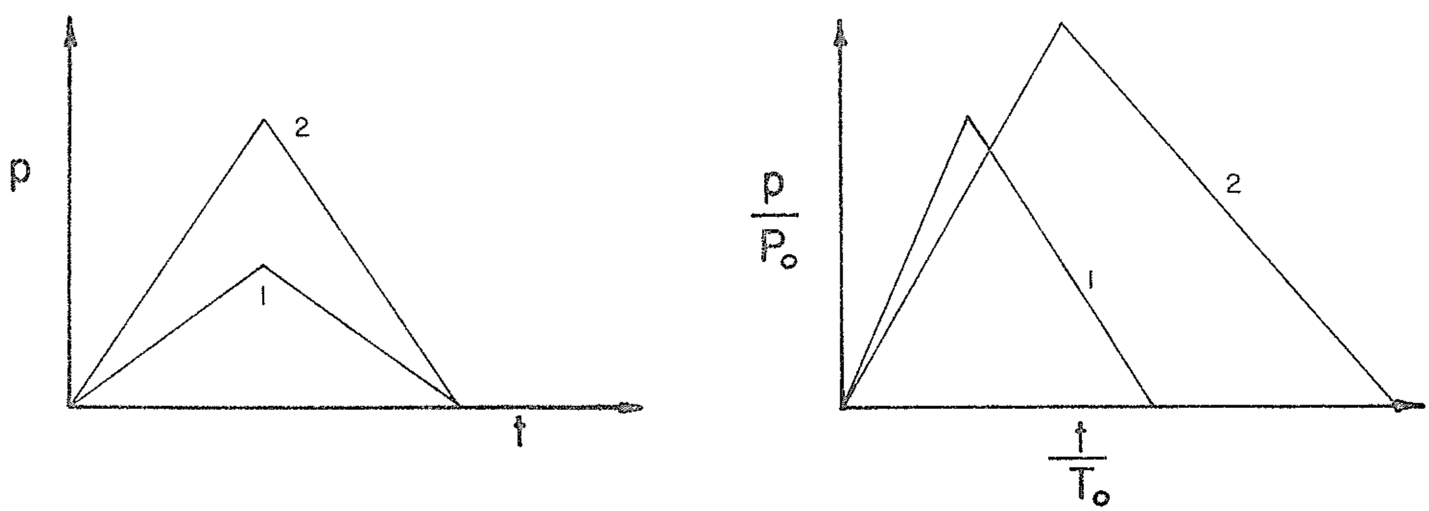


Fig. 2-3. Dissimilar inputs.

show that a uniformly nonlinear cascade is a special case of the kinematic cascade. If we consider a small reach (Δx) then $\frac{\partial Q}{\partial x}$ can be replaced by $\frac{\Delta Q}{\Delta x}$; for a given time interval Δt and a reach of length Δx we can write $\frac{\Delta Q}{\Delta x}$ = $(I - O)/\Delta x$, where I is upstream inflow and O outflow. The term $\frac{\partial h}{\partial t}$ can be replaced for small Δt by $\frac{\Delta h}{\Delta t}$. Thus we can write Eq. (2-9) as:

$$\frac{\Delta h}{\Delta t} - \frac{(I - O)}{\Delta x} = p(x,t) \tag{2-11}$$

Multiplying Eq. (2-11) by Δx,

$$\frac{\Delta h}{\Delta t} \cdot \Delta x - (I - O) = p(x,t) \cdot \Delta x \tag{2-12}$$

$\Delta h \cdot \Delta x$ is equal to Δs; lateral inflow $p(x,t) \cdot \Delta x$ can be combined with upstream inflow and can simply be written as I. Thus we can write:

$$\frac{\Delta s}{\Delta t} = I - O \tag{2-13}$$

Then Eq. (2-13) can be written as:

$$I = O + \frac{ds}{dt} \tag{2-14}$$

Equation (2-14) is the same as Eq. (2-1). Similarly, Eq. (2-10) can be reduced to Eq. (2-2). Multiply and divide the right-hand side of Eq. (2-10) by $(\Delta x)^c$,

$$q = \frac{\alpha}{(\Delta x)^c} \{\Delta x \cdot h\}^c \tag{2-15}$$

We can then write:

$$q = \frac{\alpha}{(\Delta x)^c} s^c \tag{2-16}$$

Replacing $\alpha/(\Delta x)^c$ by k,

$$q = ks^c \tag{2-17}$$

Equation (2-17) is the same as Eq. (2-2).

CHAPTER 3

APPLICATION TO NATURAL WATERSHEDS

For model calibration and testing, 21 natural agricultural watersheds were selected from two geographically distinct regions: 5 near Hastings, Nebraska, and 16 near Riesel (Waco), Texas. These watersheds vary in area from 1.2 to 1720 ha. Their detailed description can be found elsewhere (e.g. USDA, 1963 and subsequent publications).

3.1 DETERMINATION OF MEAN AREAL RAINFALL

Rainfall-runoff data are available for these watersheds in the USDA publications on hydrologic data (e.g., USDA, 1963). These publications are released almost yearly and consist of one rainfall-runoff event a year on a waterhsed. Although a watershed may have more than one raingage, data are normally available in these publications for only a centrally located raingage indicating that this represents the mean areal rainfall. For consistency this practice was followed on each watershed.

3.2 DETERMINATION OF RAINFALL-EXCESS

Rainfall-excess formed the lateral inflow and was obtained by subtracting infiltration from rainfall. Philip's equation (Philip, 1957) was utilized to estimate infiltration loss. His equation can be written as:

$$f = \alpha + \tfrac{1}{2}\beta\, t^{-\frac{1}{2}} \qquad (3\text{-}1)$$

where

f = infiltration rate in cm/hr;

t = time in hours;

α = a parameter depending on soil characteristics and initial soil moisture conditions; and

β = a parameter depending on soil characteristics and initial soil moisture conditions.

Parameter α was considered roughly identical to saturated hydraulic conductivity and was therefore determined from soil characteristics. Parameter β was allowed to vary with each rainfall episode and was determined such that for each rainfall episode volume of rainfall-excess was equal to the volume of observed runoff. Henceforth, rainfall will imply rainfall-excess.

3.3 HYDROLOGIC CASCADE

We consider a special case of the uniformly nonlinear hydrologic cascade formulated in the preceding chapter. The special, simple case is shown in Fig. 3-1, where the values of p_2, p_3,......... p_n would all be zero; only p_1 will be greater than zero. This represents a case of lumped input rather than distributed input of Fig. 2-1. The operation of the cascade can be summarized as follows:

(1) specify the parameters k, x and n.

(2) select a time interval Δt.

(3) Given the input pattern, use Eq. (2-5) to compute $\dot{S}$. Note that at the beginning of time t = 0, S and B are zero. In vector P only p_1 is a positive quantity and the rest of the elements p_2, p_3, ... p_n are all zero. These define the initial condition.

(4) Input into the jth storage element (j = 1, 2, ...n) at the beginning of a particular time interval Δt is an impulse of a magnitude equal to the sum of input p_j, the surface inflow q_{j-1} and the surface outflow q_j. The impulse input causes an instantaneous change in the storage of the jth element.

(5) Responses of the elements to the impulse inputs determine the outputs and the states of the elements at the end of the time interval Δt; the new states are the initial conditions for the following time increment.

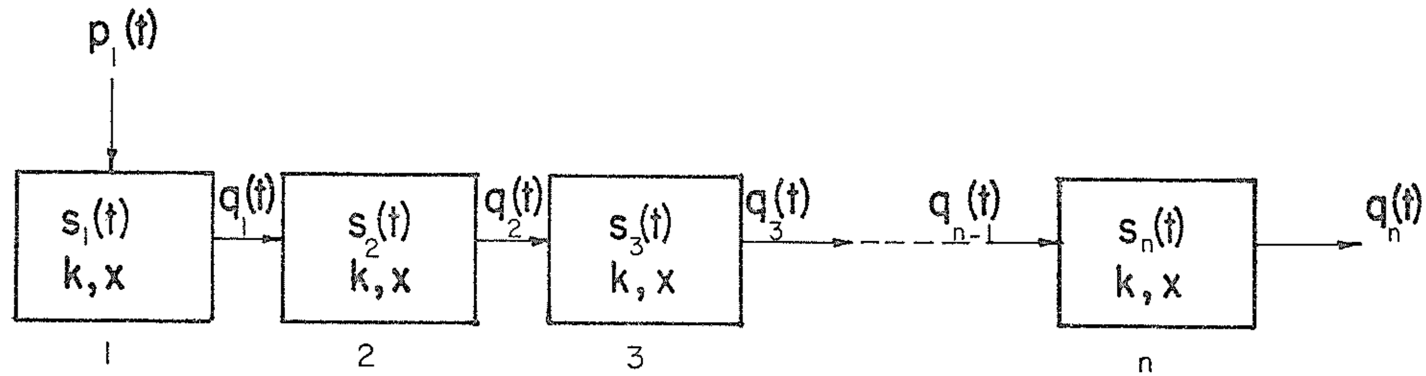


Fig. 3-1. Uniformly nonlinear hydrologic cascade with lumped input.

