Bulletin 15:

INSTANTANEOUS UNIT HYDROGRAPH RESPONSE BY HARMONIC ANALYSIS

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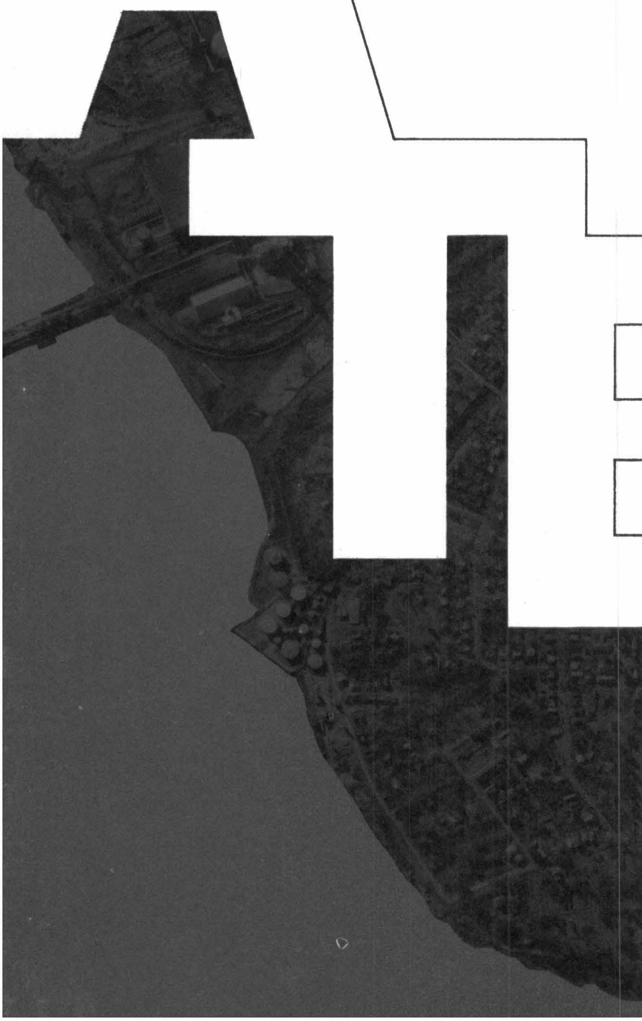

INSTANTANEOUS UNIT HYDROGRAPH RESPONSE
BY
HARMONIC ANALYSIS

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PREFACE

Prediction of streamflow is a primary need in hydrological application. The time span of the prediction can range from several hours or days for flood and outlet work studies to months or years for water-supply determination. The research reported here was proposed to test the efficacy of a particular method of short-term analysis and prediction.

The method studied was suggested by work done by O'Donnell, who proposed the use of harmonic analysis to describe the Instantaneous Unit Hydrograph. The Instantaneous Unit Hydrograph can be described as the curve that results when the discharge past a given point in the stream is plotted versus time, as a unit quantity of rainfall excess is released instantaneously over the watershed. Harmonic analysis is a mathematical technique which expresses the form of the hydrograph function in a finite series.

The basic problems presented in the course of the research show that some further definition work is necessary because the study results indicate inadequacies in the present details of the method. The results of the prediction calculations are highly dependent on the number of data presented in the computation. The watersheds do not always exhibit consistency in the predictor forms.

Despite the problems, the method offers promise in operational streamflow prediction. The method is particularily adaptable to the digital computer, and it lends itself to continuous, real-time hydrograph computation if given data from remote sensing devices such as rain gages, weirs and ground-water monitors.

William R. Walker
Director

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INTRODUCTION

Since the dawn of civilization man has recognized a relationship between rainfall and runoff. Only within the last century, however, have hydrologic principles been developed into a science.

The runoff hydrograph can be defined as a graph of stream discharge versus time. The runoff hydrograph consists primarily of two parts: direct runoff, which is the runoff directly associated with causative rainfall or snowmelt, and base flow, represented by the sustained or dry weather flow of streams.

In 1932, L. K. Sherman (11), a consulting engineer, introduced the concept of the unit hydrograph, often referred to as the unit graph. The unit hydrograph may be defined as the typical basin discharge hydrograph resulting from one inch of direct runoff generated uniformly over the area at a uniform rate during a given period of time. It is to be noted that the unit-graph concept applied only to the direct surface runoff portion of the runoff hydrograph. Since the first proposal of this intrinsically empirical tool some 35 years ago, the unit graph approach to direct surface runoff flow prediction has developed into one of the best known and most successful tools in the hands of the applied hydrologist. It is still widely used in essentially the same form as first proposed by Sherman.

Hydrologists have become quite familiar with the underlying assumption of the unit hydrograph theory--the linearity of the drainage basin response with respect to the magnitude of applied precipitation. This assumption leads directly to the three principles of the unit hydrograph. First, for a given drainage basin, and for storms of equal duration, but regardless of the intensity of the rainfall, the base times of surface runoff hydrographs are the same. Second, for a given drainage area and for similar storms of equal duration, the ordinates of the surface runoff hydrographs are proportional to the volume of the precipitation excess. Third, the time distribution of surface runoff from a given storm period is unaffected by the time distribution of surface runoff produced by any other successive storm period for a given drainage basin. Thus, a strength of the unit hydrograph is the principle of superposition, whereby ordinates of runoff hydrographs produced by successive periods of precipitation excess can be added to produce an overall surface runoff hydrograph for the entire storm. The main agrument against the unit-graph concept is that it is an approximation and does not truly represent the relationship between precipitation excess and direct surface runoff. The argument is valid, for it is true that the assumption of linearity is a simplification. The proponents of the unit graph concept maintain that although it is an approximation, it is a good first approximation in the prediction of direct surface runoff, and it is a hydrologic tool that can be put into operation easily. A few opponents of the unit hydrograph approach have proposed elaborate hydrologic theories. Although the theories may be sound, they usually involve the measurement of parameters not currently measured, or measurable.

Subsequent to the introduction of the concept of the unit hydrograph into the literature by L. K. Sherman, the idea of an instantaneous unit hydrograph emerged. The instantaneous unit hydrograph, or IUH, can be defined as the hydrograph of direct surface runoff caused by one inch of precipitation excess being released instantaneously and uniformly over a catchment basin.

According to Eagleson, Mejia, and March (4) there are three fundamental approaches to determining the IUH. The first method, which they called "general system synthesis," involves studing the drainage basin in detail and writing equations that describe its dynamic processes accurately. Having thus simulated physical processes mathematically, the IUH can be obtained as the solution to the derived equations. The second method they called "parametric system synthesis." In the second method, a simple analog model of the drainage basin containing a series of linear elements, such as linear reservoirs, is assumed to accurately depict the routing of flow through the basin. The model contains a number of system parameters which are measurable or can be determined. A form of the IUH, with parameters to be determined, is known accordingly. The parameters may be found empirically from any single measurement of input, such as effective rainfall, and output, such as direct surface runoff. The third method they called appropriately "black box analysis." In the third method, the IUH is found from the measurement of imput and output data for a drainage basin. Little consideration is given to actual dynamic processes within a basin, except to assume that they are linear.

The purpose of this paper is to examine one of the "black box" techniques. In 1960, O'Donnell (10) showed that rainfall excess and surface runoff can be related by the IUH through harmonic analysis. By treating each storm as a repeating event, harmonic coefficients for effective rainfall, surface runoff, and the associated IUH can be obtained. O'Donnell asserted that once the harmonic coefficients for the IUH from one storm event have been determined, that IUH can be used with any subsequent rainfall excess data for the drainage basin for the purpose of predicting direct surface runoff. The procedure is to combine the harmonic coefficients of the resulting surface runoff curve. From the latter, the resulting surface runoff curve can be developed readily.

O'Donnell demonstrated his technique using a set of data that Nash (7) has used previously in deriving an IUH by a different method. By means of an electronic computer, O'Donnell obtained harmonic coefficients for a rainfall excess curve, and for the IUH curve relating them. He then took the rainfall excess harmonic coefficients and the IUH coefficients, combined them, and obtained the surface runoff coefficients. These were then used to develop the surface runoff curve. The curve obtained was essentially the same as the curve with which he had started. Although he had claimed that a derived IUH could be used with subsequent rainfall excess data to predict surface runoff, he did not try this. Consequently he made no comparison between predicted and observed direct surface runoff for storms other than the one from which the IUH was obtained.

The purpose of this study in particular was to examine the use of O'Donnell's theory with application to a number of sets of effective rainfall-surface runoff data from several drainage basins. Consideration is given to the questions of limitations in the basic data which can reasonably be expected to be found available.