(6) Thus, given the initial states of the system, B_o and S_o, and the input vector P_o at the beginning of a particular time interval Δt, Eq. (2-5) is used to solve for $\dot{S}_o$. The state of the system at the end of the time interval Δt_o is the initial state at the beginning of the next time interval Δt , and is given by $S = S_o + \dot{S}_o$. Outflows from the n elements at the end of Δt_o are given by Eq. (2-6).

3.4 CHOICE OF OBJECTIVE FUNCTION

The concept of determining optimal model parameters requires that the objective function be compatible with the intended use. There is, however, a difficulty in defining an error criterion that, upon minimization, will correspond to model parameter values without an undesirable bias. The following objective function was first investigated regarding its suitability:

$$F = \gamma f_1 + (1-\gamma)\ f_2 \ \Rightarrow \min \qquad (3\text{-}2)$$

where

α = weighting factor taking values from o to 1;

$$f_1 = \sum_{j=1}^{M} [Q_{p_o}(j) - Q_{p_e}(j)]^2;$$

and

$$f_2 = \sum_{j=1}^{M} [t_{p_o}(j) - t_{p_e}(j)]^2$$

where

$Q_{p_o}(j)$ = observed hydrograph peak in cm/hr for jth event;

$Q_{p_e}(j)$ = estimated hydrograph peak in cm/hr for jth event;

$t_{p_o}(j)$ = observed hydrograph peak time in min for jth event;

$t_{p_e}(j)$ = estimated hydrograph peak time in min for jth event; and

M = number of runoff events in the optimization set.

Suitability of Eq. (3-2) was examined on watershed SW-17, Riesel (Waco), Texas. This is a small watershed of 1.2 ha in area. Nine rainfall-runoff events were available on this watershed. These events were divided into two sets; one set, called optimization set, consisted of five events; another set, called prediction set, consisted of four events. These two sets did not have any events in common. By taking n equal to 3 in the cascade the parameters k and x were optimized by the modifed Rosenbrock algorithm (Rosenbrock, 1960; Palmer, 1969; Himmelblau, 1972) for the optimization set of events with γ varying from o to 1 in Eq. (3-2). The values of F, and optimized k and x are given in Table 3-1 for various values of γ. It is interesting to note that when $\gamma \leq 0.6$, there is no change in the optimized values of parameters k and x. From this table, however, the value of γ for most suitable F is not clear. For this reason hydrographs were predicted for the events in the prediction set by using optimized parameters. Table 3-2 gives observed and predicted hydrograph peak and its time for three different values of γ. It is interesting to observe that giving greater weight to f_2 in Eq. (3-2) does not necessarily lead to better matching of hydrograph peak time. Indeed the value of γ equal to 1 provides just as good a fit of hydrograph peak time as γ equal to 0.8 and better when γ is less than or equal to 0.6. It was therefore concluded that the following objective function, a special case of Eq. (3-2), was most suitable:

$$F = \sum_{j=1}^{M} [Q_{p_o}(j) - Q_{p_e}(j)]^2 \Rightarrow \min \qquad (3\text{-}3)$$

Henceforth, Eq. (3-3) will be employed as an objective function in the ensuring discussion.

Table 3-1. Values of objective function and optimum parameters for watershed SW-17, Riesel (Waco), Texas, for various values of weighting factor.

Number of storage elements n = 3

Case number	Weighting factor Y	Parameters		Objective function F
		x	k	
1	1	1.4	0.22	0.162
2	0.8	1.38	0.15	117.01
3	0.6	1.1	0.12	208.21
4	0.4	1.1	0.12	311.05
5	0.2	1.1	0.12	413.90
6	0.0	1.1	0.12	516.75

Table 3-2. Observed and predicted hydrograph peak and its time on watershed SW-17, Riesel (Waco), Texas for three sets of optimized parameters.

Number of storage elements n = 3

Date of event	Observed hydrograph peak (cm/hr)	Observed hydrograph peak time (min)	γ=1.0, x=1.4, k=0.22				γ=0.8, k=0.15, x=1.38				γ=0.6, k=0.12, x=1.1			
			Q_{P_e} (cm/hr)	EQ	t_{P_e} (min)	Et	Q_{P_e} (cm/hr)	EQ	t_{P_e} (min)	Et	Q_{P_e} (cm/hr)	EQ	t_{P_e} (min)	Et
4-24-1957	7.366	35	7.507	-0.019	34.5	0.014	6.639	0.099	39.3	-0.123	5.948	0.192	44.2	-0.263
5-13-1957	4.420	26	4.884	-0.105	31.1	-0.196	4.419	0.000	34.1	-0.312	4.133	0.065	38.5	-0.481
6- 9-1962	9.627	33	7.123	0.260	25.5	0.227	6.855	0.288	31.5	0.046	6.301	0.345	34.0	-0.030
3-28-1965	6.196	110	8.058	-0.300	70.3	0.361	7.022	-0.133	72.7	0.339	6.049	0.024	81.6	0.258

Q_{P_e} = Estimated hydrograph peak, t_{P_e} = Estimated hydrograph peak time, EQ = Relative error in Q_{P_o},

and Et = Relative error in t_{P_e}.

3.5 MODEL CALIBRATION

The uniformly nonlinear cascade has three parameters n, k and x. It will be useful to establish if any of the parameters has physical significance. The parameter n signifies the number of storage elements in the model. The value of n must be greater than one to obtain proper hydrograph shape, and will depend on the topographic complexity of a watershed. This can essentially be called a shape parameter. A natural watershed entails a network of channels and overland flow planes. The combined action of a channel and a plane is being simulated here by a nonlinear storage element. It would then seem that a very large number of storage elements will be needed to simulate the action of a network of channels and planes and, in turn, the runoff response of a watershed. Fortunately, it so happens that only a small number of storage elements will suffice. This is because the planes and channels having more or less similar hydraulic behavior can be combined and then their combined action can be simulated by a single storage element. The exact value of n will vary from one watershed to another, but it seems plausible that n will more or less be the same for watersheds in a certain area range having the same order of drainage evolution.

The parameters x quantifies the degree of nonlinearity of surface runoff process. Although the value of x may change throughout the development of a runoff hydrograph, it is plausible that x can be fixed for watersheds in a certain area range and that this fixed value of x will provide a good approximation to the degree of nonlinearity in surface runoff.

The precise physical significance of the parameter k is not clear. It appears that it accounts for translation and attenuation effects, and consequently it may change considerably from one watershed to another. The topographic characteristics of a watershed seem to be dominant factors affecting the value of k. Although k will most likely change from event to event on the same watershed but this change may not hopefully be large. Thus it seems plausible that k can be expressed in terms of topographic characteristics.

These plausible hypotheses were tested on watershed SW-17, Riesel (Waco), Texas, using the objective function in Eq. (3-3). The parameters k and x were optimized for various values of n for the optimization set of events by the modified Rosenbrock algorithm. The values of F, and k and x are given in Table 3-3. It is interesting to note that (1) F does not change for $n \geq 7$, (2) x remains fixed for $n \geq 6$, (3) x changes little for $n < 6$, (4) x decreases as n increases, (5) k increases as n increases, and (6) for $n = 3$, F is minimum. Based on these observations, one could fix n at 3 and x at 1.4. If it can be shown that these parameter values are reasonable then the cascade will have only one parameter k to be specified. To examine further, hydrographs were predicted by using various sets of parameter values. Tables 3-4 and 3-5 compare observed and predicted hydrograph peak and its time for the prediction set of events on watershed SW-17, Riesel (Waco), Texas. It is clear from these tables that the values of $n = 3$ and $x = 1.4$ are reasonable. It must be pointed out that a higher value of n may lead to equally good prediction, but it will increase computation unnecessarily and is hence undesireable.

Now one question that must be addressed is whether the cascade, with $x = 1.4$ and $n = 3$, produces hydrographs with appropriate shape. For two sample events predicted and observed hydrographs are shown in Figs. 3-2 and 3-3. From these figures it is evident that the hydrograph shape is well preserved. These results confirm that the proposed cascade, with $x = 1.4$ and $n = 3$, is capable of representing the runoff process.

3.6 DETERMINATION OF PARAMATER k

The cascade has now only one parameter k that needs to be specified. If k can be specified apriori for a given watershed then the cascade will be

Table 3-3. Values of objective function and optimized parameters for watershed SW-17, Riesel (Waco), Texas.

Case number	Number of storage elements n	Objective function F	Optimal values of parameters x	Optimal values of parameters k (cm^{-x}/hr)
1	2	2.692	3.10	0.26
2	3	0.162	1.4	0.22
3	4	0.168	1.36	0.27
4	5	0.172	1.36	0.32
5	6	0.174	1.35	0.38
6	7	0.175	1.35	0.42
7	8	0.175	1.34	0.46
8	9	0.175	1.34	0.51
9	10	0.175	1.35	0.55

Table 3-4. Observed and predicted hydrograph peak on watershed SW-17, Riesel (Waco), Texas, using various sets of optimized parameter values.

Date of Event	Observed hydrograph peak (Q_{p_o}) cm/hr	Number of reservoirs used																	
		2		3		4		5		6		7		8		9		10	
		Q_{p_e}	EQ	Q_{p_e}	EQ	Q_{p_e}	EQ	Q_{p_e}	EQ	Q_{p_e}	EQ	Q_{p_e}	EQ	Q_{p_e}	EQ	Q_{p_e}	EQ	Q_{p_e}	EQ
4-24-1957	7.366	8.405	-0.141	7.507	-0.019	7.478	-0.015	7.519	-0.021	7.543	-0.024	7.548	-0.025	7.556	-0.026	7.567	-0.027	7.584	-0.030
5-13-1957	4.420	5.138	-0.163	4.884	-0.105	4.894	-0.107	4.924	-0.114	4.944	-0.119	4.961	-0.123	4.974	-0.126	4.981	-0.127	4.986	-0.128
6-9-1962	9.627	7.186	0.254	7.123	0.260	7.162	0.256	7.206	0.251	7.234	0.249	7.252	0.247	7.267	0.245	7.271	0.245	7.275	0.244
3-29-1965	6.196	9.131	-0.474	8.058	-0.300	7.953	-0.283	7.971	-0.287	7.984	-0.288	7.973	-0.287	7.977	-0.287	7.989	-0.289	8.019	-0.294

Q_{p_e} = Estimated hydrograph in cm/hr, $EQ = (Q_{p_o} - Q_{p_e})/Q_{p_o}$

able 3-5. Observed and estimated hydrograph peak time on watershed SW-17, Riesel (Waco), Texas, using various sets of optimized parameter values.

Date of event	Observed hydrograph peak time t_{p_o} (min)	Number of Reservoirs used																	
		2		3		4		5		6		7		8		9		10	
		t_{p_e}	Et	t_{p_e}	Et	t_{p_e}	Et	t_{p_e}	Et	t_{p_e}	Et	t_{p_e}	Et	t_{p_e}	Et	t_{p_e}	Et	t_{p_e}	Et
4-24-1957	35	31	0.114	34.5	0.014	36.6	-0.046	38.0	-0.086	39.3	-0.123	40.6	-0.160	41.7	-0.191	42.7	-0.22	43.6	-0.246
5-13-1957	26	29.4	-0.131	31.1	-0.196	32.4	-0.246	33.2	-0.277	34.0	-0.308	35.0	-0.346	36.0	-0.385	37.1	-0.427	38.0	-0.462
6-9-1962	33	10.3	0.688	25.5	0.227	26.2	0.206	26.4	0.200	27.3	0.173	28.4	0.139	29.4	0.109	30.4	0.079	31.2	0.055
3-29-1965	110	68.1	0.381	70.3	0.361	72.2	0.344	73.5	0.332	74.8	0.320	76.1	0.308	77.2	0.298	78.2	0.289	79.1	0.281

t_{p_e} = Estimated Hydrograph peak time, min; $Et = (t_{p_o} - t_{p_e})/t_{p_o}$

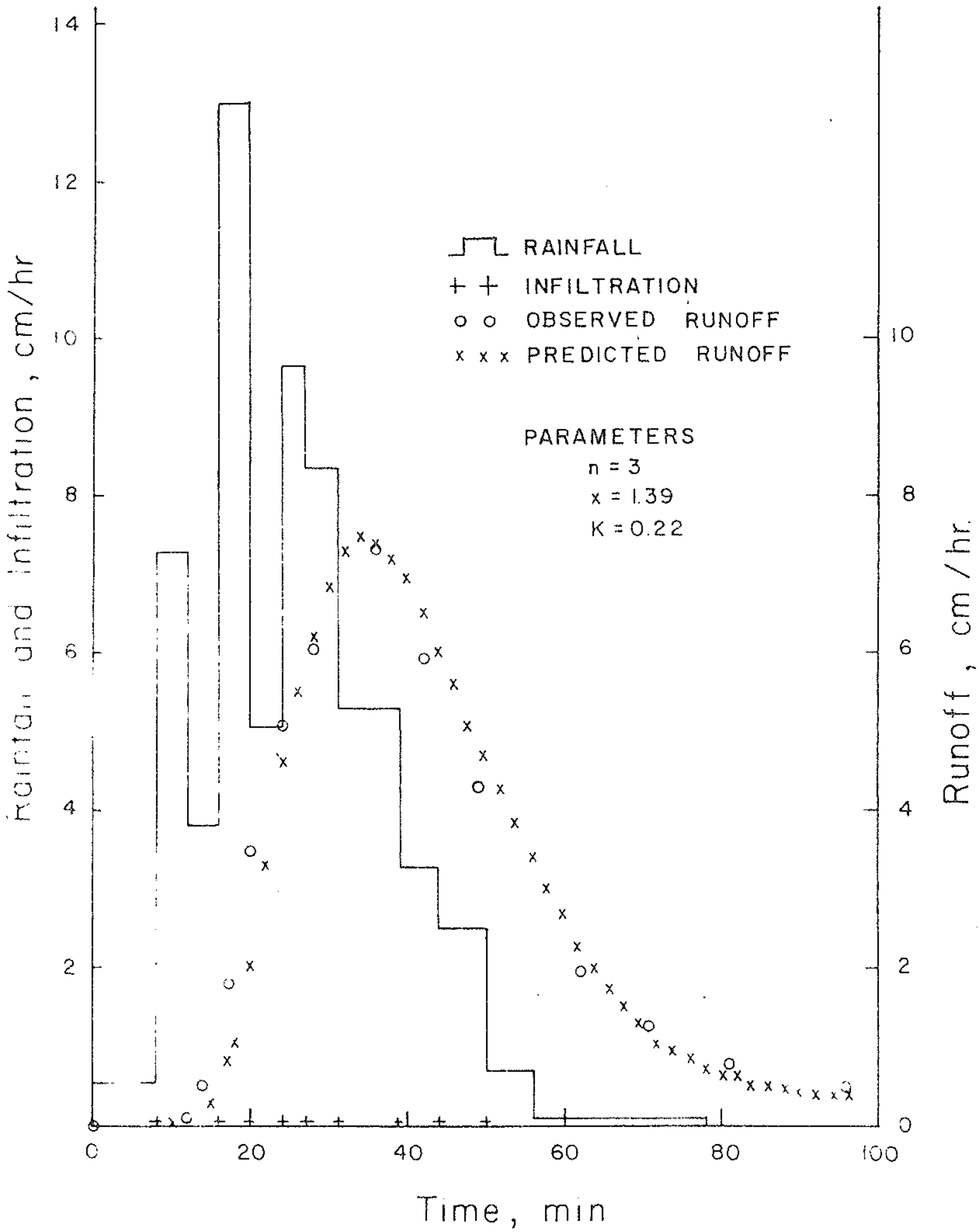


Fig. 3-2. Hydrograph prediction by the model for rainfall event of 4-24-1957 on Watershed SW-17, Riesel (Waco), Texas.