LITERATURE REVIEW

In 1923, L.K. Sherman (11) gave applied hydrology a new tool, the "unit-hydrograph." The literature is well supplied with articles concerning the use of the unit hydrograph as propounded by Sherman. Any good, modern textbook in hydrology also treats the subject in some length. (1), (6), (13).

Eagleson, Mejia, and March (4) in 1965 indicated three fundamental approaches to finding the instantaneous unit hydrograph. The first they called "general system synthesis"; the second they called "parametric system synthesis"; and the third they called "black box analysis."

C.O. Clark (2), in 1945, was one of the first to use the general system synthesis approach to finding the instantaneous unit hydrograph. His approach was to consider that a unit rainfall excess over a drainage basin reaches the basin outlet in the form of a time-area concentration curve. He routed the time-area concentration curve through a single element of storage using the Muskingum-x method of flood routing. In his study he considered the basin to provide pure outlet control over storage. The resulting unit hydrograph thus derived was then used to produce unit hydrographs for rains with durations of particular interest. The unit hydrographs were then combined with precipitation excess data to produce surface runoff hydrographs. The surface runoff hydrographs produced in this manner showed remarkable correlation with the corresponding hydrographs observed.

In 1957, J. E. Nash (7) derived a general IUH using the second approach--parametric system synthesis. To derive the general equation, Nash assumed that the basin could be replaced by a series of n reservoirs of linear storage with equal delay time. The form of storage equation that he assumed was $S = KQ$, where S is storage, Q is outflow, and K is the linear constant of proportionality. He derived a two-parameter, exponential equation for the instantaneous unit hydrograph. The two parameters involved are n, the number of equivalent linear reservoirs by which the basin could be replaced, and k, the constant of proportionality in the linear storage equation. The general equation for the ordinates of the unit hydrograph, u, is,

$$u = \frac{v}{k\,\Gamma(n)}\; e^{-t/k}\;(t/k)^{n-1}$$

where v is the volume of the instantaneous unit hydrograph, and the other items are as defined above. The general equation is statistically correlated with the observed rainfall excess and surface runoff data by means of the first and second moments of the curve described by the equation. In 1959, Nash (8) amplified the subject of IUH parameters, particularly their correlation with drainage basin characteristics. He indicated that statistical correlation should be made using the first, second, third, and any significant moments of the effective rainfall, surface runoff, and IUH curves. Nash pointed out that it is to a hydrologist's advantage if the IUH can be expressed as an algebraic equation, u = u(t), where the ordinates of the IUH, u, are a function of time.

J. C. I. Dooge (3), in 1959, following the work of Clark, Nash, and others, prepared a general theory of the unit hydrograph. Dooge made the physical assumption that the reservoir action performed by a drainage basin an excess rainfall could "be separated from the translatory action and lumped into a number of reservoirs unrestricted in number, size, or distribution." He suggested that the general equation for the unit hydrograph that he derived could be made "a simple integral by assuming that above each confluence in the catchment the reservoirs are equally distributed for equal lengths of tributary." This assumption, he claimed, reduced the complexity of the computation required. He also claimed that the ordinates of the unit hydrograph could then be obtained by integrating the product of the time-area concentration curve and an ordinate of the Poisson probability function. His expression for the ordinate of the instantaneous unit hydrograph is:

$$u(0,t) = \frac{V_o}{T} \int_o^{t \leq T} P(m,n-1)\, \omega(\tau')\, dm$$

where V_o is the volume of rainfall excess, T is the maximum translation time in the drainage basin, P(m,n-1) is the Poisson probability function, $\omega(\tau')$ is the ordinate of the time-area concentration curve, m is a dimemsionless time variable, and n is the number of reservoirs. Dooge asserted that the solutions of Clark (2), and Nash (7) are special cases of his general equation. Gray (5) and Wu (14) applied the use of instantaneous unit hydrograph theory to construction of design curves for small watersheds. Singh (12) attempted to extend the theory of Dooge to cases where the drainage basins do not offer linear storage.

In 1960, O'Donnell (10), basing his work on that of Nash, made use of harmonic analysis to derive the ordinates of the IUH. In his black-box analysis method, O'Donnell devised a computer program for calculating the ordinates of the instantaneous unit hydrograph from rainfall excess and surface runoff data. He demonstrated the validity of these theoretical considerations by first taking a set of effective rainfall and corresponding surface runoff data for a drainage basin, and processing them through a computer to produce the ordinates of the related IUH. He then was able to take the original rainfall excess data and the ordinates of the IUH to reproduce the original surface runoff data exactly (within the margin of computer round-off). O'Donnell's theory will be explored more thoroughly in the section on theoretical considerations in this study.

More recently, attempts have been made to construct unit hydrographs for various time durations by means of computers. D. W. Newton and J. W. Vinyard (9) with the Tennessee Valley Authority, developed a computer procedure to derive a unit hydrograph from rainfall and streamflow records of complex floods using matrix theory. Their approach was basically that of Sherman. One interesting innovation was a correction for improperly determined rainfall excess. Their computer program is designed to allow sufficient successive approximations to adjust the rainfall excess amounts so that they correspond properly to the observed runoff values when combined with the applicable unit hydrograph.

Eagleson, Mejia, and March (4), of M.I.T., have also summarized other methods of obtaining the unit hydrograph, although not necessarily making use of the instantaneous unit hydrograph. In addition, they introduced a computer solution for the unit hydrograph, using linear programming techniques.

THEORETICAL CONSIDERATIONS

For a given drainage basin, the curve of the distribution of precipitation excess with time is related to the resultant direct surface runoff (DSRO) hydrograph by the instantaneous unit hydrograph according to presently accepted unit hydrograph theory. These three curves can be related by the convolution integral (Duhamel's integral):

$$Q(t) = \int_{o}^{t} i(\tau)\, d\tau\, U(t-\tau) \tag{1}$$

Q(t) is the rate of direct surface runoff at time, t, after the beginning of rainfall excess. U(t-τ) is the ordinate of the IUH at time, t - τ, from the origin of the IUH. τ represents a variable time, $0 \leq \tau \leq T_i$, where T_i is the duration of the precipitation excess. i(τ) denotes the intensity of rainfall excess at time, τ, from the beginning of rainfall excess.

According to O'Donnell (10), if the duration of direct surface runoff is taken to be T, and DSRO begins with the inception of precipitation excess, equation (1) above may be written as:

$$Q(t) = \int_{t-T}^{t} i(\tau)\, d\tau\, U(t-\tau) \tag{2}$$

This manipulation allows the expression above to be considered as an event that successively repeats itself at intervals of T, thus lending the previously discrete rainfall-runoff event to harmonic analysis.

The surface runoff at any time, t, after initiation of effective rainfall, can be expressed by the infinite Fourier expansion:

$$Q(t) = \sum_{r=0}^{\infty} A_r \cos r \frac{2\pi t}{T} + \sum_{r=1}^{\infty} B_r \sin r \frac{2\pi t}{T} \tag{3}$$

where Q(t) = Q (t + T) = Q(t + 2T) = ...

Similar expressions exist for i(τ) and U(t-τ):

$$i(\tau) = \sum_{n=0}^{\infty} a_n \cos n \frac{2\pi\tau}{T} + \sum_{n=1}^{\infty} b_n \sin n \frac{2\pi\tau}{T} \tag{4}$$

and

$$U(t-\tau) = \sum_{m=0}^{\infty} \alpha_m \cos m \frac{2\pi(t-\tau)}{T} + \sum_{m=1}^{\infty} \beta_m \sin m \frac{2\pi(t-\tau)}{T} \tag{5}$$

O'Donnell (10) substituted equations (4) and (5) into equation (2) and found that Q (t) was the sum of an infinite number of integrals, all of which belonged to one of four types. He found that these four types of integrals reduced to zero when $m \neq n$ but had the following values when $m = n$:

Type I $= +\frac{T}{2} a_n \alpha_n \cos n \frac{2\pi t}{T}$, but $= +T\, a_o \alpha_o$ if $m = n = 0$

Type II $= +\frac{T}{2} a_n \beta_n \sin n \frac{2\pi t}{T}$

Type III $= +\frac{T}{2} b_n \alpha_n \sin n \frac{2\pi t}{T}$

Type IV $= -\frac{T}{2} b_n \beta_n \cos n \frac{2\pi t}{T}$

Thus it can be easily seen that Q(t) becomes:

$$Q(t) = T a_o \alpha_o + \sum_{n=1}^{\infty} \frac{T}{2} (a_n \alpha_n - b_n \beta_n) \cos n \frac{2\pi t}{T} + \sum_{n=1}^{\infty} \frac{T}{2} (a_n \beta_n + b_n \alpha_n) \sin n \frac{2\pi t}{T} \tag{6}$$

Comparing equation (6) with equation (3), the coefficients of the nth harmonics of Q(t) are:

$$A_o = T a_o \alpha_o \tag{7}$$

$$A_n = \frac{T}{2} (a_n \alpha_n - b_n \beta_n) \qquad \text{where } n \geqslant 1 \tag{8}$$

$$B_n = \frac{T}{2} (a_n \beta_n + b_n \alpha_n) \tag{9}$$

Equations (7), (8), and (9) are the basis of the harmonic linkage between effective rainfall and direct surface runoff. Solving equations (7), (8), and (9) for the coefficients of the IUH, α_n and β_n, gives:

$$\alpha_o = \frac{1}{T} \frac{A_o}{a_o} \tag{10}$$

$$\alpha_n = \frac{2}{T} \frac{a_n A_n + b_n B_n}{a_n^2 + b_n^2} \tag{11}$$

$$\beta_n = \frac{2}{T} \frac{a_n B_n - b_n A_n}{a_n^2 + b_n^2} \tag{12}$$

O'Donnell's work (10) centered around the last three equations. With these three equations, the harmonic components of the IUH can be found from the rainfall excess and direct surface runoff curves by harmonic analysis. If the actual ordinates of the IUH are desired, they can be calculated by applying the harmonic coefficients of the IUH to a unit rainfall excess. If, however, all that one is concerned with is deriving a direct surface runoff hydrograph from new, specified rainfall excess data, then the harmonic coefficients of the IUH can be applied directly to the new rainfall excess data. Quoting O'Donnell (10), "As long as one has rainfall excess and runoff data from one storm on the catchment equations [(10), (11) and (12)] can be used to find the α and β coefficients of the IUH for that catchment." "These can be used immediately in equations [(7), (8) and (9)] along with the coefficients of the new rainfall excess curve to find the coefficients of the new runoff hydrograph."

Theoretically, any measurement of rainfall excess input and direct surface runoff output may be used to obtain the harmonic coefficients of the IUH. For a given set of rainfall excess and DSRO data, the IUH coefficients are unique without regard to the linearity of the basin response. If the basin response is truly linear, then the IUH coefficients derived, as above, are valid for all rainfall excess inputs and are independent of such inputs. Of course, present unit hydrograph theory indicates that different unit hydrographs should be used for effective rainfall patterns which approach the drainage basin from other than a standard direction. O'Donnell (10) drew a distinction between infinite and finite harmonic series, of which rainfall and runoff data are representative of the latter. With a finite set of data, only a finite harmonic series can be fitted, having a finite number of coefficients. More specifically, if there are only p observations, then only p coefficients can be found divided between sine and cosine terms. Instead of summing terms to infinity, cosine and sine terms are now summed respectively to (p-1)/Z. According to O'Donnell (10) "With p items of data, a finite harmonic series of p terms not only sums exactly to be observed values at the relevant points-it also gives the smoothest possible continuous function fitting that data."

O'Donnell (10) indicated two major sources of error in his method. The first is more critical; it is errors in basic data, leading to harmonic coefficients of the IUH which are in error. The second is the use of harmonic coefficients in the place of Fourier coefficients, which apply to the infinite series. Errors of the second type are inherent in the use of harmonic analysis, but can be reduced by the use of larger numbers of data points. Errors of the second type decrease in proportion to $\sqrt{p}$, where p is the number of data points used. To use this method of IUH derivation, one should have a sufficient number of data points. Just what number can be considered sufficient for a reasonable IUH derivation is a question that was not answered explicitly by O'Donnell.

PROCEDURE

Computer programs compatible with the IBM 7040-1401 computer were written in order to make practical application of the theoretical considerations previously discussed. The computer programs written, utilized the same basic routines and subroutines as much as possible. Each program was modified somewhat from the others to achieve different ends.

The first basic program (Appendix A) was written to calculate the harmonic coefficients of the IUH from rainfall excess and direct surface runoff data and to check the coefficients against the source data. The latter was performed by using the derived harmonic coefficients of the IUH with the source rainfall excess data to compute the direct surface runoff, which was then compared with the source DSRO data for closeness of fit.

The second basic computer program (Appendix B) was written to compute direct surface runoff, using perviously derived IUH harmonic coefficients with rainfall excess data entered by intervals. The second program computes direct surface runoff progressively as rainfall excess data becomes available, interval by interval. When the second program is used with a unit rainfall excess in a standard interval, with each set of IUH harmonic coefficients a unit hydrograph is obtained for the rainfall coefficients. In the process, each resulting unit hydrograph was examined for overall shape and also to verify its volume to be unity.

Data were taken from a number of small drainage basins in Macomb, Oakland, and Wayne Counties in the area around Detroit, Michigan. Precipitation data was weighted using the Thiessen method and became the rainfall data used as the basis of the present study. The runoff data were divided into intervals of two hours (in a few cases, the intervals were one hour) and were measured in cubic feet per second. Although the original data were not available to the authors, the data made available were reliable, since the compilation had been made under careful supervision. The data had been selected from precipitation and runoff data of reasonably well defined storm patterns. It should be noted that in the area around Detroit, Michigan, the usual storm proceeds from west to east and, so far as is known, such is the pattern of storms which the data represent.

In the present study, the data were plotted so that the authors could ascertain which data could be used most effectively. Storms with extremely complicated rainfall patterns were omitted from further consideration. For the remaining storms, either smooth, single-peaked hydrographs, or those that could be divided into smooth, single-peaked curves, were selected for further study. In the latter case, hydrographs that showed additional humps caused by minor precipitation, subsequent to the main storm, were reduced to smooth, single-peaked hydrographs by using accepted methods of recession curve separation techniques [(1), (6), (13)].

The hydrographs finally selected and adjusted were then divided into base flow and direct surface runoff. This was done by connecting a straight line between the point of maximum curvature of the rising limb and the point of maximum curvature of the falling limb of the storm hydrograph. Although this method of separation cannot be considered ideal, it is an acceptable first approximation to more precise separation [(1), (6), (13)]. The authors felt that the data in hand were so limited as to warrant no further refinement in separation.

The separation of the total precipitation data into precipitation excess on the one hand, and infiltration and other losses on the other, posed a more difficult problem. Examination of the data available showed surface runoff, for the various storms being considered, to vary from 5 to 50 per cent of the total causative rainfall. No accepted method of separation seemed to apply precisely to the situation encountered. Although the f-capacity method is perhaps the most realistic, it was felt that insufficient data was available to justify or even allow the use of this method [(1), (6), (13)]. After trying the Φ-Index method [(1), (6)] with little success, the authors chose a runoff-percentage method (1), realizing that although an infiltration curve was practically unattainable with the data available, such a curve depicts reality better than curves from most other available methods. The volume of direct surface runoff for each storm was matched with its causative rainfall volume to arrive at a runoff percentage. Each item of the original weighted precipitation data for the storm was multiplied by the runoff percentage to give the rainfall excess items used in further analysis.

Finally, all units were converted to cubic feet per second per square mile. These units have the advantage of permitting ready comparison of rainfall and runoff from basin to basin at a glance.

RESULTS

A total of 38 storms were selected from the data supplied, the hydrographs of which appeared to be readily reducible to smooth, single-peaked hydrographs. Table 1 gives the distribution of the storms selected from the various data available. For each storm, precipitation excess was divided into baseflow and direct surface runoff, as outlined before. All values were expressed in units of cubic feet per second per square mile.

Each set of rainfall excess and direct surface runoff data was entered into the computer with a program (Appendix A) which developed a set of harmonic coefficients for the IUH relating the rainfall excess and DSRO for each storm. The program then made use of the set of IUH coefficients previously generated, with the original rainfall excess data, to reproduce the direct surface runoff data for the same storm. Within the limits of computer round-off; the direct surface runoff data synthesized from rainfall excess and corresponding IUH harmonic coefficients were an exact reproduction of the original data from which the IUH coefficients had been derived. In essence, such was the limit of accomplishment reported by O'Donnell (10).

Unit hydrographs were produced from the IUH harmonic coefficients by introducing a unit of rainfall excess in the second computer program (Appendix B) with each set of IUH coefficients. Each unit hydrograph so developed was examined carefully with regard to its overall shape.