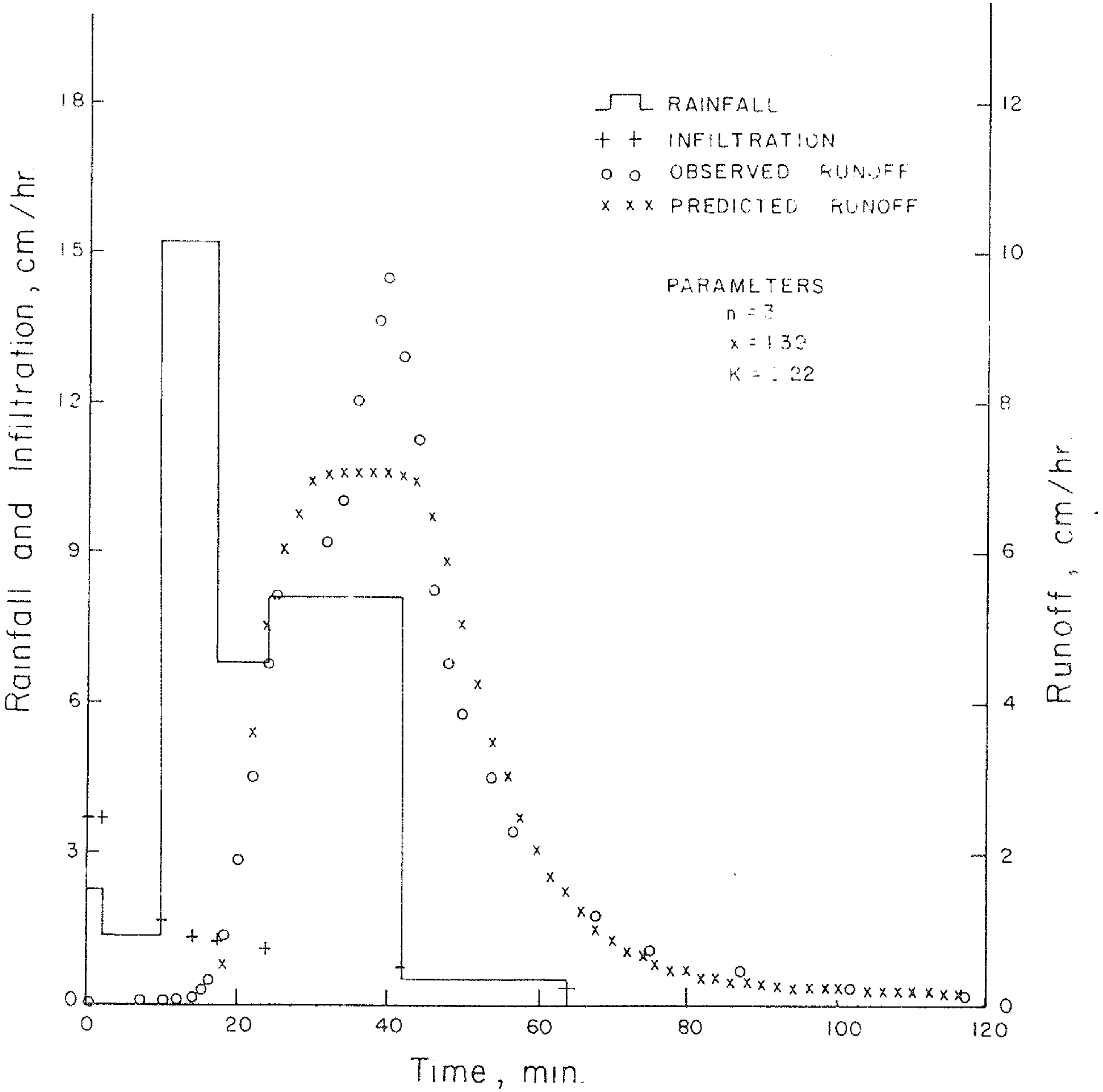


Fig. 3-3. Hydrograph prediction by the model for rainfall event of 6-9-1962 on watershed SW-17, Riesel (Waco), Texas.

Table 3-6. Watershed characteristics.

Watershed	Area (ha)	Width (m)	Length of main stream (m)	Slope (%)	Shape	Parameter k $\frac{(cm)^{-0.4}}{sec}$
Riesel (Waco)						
C	234.21	1402	2366	2.04	1.8761	0.0245
D	449.22	1892	3567	2.10	2.2246	0.0382
G	1772.59	2592	7829	2.05	2.7160	0.0134
Y	125.05	915	1537	2.40	1.4830	0.0400
Y-2	53.42	854	1000	2.58	1.4702	0.0794
Y-4	32.30	595	610	2.85	0.9031	0.0700
Y-6	6.50	259	338	3.22	1.0634	0.0800
Y-7	16.19	381	543	1.86	1.4289	0.1340
Y-8	8.418	183	244	1.94	0.5550	0.1667
Y-10	7.53	381	338	2.37	1.0584	0.1620
W-1	71.23	610	1646	2.18	2.9887	0.1594
W-2	5.26	823	945	2.55	1.3335	0.0989
W-6	17.12	457	445	2.02	0.9090	0.1312
W-10	7.97	305	323	1.62	1.0289	0.2100
SW-12	1.20	119	116	3.95	0.8770	0.3021
SW-17	1.21	122	116	1.83	0.8712	0.1975
Hastings						
2-H	1.21	76	189	6.13	2.0395	0.2501
4-H	1.47	107	162	5.96	1.3921	0.4462
W-3	194.66	1207	2720	5.30	2.9861	0.0700
W-8	844.20	1811	7953	5.50	5.8850	0.0179
W-11	1412.40	2012	11673	5.09	7.5763	0.0005

completely specified and can be readily applied to gaged or ungaged watersheds. A logical way is to relate k to topographic characteristics of a watershed. To accomplish this rainfall events of each of 21 watersheds were divided, as before, into optimization set and prediction set. Then k was optimized with $n = 3$ and $x = 1.4$ for optimization set of events on each watershed by the modified Rosenbrock algorithm in conjunction with Eq. (3-3) The optimized k values are given in Table 3-6. Topographic characteristics, selected for correlating them with k, included area, width, length of the main stream, weighted slope and shape factor (Chorley, Malm and Pagorzelski, 1957). These characteristics are given for each watershed in the USDA publications (see Table 3-6). The shape factor of Chorley, Malm and Pagorzelski (1957) can be written as:

$$\text{Shape} = \frac{\pi L^2}{4A}$$

where

L = length of the mainstream; and

A = area of the watershed.

This shape factor is a dimensionless parameter and quantifies the watershed shape.

To correlate k with topographic characteristics a multiple linear regression analysis was used. k was obviously the dependent variable in the analysis. The linear regression analysis yielded a correlation coefficient of 0.9208 and a standard error of estimate of 0.0746 where slope was most highly correlated with a correlation coefficient of 0.8080, then were length of mainstream, area and shape factor respectively.

The regression equation can be written as:

k = 0.00044 Area - 0.00014 Length + 0.03828 Slope + 0.08511 Shape (3-4)

To check the suitability of Eq. (3-4) residuals between optimized k and k estimated from Eq. (3-4) were computed for all the watersheds. As evident from Fig. 3-4 there is considerable scattering of points around the regression-fit line, and we would naturally like to minimize this scattering.

In the hope of improving the correlation all variables, dependent as well as independent, were transformed logarithmically to the base 10. Henceforth, we will deal with these transformed variables only. Then the regression analysis was performed. A correlation coefficient of 0.9890 and a standard eror of estimate of 0.1827 were obtained. This time length of mainstream provided the highest correlation with a correlation coefficient of -0.9763, then did area, shape factor and slope respectively. The regression equation can be written as:

Log k = -0.30871 Log Area -0.21608 Log Length - 0.08328 Log Slope +
0.30379 Log Shape (3-5)

To check the reliability of Eq. (3-5) residuals between optimized k and k estimated from Eq. (3-5) were computed. As shown in Fig. 3-5, the scattering of points is considerably reduced and consequently the relationship is much improved.

In the multiple linear regression analysis the independent variables are assumed to be independent in a statistical sense; they are seldom so, as clearly seen from the partial correlation matrix for the transformed variables given in Table 3-7. It then appears that a fewer number of independent variables may suffice to develop a reasonable equation for k. To accomplish this, shape factor was removed from independent variables, and then regression analysis was performed. A correlation corfficient of 0.9885 and a standard error of estimate of 0.1757 were obtained. Now width was most highly cor-

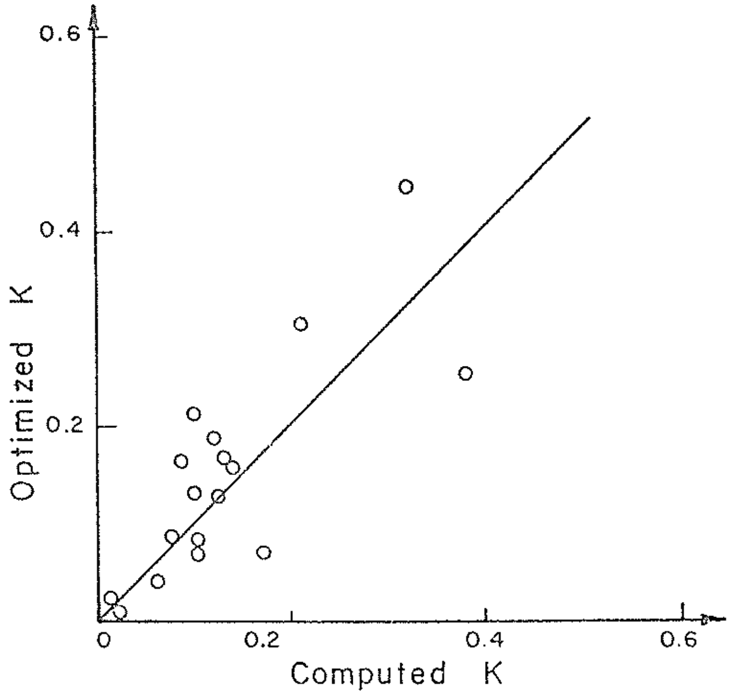


Fig. 3-4. Optimized k versus computed k using Eq. (3-4).

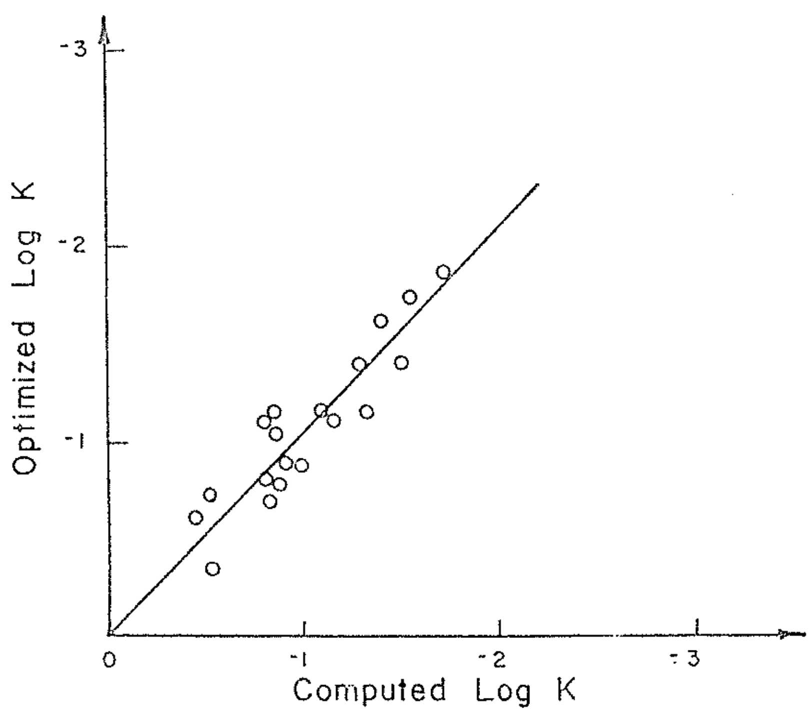


Fig. 3-5. Optimized k versus computed k using Eq. (3-5).

related with a correlation coefficient of -0.972, and then was area. The regression equation can be written as:

$$\text{Log } k = -.027271 \text{ Log Area} - 0.24309 \text{ Log Width} \quad (3\text{-}6)$$

To evaluate the goodness of Eq. (3-6) the residuals of k were plotted as shown in Fig. 3-6. It is clear that the relation for k is nearly as good as Eq. (3-5).

To find out a different combination of independent variables that will give an equally good relationship for k, shape factor and width were deleted from independent variables and then regression analysis was performed. A correlation coefficient of 0.9882 and a standard error of estimate of 0.1834 were obtained. Length of mainstream alone gave a correlation coefficient of -0.9763, and it was further improved by area and slope respectively. The regression equation can be written as:

$$\text{Log } k = -0.22889 \text{ Log Area} -0.26395 \text{ Log Length} + 0.10079 \text{ Log Slope} \quad (3\text{-}7)$$

Again, residuals of k were computed to determine the reliability of Eq. (3-7), as shown in Fig. 3-7. This provides, as clear from the figure, just as good a relationship for k.

Thus we have three different relationships for k given by Eqs. (3-5) - (3-7) which are comparable. Any one of the three relationships can be used to estimate k. However, one may prefer to choose Eq. (3-6) or Eq. (3-7) because of fewer variables involved therein. Since the ultimate objective of the model is to predict surface runoff, we would like to see how good these estimates of the parameters are.

Table 3-7 Partial correlation matrix for variables after transformation.

	Area	Width	Length	Slope	Shape	k
Area	1.00	0.900	0.913	0.732	0.796	-0.954
Width		1.000	0.998	0.899	0.660	-0.972
Length			1.000	0.905	0.699	-0.976
Slope				1.000	0.667	-0.843
Shape					1.000	-0.717
k						1.000

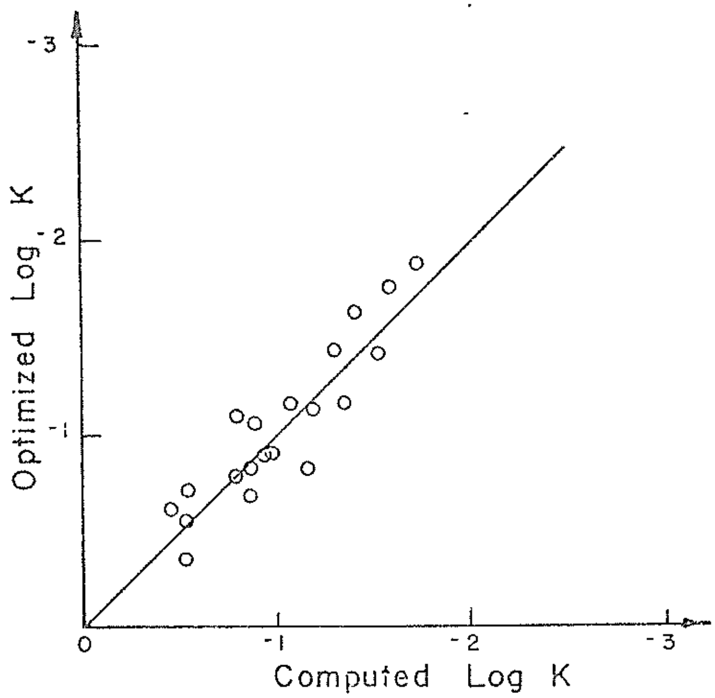


Fig. 3-6. Optimized k versus computed k using Eq. (3-6).

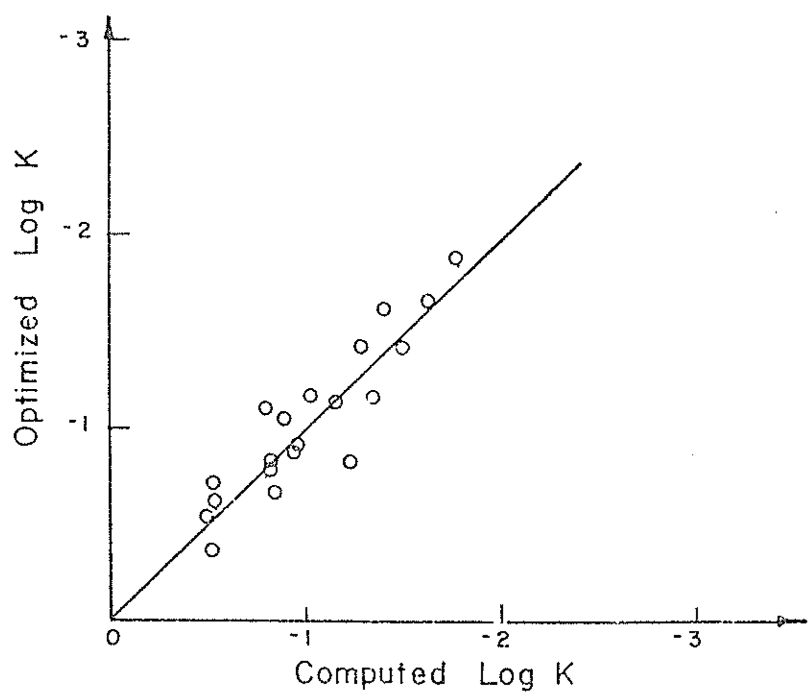


Fig. 3-7. Optimized k versus computed k using Eq. (3-7).

CHAPTER 4

RESULTS AND DISCUSSION

4.1 HYDROGRAPH PREDICTION

Hydrographs were predicted for the events in the prediction set of each of the 21 watersheds, utilizing $x = 1.4$, $n = 3$ and k estimated by Eq. (3-7). Table 4-1 provides observed and predicted hydrograph peak characteristics for the prediction set of events on each watershed. Figures 4-1 and 4-2 show observed and predicted runoff hydrographs for two sample events. It is evident that k estimated by Eq. (3-7) is nearly as good as obtained by optimization (see the results for watershed SW-17, Riesel (Waco), Texas). It is also clear from the table and figures that observed and predicted runoff hydrographs are in close agreement. These results not only indicate that we can completely specify the uniformly nonlinear cascade but also the cascade is a useful surface runoff simulator. Further, these results confirm that it is reasonable to take $x = 1.4$, $n = 3$ and k estimated by Eq. (3-7), and that with these values of the parameters the model predicts surface runoff well.

4.2 RESULTS AND DISCUSSION

From Table 4-1 it is clear that errors in predictions of runoff peak and its time are relatively small (less than 30%) in most cases. In some cases, although only a few, these errors are large. An examination of rainfall-runoff records indicated that these errors were high in those cases where (1) synchronization between rainfall and runoff observations was poor, and (2) infiltration was high so that rainfall-excess was not adequately represented. Errors in determination of rainfall-excess seem to be a major problem in most rainfall-runoff models (Singh and Woolhiser, 1976).