Several writers (4), (10), have commented on the oscillations noted in the tail of the unit hydrograph which was cited by O'Donnell (10) as an example. O'Donnell attributed the observed oscillations to being "an effect inherent in the harmonic synthesis of a function which has a discontinuity in its derivative, such as occurs at the start of the IUH." Of the 38 harmonic IUH's developed in this study, many were observed to have rather pronounced oscillations throughout the entire IUH as well as in the tail portion. See Figures 1 and 2 as examples. However, a few of the harmonic IUH's developed were found to be smooth curves with little oscillation in the tail portion or elsewhere. See Figure 5, in particular, as an example. Figures 3 and 4 also seem to be fairly good IUH's for the basin to which they pertain. It was observed in this study that storms which have rainfall excess confined to one, two, or three intervals (of runoff measurement) exhibited the smoothest harmonic IUH's. This was true in the case of the storm occuring in Oakland County, basin 5, on June 9, 1963 (see Figure %). For the same basin, the bulk of the rainfall for the storms of June 14, and June 16, 1960 was concentrated within two or three intervals, but significant rainfall did occur in additional intervals. The IUH's (see Figures 3 and 4) for these storms show some

TABLE I

County	Drainage Basin	No. of Storms Used
Macomb	2	7
Macomb	3	12
Oakland	5	13
Wayne	3	6
		Total: 38

Distribution of storms selected for harmonic analysis (area near Detroit, Michigan).

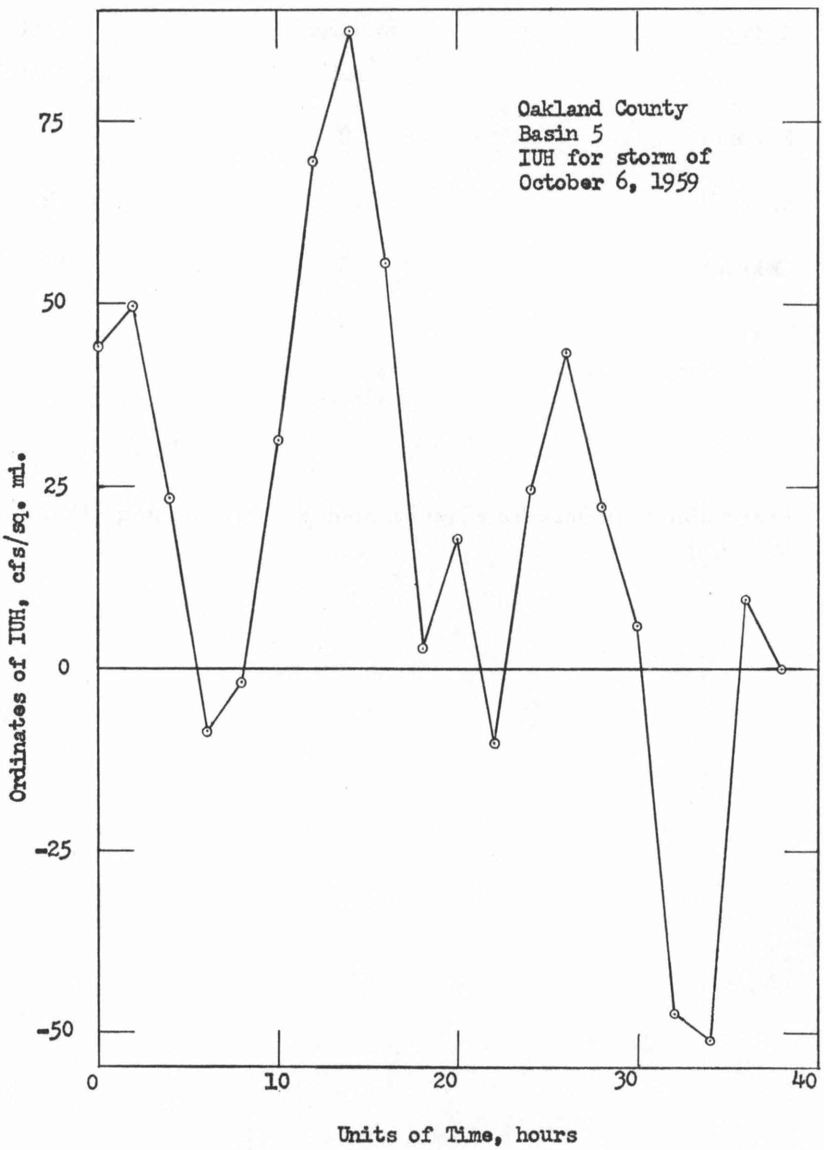

Figure 1: IUH for Storm of October 6, 1959, Oakland County, Michigan, Basin 5.

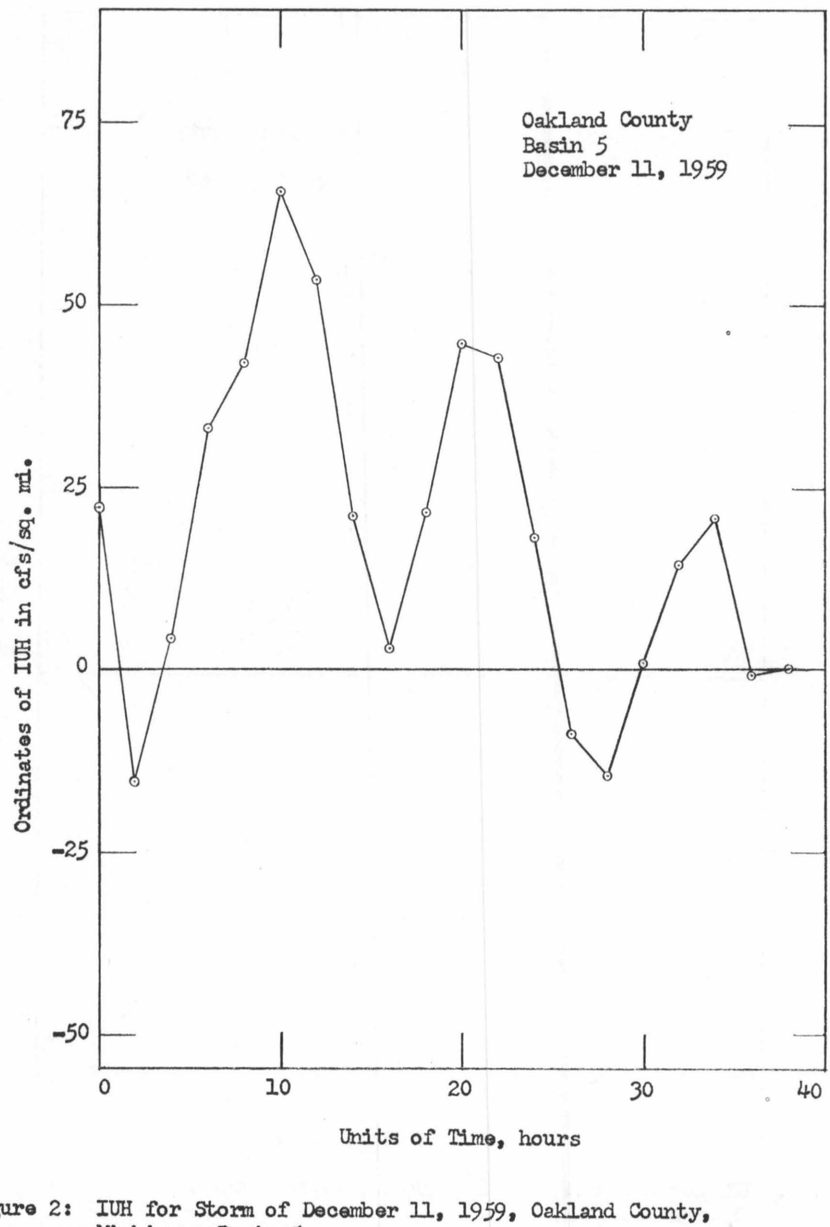

Figure 2: IUH for Storm of December 11, 1959, Oakland County, Michigan, Basin 5.