Table 4-1. Hydrograph peak predictions on agricultural watersheds, using estimated parameter k.

Watershed and location	Date of event	Observed hydrograph peak (cm/hr)	Estimated hydrograph peak (cm/hr)	Relative error in peak prediction	Observed hydrograph peak time (min)	Estimated hydrograph peak time (min)	Relative error in peak time prediction
Riesel (Waco), Texas	6-10-1941	2.240	2.195	0.020	81	77.6	0.042
C	6-23-1949	1.588	2.651	-0.670	107	65.4	0.389
	7- 9-1961	0.127	0.126	0.002	30	95.4	-2.181
	5-10-1965	3.512	3.947	-0.124	45	52.0	-0.156
D	5- 6-1955	0.693	0.111	0.840	33	114.6	-2.473
	7-10-1961	0.417	0.543	-0.302	86	89.8	-0.044
	5-10-1965	2.272	3.679	-0.619	41	57.8	-0.410
G	7-14-1941	0.230	0.273	-0.185	163	139.2	0.146
	11- 4-1959	0.189	0.260	-0.380	198	216.6	-0.094
	3-29-1965	2.414	4.086	-0.693	145	147.8	-0.019
Y	4-24-1957	4.597	2.783	0.395	31	59.6	-0.923
	6- 4-1957	3.632	2.189	0.397	37	52.4	-0.416
	3-29-1965	5.198	5.623	-0.082	80	118.2	-0.448
Y-2	4-24-1957	4.267	4.346	-0.018	37	49.6	-0.341
	5-13-1957	3.150	3.165	-0.005	35	44.4	-0.269
	6- 4-1957	4.547	3.423	0.247	34	41.8	-0.229
	3-23-1965	5.975	5.837	0.023	82	115.4	-0.407
Y-4	4-24-1957	4.089	5.263	-0.287	48	44.0	0.083
	5-13-1957	2.896	3.836	-0.325	47	42.0	0.106
	6- 4-1957	4.039	4.574	-0.133	37	36.6	0.011
	3-29-1965	6.344	6.447	-0.016	66	79.6	-0.206
Y-6	4-24-1957	2.667	6.176	-1.316	44	38.0	0.136
	5-13-1957	2.040	4.906	-1.405	42	31.0	0.262
	6- 9-1962	2.540	1.801	0.291	41	36.6	0.122
	3-29-1965	6.838	6.644	0.028	78	82.0	-0.051
Y-7	4-24-1957	5.994	5.352	0.107	31	37.0	-0.194
	5-13-1957	5.156	4.203	0.185	33	40.6	-0.230
	6-23-1959	4.470	3.398	0.240	57	114.0	-1.000
	3-29-1965	5.778	6.491	-0.124	79	77.8	0.015
Y-8	5-13-1957	5.664	4.963	0.123	19	31.4	-0.653
	6-18-1961	0.199	1.236	-5.24	100	83.0	0.170
	3-29-1965	5.712	6.347	-0.111	61	76.0	-0.246

Table 4-1. Con't.

Watershed and location	Date of event	Observed hydrograph peak (cm/hr)	Estimated hydrograph peak (cm/hr)	Relative error in peak prediction	Observed hydrograph peak time (min)	Estimated hydrograph peak time (min)	Relative error in peak time prediction
Y-10	4-24-1957	6.858	6.321	0.078	35	37.0	-0.08
	5-13-1957	4.851	4.184	0.138	26	35.6	-0.369
	6- 9-1962	1.001	1.300	-0.299	38	39.4	-0.037
	3-29-1965	6.925	7.618	-0.100	69	82.4	-0.194
W-1	3-12-1953	2.870	1.222	0.574	59	48.8	0.173
	6-23-1959	4.800	3.246	0.324	87	122.6	-0.409
	3-29-1965	5.875	5.225	0.111	101	99.6	0.014
W-2	4-24-1957	5.182	5.882	-0.135	56	42.8	0.236
	5-13-1957	3.912	4.545	-0.162	56	37.2	0.336
	3-29-1965	4.653	5.771	-0.240	131	75.0	0.428
W-6	4-29-1949	1.113	3.007	-1.703	24	31.4	-0.308
	4-24-1957	5.588	5.772	-0.033	35	44.2	-0.263
	3-29-1965	4.874	5.526	-0.134	110	76.4	0.306
W-10	4-24-1957	7.087	6.025	0.150	28	33.8	-0.207
	4-24-1957	7.087	5.409	0.237	28	34.4	-0.229
	6-23-1959	4.978	4.839	0.028	63	56.2	0.108
	3-29-1965	4.495	6.673	-0.485	71	72.4	-0.020
SW-12	6- 4-1957	1.549	1.825	-0.178	29	17.8	0.386
	3-29-1965	10.172	9.566	0.060	103	63.0	0.388
SW-17	4-24-1957	7.366	7.963	-0.081	35	33.0	0.057
	5-13-1957	4.420	5.377	-0.217	26	21.4	0.177
	6- 9-1962	9.627	7.527	0.218	33	15.4	0.533
	3-29-1965	6.196	8.621	-0.391	110	69.4	0.369
Hastings, Nebraska							
W-3	9- 5-1946	1.298	0.957	0.263	148	171.2	-0.173
	6- 5-1949	0.358	0.176	0.509	74	100.8	-0.362
	6-25-1951	1.697	1.173	0.307	129	172.2	-0.335
	7-13-1952	3.378	2.602	0.230	124	151.8	-0.224
	6-15-1957	2.487	1.616	0.350	64	81.6	-0.275
	6-16-1957	1.727	1.194	0.309	154	165.6	-0.075

Table 4-1. Con't.

Watershed and location	Date of event	Observed hydrograph peak (cm/hr)	Estimated hydrograph peak (cm/hr)	Relative error in peak prediction	Observed hydrograph peak time (min)	Estimated hydrograph peak time (min)	Relative error in peak time prediction
W-8	6-27-1942	0.272	0.215	0.290	64	125.0	-0.953
	5-11-1944	0.452	0.113	0.750	75	191.0	-1.547
	7-16-1948	0.427	0.350	0.181	71	111.0	-0.563
	6- 1-1951	1.133	1.405	-0.24	154	202.0	-0.312
	6-26-1952	0.449	0.625	-0.391	106	123.0	-0.160
	7- 6-1952	0.220	0.097	0.559	62	179.0	-1.887
	6-16-1957	0.658	1.092	-0.662	212	191.0	0.099
W-11	6- 1-1951	0.831	0.646	0.222	247	233.0	0.057
	6-26-1952	0.205	0.310	-0.514	555	194.0	0.651
	7-13-1952	0.772	0.967	-0.252	256	175.0	0.316
	6-16-1957	0.583	0.890	-0.529	349	194.0	0.444
	5-21-1965	1.064	2.342	-1.201	560	155.0	0.723
2-H	8-11-1939	2.819	0.795	0.718	10	17.0	-0.700
	8- 7-1942	2.520	1.295	0.486	10	28.0	-1.800
	9- 7-1942	3.531	1.493	0.577	15	17.0	-0.133
	8- 7-1946	3.759	1.689	0.689	7	17.0	-1.429
	9- 5-1946	2.289	1.345	0.412	9	19.0	-1.111
	5-20-1949	0.704	0.126	0.822	3	30.0	-9.000
	6-12-1965	8.814	7.557	0.143	9	28.0	-2.111
	6-12-1965	2.157	1.777	0.176	13	20.0	-0.539
	6-29-1965	2.068	1.368	0.338	9	16.0	-0.778
4-H	8-11-1939	4.521	1.897	0.580	5	15.0	-2.000
	6-20-1942	5.817	3.488	0.400	8	17.0	-1.125
	9- 5-1946	3.886	1.561	0.598	12	19.0	-0.583
	6- 1-1951	6.756	4.756	0.296	123	135.0	-0.098
	7-13-1952	9.195	8.605	0.064	21	21.0	0.000
	6-12-1958	0.920	0.253	0.725	14	24.0	-0.714
	6-12-1965	9.703	6.588	0.321	19	19.7	0.000
	6-12-1965	6.147	3.474	0.435	7	18.0	-1.571