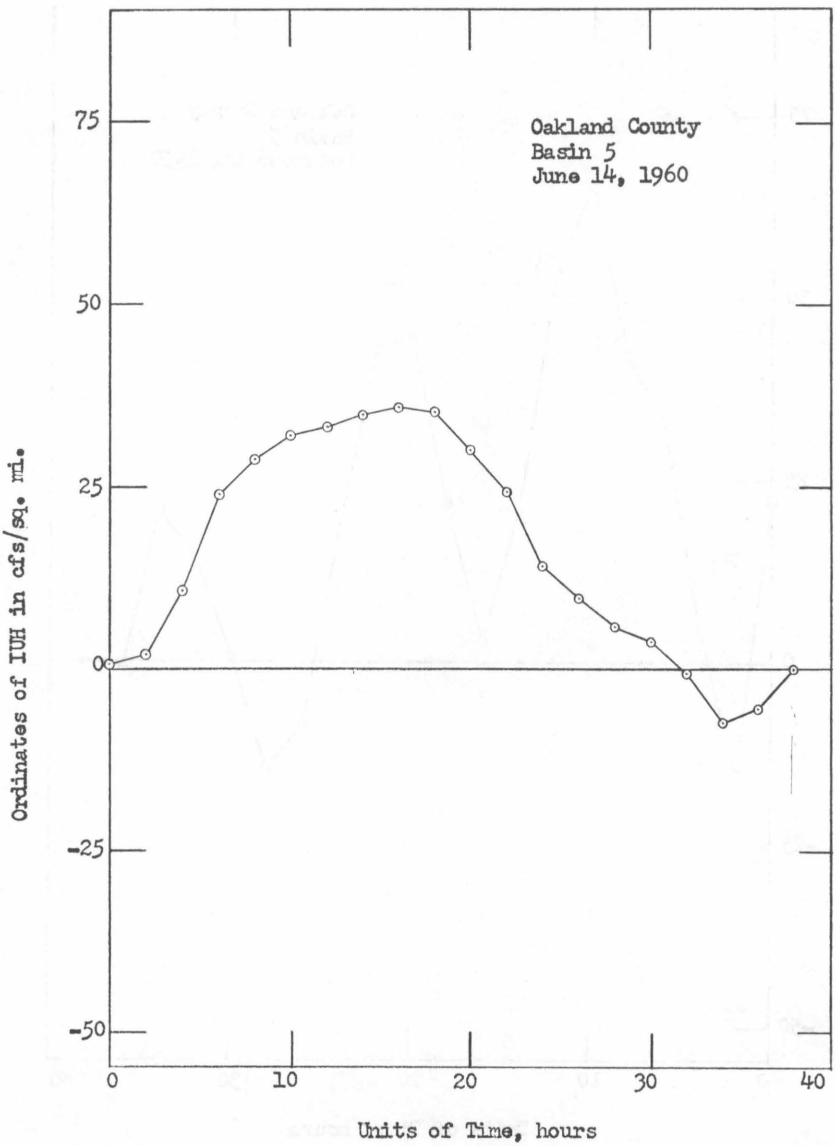

Figure 3: IUH for Storm of June 14, 1960, Oakland County, Michigan, Basin 5.

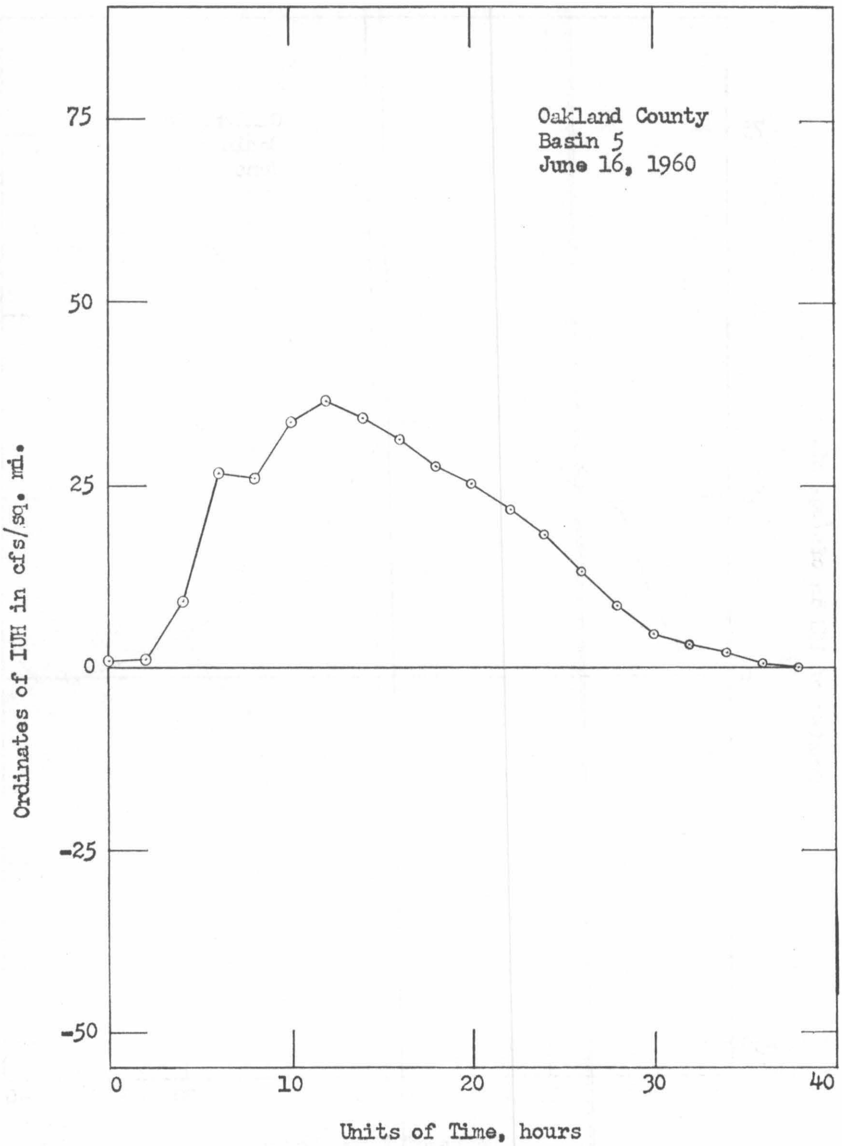

Figure 4: IUH for Storm of June 16, 1960, Oakland County, Michigan, Basin 5.

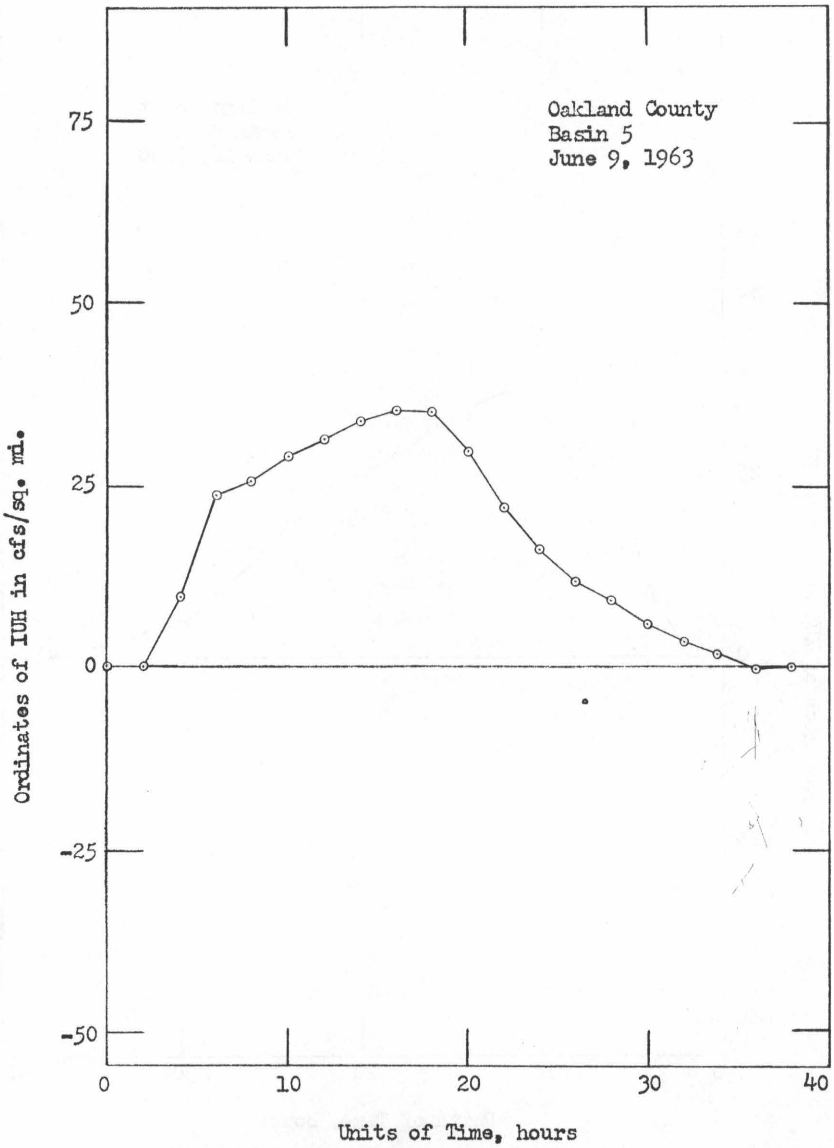

Figure 5: IUH for Storm of June 9, 1963, Oakland County, Michigan, Basin 5.

consequent oscillation. Storms with protracted periods of rainfall excess were observed to produce harmonic IUH's with rather violent oscillations. Figures 1 and 2 are somewhat extreme examples of the degree of oscillation observed. In the example cited by O'Donnell (10), the rainfall excess was pronounced and occurred within three runoff intervals as opposed to 38 intervals of direct surface runoff. Thus, in his example, a well-developed rainfall excess pattern of short duration produced a relatively smooth IUH with only minor oscillation in the tail portion, as could be expected. Perhaps the explanation offered by O'Donnell for the oscillation noted in the IUH for his example is valid as far as it goes; however, it would seem that other factors also enter into the explanation. These will be discussed in the conclusions.