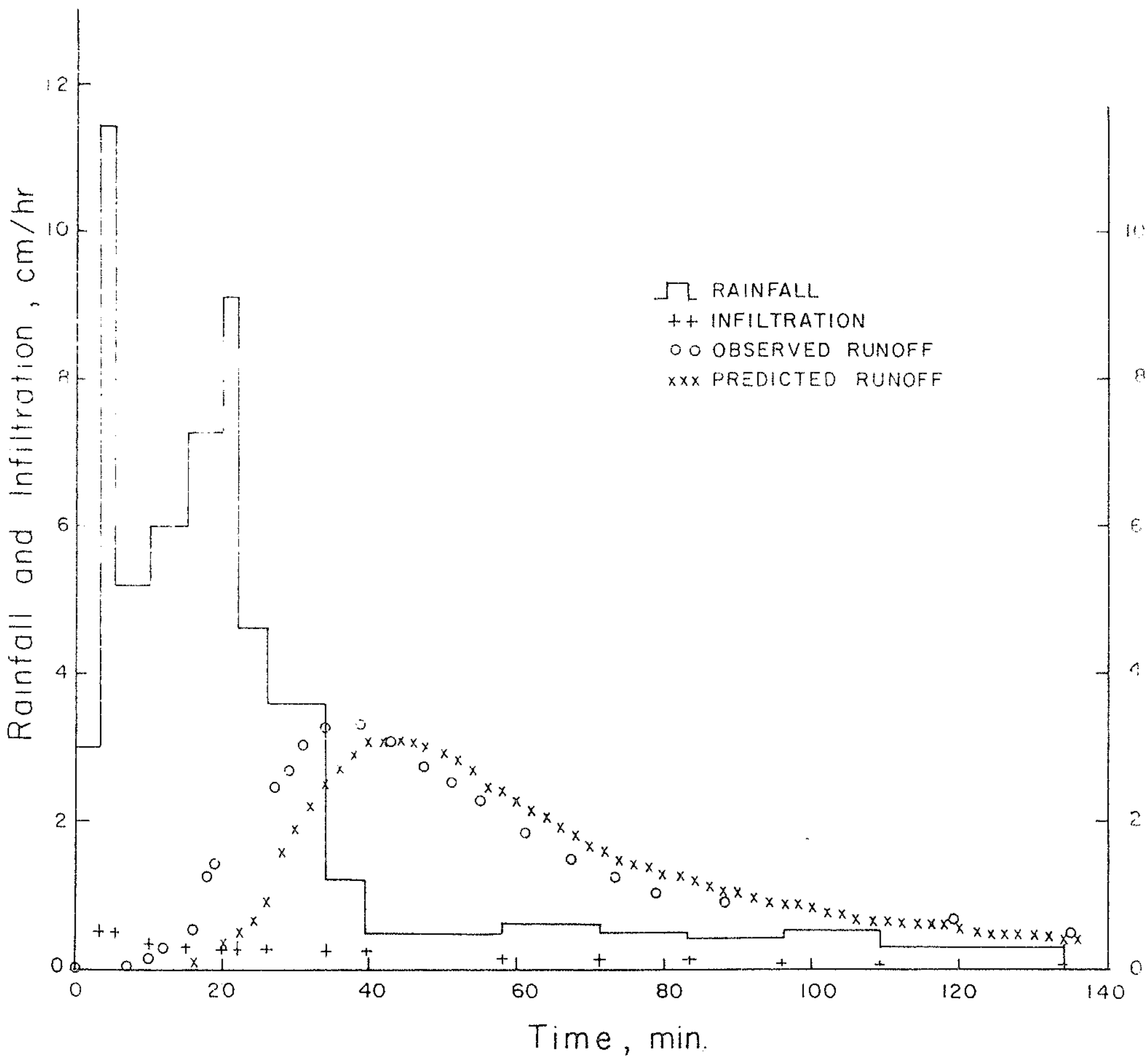


Fig. 4-1. Hydrograph prediction by the model for rainfall event of 5-13-1957 on watershed Y-2, Riesel (Waco), Texas.

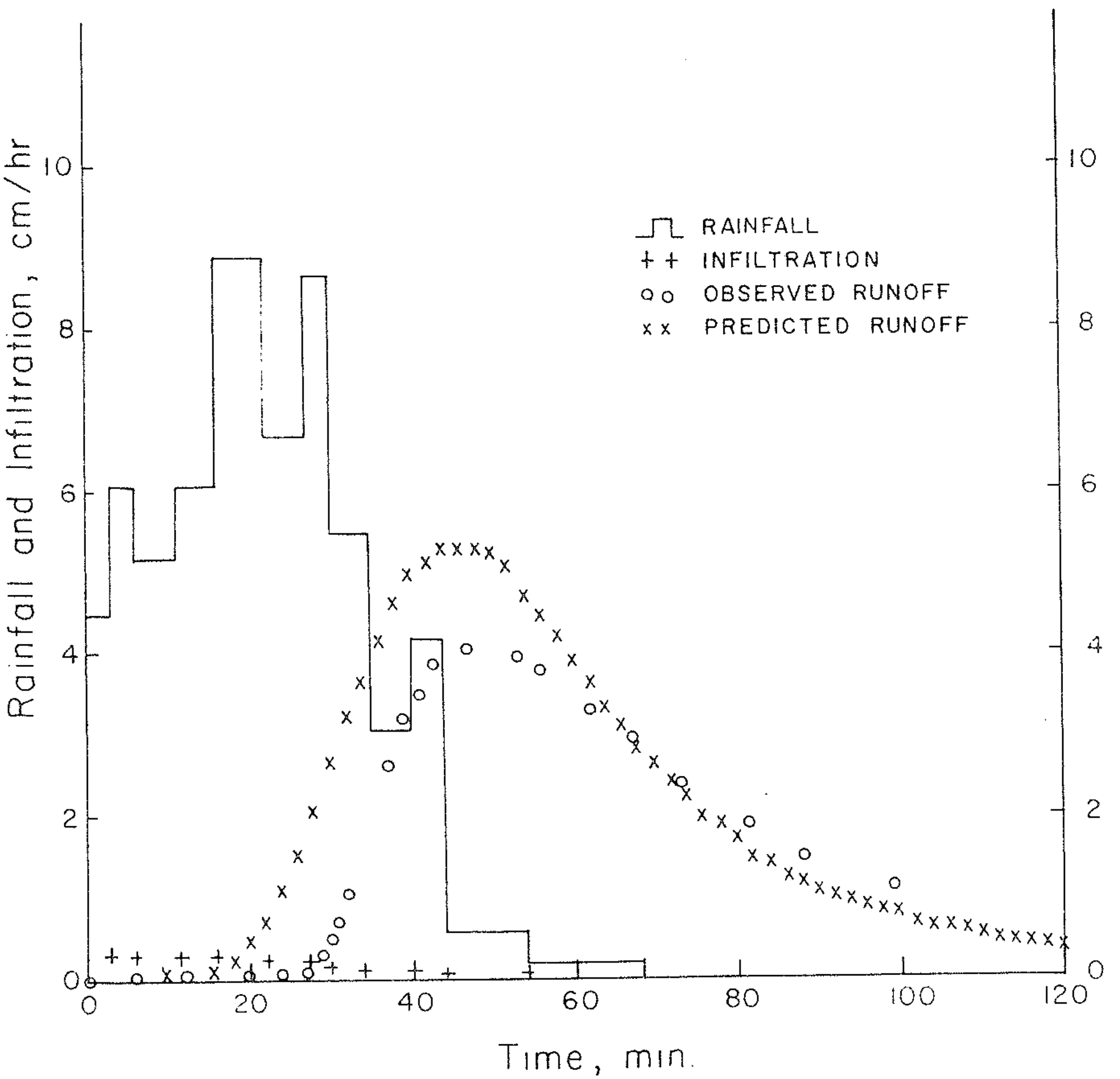


Fig. 4-2. Hydrograph prediction by the model for rainfall event of 4-24-1957 on watershed Y-4, Riesel (Waco), Texas.

It is well recognized that no matter how sophisticated a runoff model is, its output accuracy cannot exceed the accuracy of the input that goes into it. It goes without saying that rainfall-excess can never be accurately estimated for two reasons: (1) the very concept of rainfall-excess is erroneous, (2) most infiltration equations do not provide true representation of infiltration phenomenon and furthermore, there is the difficulty in estimating their parameters. Philip's equation, used in this study, suffers from these same handicaps.

To further examine the errors in hydrograph predictions, observed values were plotted against predicted values for both hydrograph peak and its time as shown in Figs. 4-3 and 4-4 respectively. On these figures ±50% relative error limits have also been drawn. Although these figures manifest a wide scattering, a close examination will reveal that the fit is not as bad as it looks because a large number of events have been plotted and only relatively a few events are far off the plot for the reasons cited above. Preliminary statistical calculations indicated that (1) mean and standard deviation of observed peaks were 3.606 and 4.425 respectively, (2) mean and standard deviation of predicted values were 3.4 and 4.208 respectively, (3) correlation coefficient and standard error of estimate for observed and predicted peaks were 0.9604 and 1.2391 respectively, (4) mean and standard deviation of observed peak time were 79.43 and 123.24 respectively, (5) mean and standard deviation of predicted peak time were 76.8 and 95.47 respectively, and (6) correlation coefficient and standard error of estimate for observed and predicted peak time were 0.845 and 66.273 respectively. These statistics point again toward a close agreement between observations and model results.