Only the smoother harmonic IUH's were retained for further study. For each of the four basins there was at least one smooth harmonic IUH. Each IUH retained was matched with rainfall excess data for each of the storms in the respective basins. The second computer program (Appendix B) was used for this purpose. The program predicted direct surface runoff for each storm. As could be expected, the total volume of direct surface runoff predicted for each storm using the respective harmonic IUH's was essentially equal to the total volume of DSRO actually observed. However, the time distribution of the ordinates of DSRO in most cases varied considerably from those actually observed.

In order to develop an IUH for each set of data, limits for the development of harmonic coefficients for the finite series involved had to be set by entering the total number of data points involved into the computer program. Referring to the computer program (Appendix A) we find the quantity K defined. K is given as:

$$K = \frac{TE - TB}{TI} + 1 \tag{13}$$

where TE is the time of ending of direct surface runoff referenced to midnight of the day rainfall excess began, TB is the time of beginning of rainfall excess similarly referenced to midnight, and TI is the time interval at which runoff readings are taken. All time is in hours. K then represents the total number of points to which an IUH is to be fitted. There are a total of K terms divided between sine and cosine terms. Consequently, the term N, where N = (K-l)/2, is the number of sine harmonic terms, there being one more cosine term.

Figure 6 is representative of some of the predicted direct surface runoff curves obtained in this study. In figure 6, the curve of reference is that of the direct surface runoff observed from the storm of June 14, 1960 in Oakland County, Michigan, basin 5. Plotted with it are the predicted DSRO curves for the same storm using the IUH's derived from storms on the same basin occurring on June 14, 1960 and June 9, 1963 respectively. It should be noted that the rainfall excess of the storm of June 9, 1963 was concentrated in one interval. The rainfall excess of the storm of June 14, 1960 occurred over several intervals. The correspondence between the observed storm DSRO curve and the runoff predicted using IUH's from the other mentioned storms is seen to be quite good. The time lag seen between the start of the actual DSRO curve and the start of the predicted DSRO curves (obtained through the use of the second computer program) can probably be attributed to a slight difference in handling the rainfall excess input in the second program (Appendix B) as opposed to the first program (Appendix A). In the first program, hourly rainfall excess data was summed into two-hourly intervals beginning with the first period of rainfall excess, whether that occurred on an odd hour or not. In the second program, the objective was to be able to predict increments of runoff, interval by interval, as the rainfall excess data became available. Consequently, the program was written to sum rainfall excess data into two-hour intervals beginning with the first hour (odd hour) each day. Therefore, the IUH's were not used with precisely the same two-hourly intervals, for prediction purposes, as those from which they had been derived. Referring to Figure 7, one observes this effect as a slight lead, in this case, in the beginning of the DSRO curve.

A problem of some interest is that of making the proper allowance for the fact that the inception of direct surface runoff seems to lag the beginning of precipitation excess by varying time intervals from storm to storm. Table 2 summarizes the lag observed in the four basins studied. Since the IUH related rainfall excess and direct surface runoff, the lag will have an effect on the IUH derived. This effect can be seen on a subsequent DSRO prediction, with the DSRO curve lagging the precipitation excess by the same time difference that existed between the rainfall excess and DSRO from which the IUH was derived. O'Donnell (10) also noted and commented briefly on this lag.

One further observation should be made. Referring to Figure 6, a close correspondence is found between the observed and predicted DSRO curves using IUH's from three different storms. Referring to Figure 7, the correspondence between the actual DSRO curve and that predicted by using

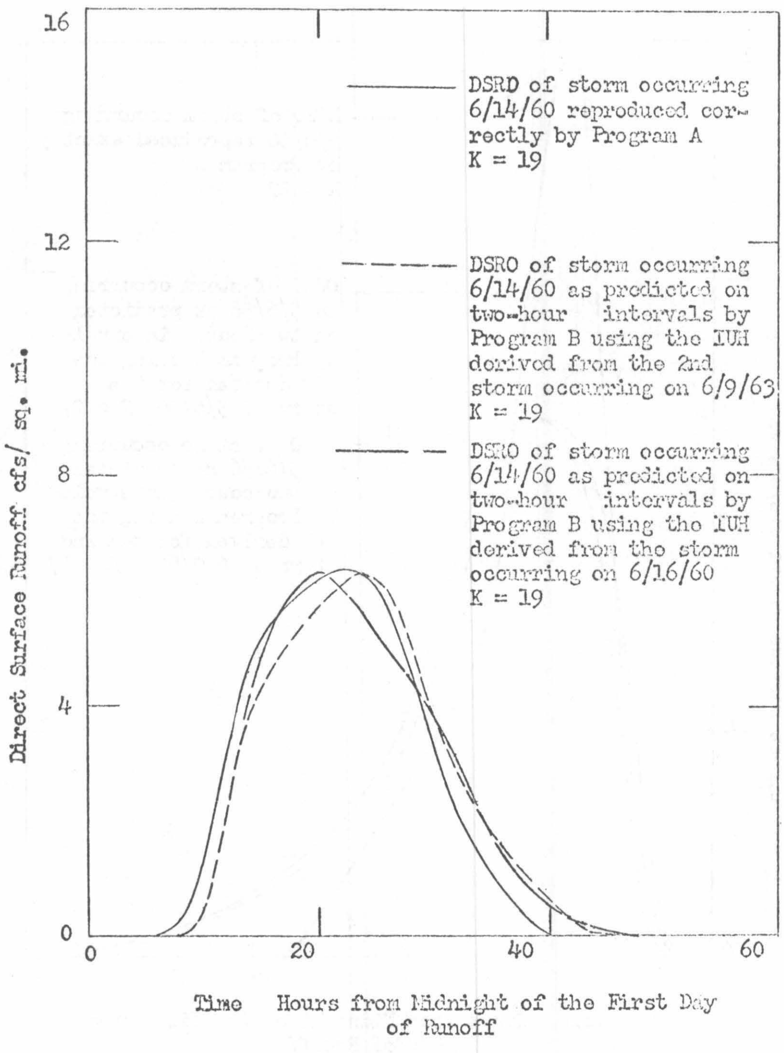

Figure 6: Representative Direct Surface Runoff Hydrographs from Oakland County, Michigan, Basin 5.

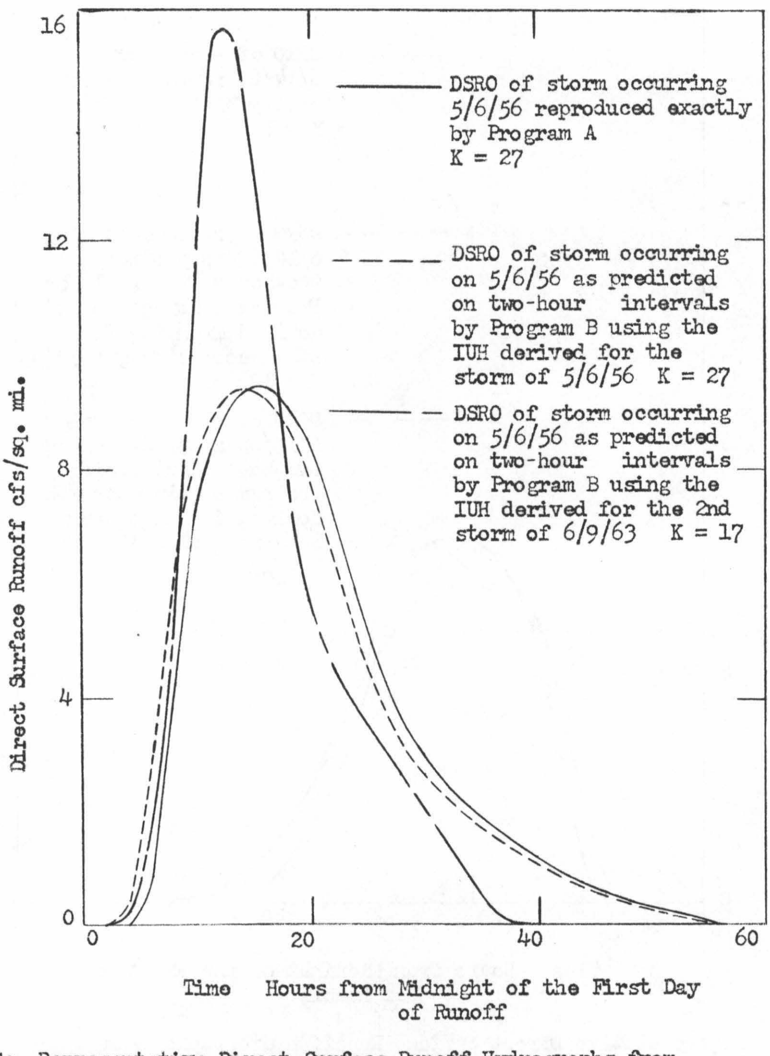

Figure 7: Representative Direct Surface Runoff Hydrographs from Macomb County, Michigan, Basin 3.

TABLE II

Drainage Basin	Average lag* of DSRO from inception of rainfall excess	Range of lag
Macomb, 3	1 hour	0-5 hours
Macomb, 3	3 hours	0-12 hours
Oakland, 5	3 hours	0-9 hours
Wayne, 3	2 hours	0-4 hours

*To the nearest hour

Average lag between the beginning of DSRO and the beginning of rainfall excess for four drainage basins.

the IUH from the storm of June 9, 1963 with the rainfall excess of the storm of May 6, 1956 is obviously poor. It is seen that the volume of the DSRO under all three curves in Figure 6, and likewise in Figure 7, is the same. The shape of the predicted DSRO curve reflects the shape and duration of the IUH, which in turn reflects the shape and duration of the rainfall excess and associated DSRO curves of the storm from which the IUH was derived. Consequently, an IUH derived from a storm with rainfall excess and DSRO of combined shorter duration than the average duration found in a drainage basin will produce a sharply-peaked predicted DSRO curve. The curve will have a shorter duration than the average for the basin when used with rainfall excess data from a storm of average duration. No matter which IUH is used, however, the total volume under the predicted DSRO curve will be the same. An IUH produced from a set of rainfall excess data from a storm of average duration, will produce a predicted DSRO curve with smaller ordinates and longer duration than that from an IUH from a storm of average duration.

Finally, something should be said about the effect of the number data points and the shape of the IUH. As indicated previously, a brief study was made of a sine curve. The curve was divided into various degree increments. The degree increments were 45, 30, 20, 15, 10, 5, 4, 3, and 2. It was found that the first and second cosine terms, the most significant harmonic coefficients, tended to stable values if 20 or more data points were used. Referring to Figures 8 and 9, it is seen that below 20 data points, the cosine and sine terms show substantial variation in values. The remainder of the sine and cosine terms, although smaller in numerical value, followed the same general pattern. Although the rainfall excess, DSRO, and IUH curves are not sine curves, the results of this brief study would seem to indicate that stability of sine and cosine harmonic coefficient terms can be reached with 20 to 30 items of data. Having more data points available is, of course, desirable. It may, therefore, reasonably be expected that IUH's derived from storms with equal numbers of data points, all other factors being disposed of properly, will bear a close resemblance for a drainage basin. Referring to Figures 8 and 9 again, two values in each figure appear to be displaced with reference to the general trend. The first of these points corresponds to 20 degree intervals (10 data points). The second corresponds to 4 degree intervals (46 data points). Both of these divisions miss the 90 degree value which is the peak value of the sine curve, since 20 degrees and 4 degrees do not divide evenly into 90 degrees. This observation would suggest that it is important to refine the intervals of data collection so that all peak values are included.

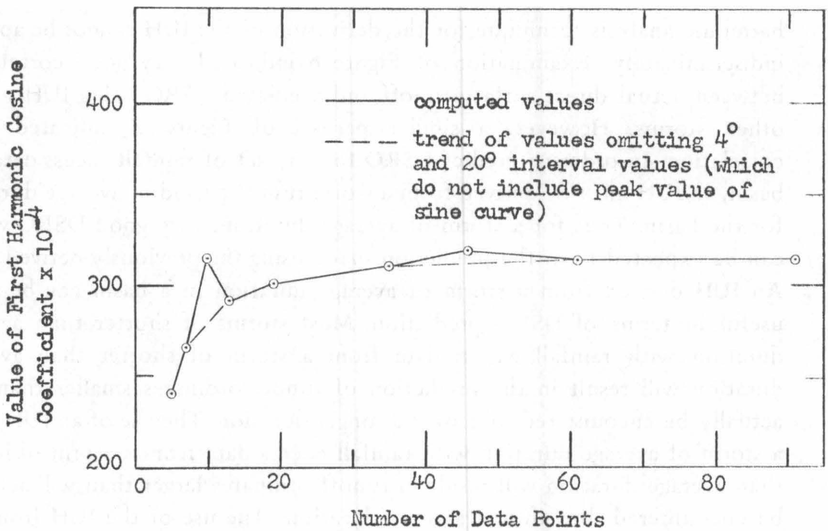

Figure 8: Value of 1st Harmonic Cosine Coefficient Describing the Shape of a Sine Curve.

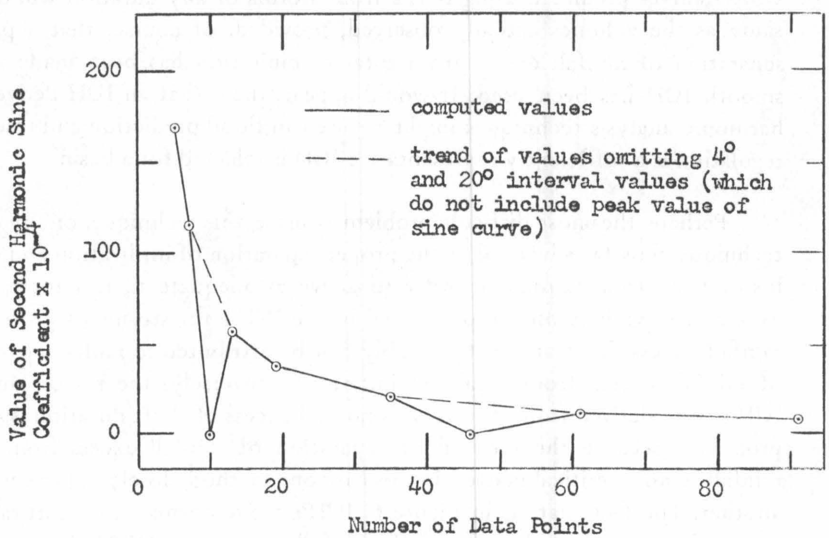

Figure 9: Value of 2nd Harmonic Sine Coefficient Describing the Shape of a Sine Curve.

CONCLUSIONS

An obvious conclusion that can be drawn from this study is that the harmonic analysis technique for the derivation of the IUH cannot be applied indiscriminately. Examination of Figure 6 indicated very good correlation between actual direct surface runoff and predicted DSRO using IUH's from other storms. However, a similar perusal of Figure 2 indicated little correlation. In order to predict DSRO for any set of rainfall excess data in a basin, an IUH must be derived from a storm-runoff period of average duration for the basin. Thus for a storm of average duration, very good DSRO values can be expected from the prediction made using the previously derived IUH. An IUH derived from a storm of average duration in a basin can be quite useful in terms of DSRO prediction. Most storms of shorter than average duration with rainfall excess data from a storm of shorter than average duration will result in the prediction of runoff ordinates smaller than will actually be encountered, but over a longer duration. The use of an IUH from a storm of average duration with rainfall excess data from a storm of longer than average duration will result in runoff ordinates larger than will actually be encountered, but over a shorter duration. The use of the IUH from the storm of average duration in DSRO prediction for flood forecast purposes will be found to err in a conservative manner. The total volumes under the DSRO curves predicted using IUH's from storms of any duration will be the same as the volumes actually observed, provided, of course, that a proper separation of rainfall excess from total precipitation has been made and a smooth IUH has been used. It would appear, then, that an IUH derived by harmonic analysis techniques might be used in flood prediction and reservior regulation is a sufficiently representative IUH is selected for a basin.

Perhaps the most difficult problem is using this technique, or any other technique thus far advanced, is the proper separation of infiltration and other losses from total rainfall in order to arrive at adequate figures for rainfall excess. The violent oscillations noted in the IUH's for storms of protracted rainfall excess duration most probably can be attributed to faulty separation of rainfall excess from total precipitation. Conversely, the reason smooth IUH's were derived from storms with rainfall excess of short duration is, most probably, because the method of separation of rainfall excess from total rainfall is not a critical matter, in this case one method closely approximating another. The fact that, as in Figure 6, IUH's from storms with short rainfall excess durations have been used successfully to predict DSRO from storms with protracted rainfall excess duration would tend to support that observation. To apply this method to a basin of particular interest then, one should select a more realistic method of rainfall separation than was used in this study. Perhaps an f-capacity type method (1) might be used.