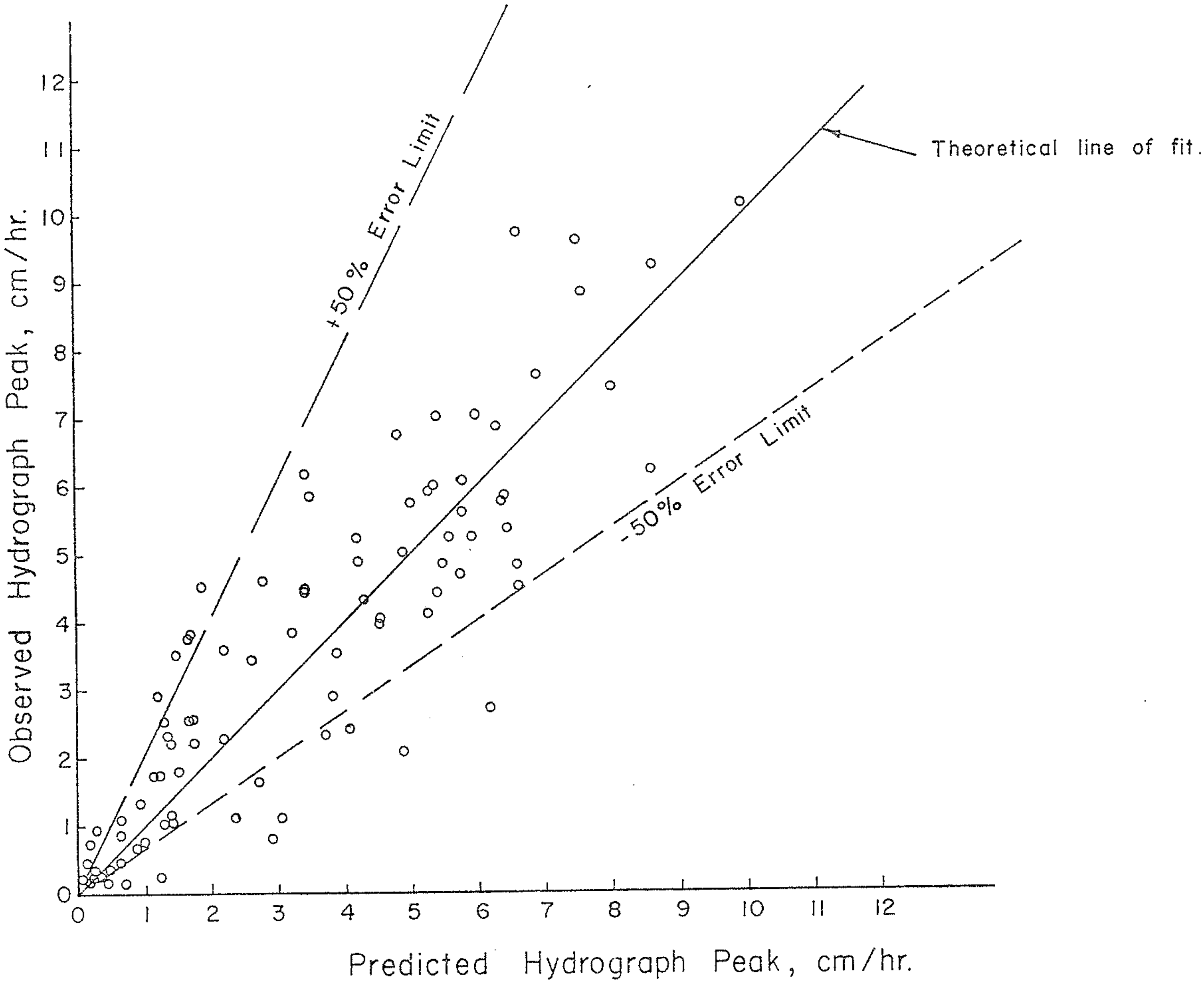
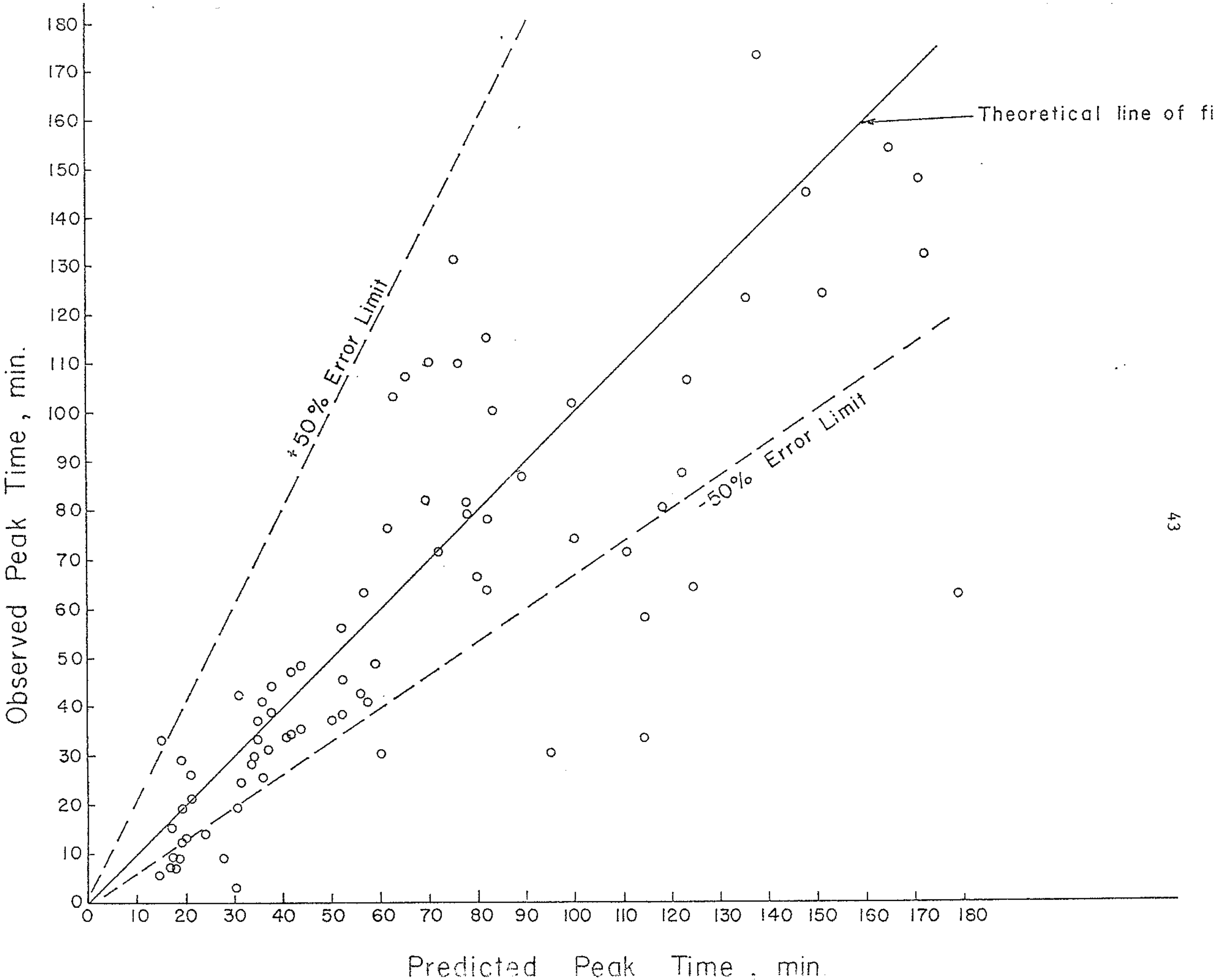


Fig. 4-3. Observed hydrograph peak versus predicted hydrograph peak from agricultural watersheds.


Observed Peak Time, min.
Predicted Peak Time, min.
+50% Error Limit
-50% Error Limit
Theoretical line of fit
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180

CHAPTER 5

CONCLUSIONS

The following conclusions are drawn from this study:

(1) The uniformly nonlinear hydrologic cascade is a useful surface runoff simulator. A close agreement between observed and predicted hydrographs suggests that the cascade does seem to account for the important nonlinear effects in surface runoff.

(2) Its relative simplicity and good predictive ability can be the basis to become an operational tool in routine hydrologic applications.

(3) Based on its application to 21 small, natural, agricultural watersheds it is concluded that the number of storage elements n can be fixed at 3 and the parameter x at 1.4, and that the parameter k can be estimated reliably from topographic characteristics of a given watershed. Thus the cascade can be completely specified.

(4) Because of smallness of n, hydrograph computations can be easily performed with a disc calculator or a minicomputer.

(5) A state-space variable representation of the cascade model is useful for computer programming.

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