A problem of somewhat lesser importance, but of significance, is the manner of separation of base flow and direct surface runoff for use in the derivation of the IUH. Some of the DSRO curves separated by the method indicated earlier in this study showed signs of not being completely realistic. In the present study, the choice of points of maximum curvature in the rising and falling limbs of the runoff hydrographs is a subjective matter and can affect the results of the separations.

In order to make practical application of this technique for flood flow prediction in a given drainage basin, consideration must be given to correctly separating total rainfall. This means that in addition to using an f-capacity type curve in rainfall separation, the effects of antecedent precipitation also must be evaluated. In some of the basins studied, infiltration and other losses amounted to anywhere between 50 and 90 percent of the total rainfall. It is imperative, if proper results are to be obtained from the application of this method, to establish some means of determining initial values for infiltration rates.

From previous discussion concerning the number of data points needed, it should be obvious that the method of harmonic analysis should not be applied to a basin where less than 20 to 30 data points per storm can be expected. Of course, one way to obtain more data points is to have rainfall and runoff readings taken in shorter intervals. If continuous records are available, the method can be applied quite readily. If runoff readings are taken only at the most a few times daily, the method will be more difficult to apply.

As indicated before, it would be advisable to use an IUH which reflects an average lag between inception of rainfall excess and direct surface runoff. Another approach might be to artifically initiate DSRO and rainfall excess at the same time. The lag could be reintroduced later as needed.

The following steps are summarized for the use of the method of harmonic analysis in flood flow prediction:

1. Select a basin for which sufficient past records exist, and in which readings of rainfall and runoff are taken in short enough intervals so that sufficient data points result for proper analysis.

2. Develop suitable infiltration rate curves so that total rainfall can be divided realistically.

3. Separate rainfall excess from total precipitation by making use of the infiltration rate curves, taking into account antecedent precipitation.

4. Separate base flow from DSRO for chosen storms by a suitable method. (Rainfall and runoff should be expressed in compatible units).
5. Use the first computer program (Appendix A) to derive IUH coefficients for each representative storm selected.
6. Produce actual IUH's by making use of the second computer program (Appendix B) by combining the IUH coefficients with a unit of rainfall excess which has been confined to one interval.
7. Examine the IUH'S produced for overall shape.
8. Use the IUH's with rainfall excess data of record to predict DSRO curves.
9. Compare the predicted DSRO curves with existing curves of record for correlation. (At this point critical examination can be made of the correctness of infiltration rate curves and of hydrograph separation techniques. Some adjustments may be in order).
10. Select the IUH that most adequately reproduces existing DSRO curves. (All are not likely to be adequate).
11. Use the second computer program (Appendix B), or a suitable modification of it, with the selected set of IUH harmonic coefficients to predict DSRO curves for successive intervals of rainfall excess data for the basin as they become available.

In conclusion, the method of harmonic analysis is not a simple panacea for hydrologic calculations. It may have useful application as a flood flow prediction device if the method is used judiciously. The main difficulties in its application seem to be in the areas of the selection of accurate infiltration rate curves and of determining the affects of antecedent precipitation. More study in these areas is recommended.

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APPENDIX A

```
ISN        SOURCE STATEMENT

  0 $IBFTC HARMSC
  1          INTEGER  TI,TB,TE
  2          CIMENSION A(100), B(100), A1(100), B1(100), ALPH(100), BETA(100)
  3          CIMENSION FNT(300), FNT1(300), FNT2(300), Q(300)
  4        1 WRITE (6,12)
  5       12 FORMAT (12H1 INPUT DATA)
  6          CO 71 I=1,300
  7          FNT(I)=0.
 10          FNT1(I)=0.
 11          Q(I)   = 0.  .
 12       71 FNT2(I)=0.
 14          CO 91  I = 1,100
 15          A(I)      = 0.
 16          B(I)      = 0.
 17          A1(I)     = 0.
 20          B1(I)     = 0.
 21          ALPH(I) = 0.
 22       91 BETA(I) = 0.
 24          READ (5,36) AR,DA,NO
 26       36 FORMAT (2A6,I3)
 27          WRITE (6,361) AR, DA, NO
 30      361 FORMAT (1X, 2A6, I5)
 31          NN=24*NO
 32          READ (5,37) (FNT(I), I=1,NN)
 37       37 FORMAT(8X,12F6.2)
 40          WRITE(6,371) (FNT(I), I = 1,NN)
 45      371 FORMAT(1X,6F6.2)
 46          READ (5,38)TB,TI,TE
 52       38 FORMAT (18X,2I3,10X,I3)
 53          K=(TE-TB)/TI+1
 54          WRITE (6,381) TB, TE, TI, K
 55      381 FORMAT (1X, 4I5)
 56          READ (5,39)(FNT1(I),I=1,K)
 63       39 FORMAT(8X,12F6.4)
 64          WRITE (6,371)(FNT1(I), I=1,K)
 71          NN=NN+1
 72          CO 60 I=NN,TE
 73       60 FNT(I)=0.
 75          I1=(NN-TB+1)/TI
 76          CO 50 I=1,I1
 77          FNT2(I)=0.
100          CO 70 J=1,TI
101          KK =   TB + ( I - 1 ) * TI   +   J-1
102       70 FNT2(I) = FNT(KK) + FNT2(I)
104       50 CONTINUE
106          I1=I1+1
107          CO 72 I=I1,K
110       72 FNT2(I)=0.
112          WRITE (6,16)(FNT2(I), I=1,K)
117       16 FORMAT(5E12.6)
120          WRITE (6,14)
121       14 FORMAT (13H0 OUTPUT DATA)
122          K=(K-1)/2
123          N=K
124          M  =  N
```

APPENDIX A (CONTINUED)

```
ISN       SOURCE STATEMENT

125           WRITE (6,13) N,M
126        13 FORMAT (1X, 2I10)
127           CALL FORIT(FNT2, N, M, A, B, IER)
130           CALL FORIT(FNT1, N, M, A1, B1, IER1)
131           M1 = M + 1
132           WRITE (6,15)
133        15 FORMAT (31H0 FOURIER VECTOR, INPUT,COSINES)
134           WRITE (6,16)(A(I), I=1,M1)
141           WRITE (6,21)
142        21 FORMAT (29H0 FOURIER VECTOR, INPUT,SINES)
143           WRITE (6,16)(B(I),I=1,M1)
150           WRITE (6,22)
151        22 FORMAT (33H0 FOURIER VECTOR, OUTPUT, COSINES)
152           WRITE (6,16)(A1(I), I=1,M1)
157           WRITE (6,23)
160        23 FORMAT (31H0 FOURIER VECTOR, OUTPUT, SINES)
161           WRITE (6,16)(B1(I), I=1,M1)
166           N2 = 2*N + 1
167           TT = N2
170           DO 400 I=1, N2
171           F = 0.
172           F1 = 0.
173           EI=I-1
174           DO 500 J = 1,M1
175           EM=J-1
176           FACT=COS(EM*6.2832*EI/TT)
177           FACT2=SIN(EM*6.2832*EI/TT)
200           F = F + A(J) * FACT + B(J) * FACT2
201       500 F1=F1+A1(J)*FACT+B1(J)*FACT2
203           FNT2(I) = F
204       400 FNT1(I) = F1
206           ALPH(1) = A1(1) / (A(1)*TT)
207           BETA(1)=0.
210           DO 100 I=2,M1
211           DENOM=TT*(A(I)*A(I)+B(I)*B(I))
212           ALPH(I)=2.*(A(I)*A1(I)+B(I)*B1(I))/DENOM
213       100 BETA(I)=2.*(A(I)*B1(I)-A1(I)*B(I))/DENOM
215           WRITE(7,32)
216        32 FORMAT(28H FOURIER VECTOR, CONVOLUTION)
217           WRITE(7,31) (ALPH(I),BETA(I), I = 1,M1)
224        31 FORMAT(2E15.8)
225           WRITE (6,17)
226        17 FORMAT (38H0 FOURIER VECTOR, CONVOLUTION, COSINES)
227           WRITE (6,16)(ALPH(I),I=1,M1)
234           WRITE (6,24)
235        24 FORMAT (36H0 FOURIER VECTOR, CONVOLUTION, SINES)
236           WRITE (6,16)(BETA(I),I=1,M1)
243           DO 600  I = 1, N2
244           EI = I-1
245           GQ = 0.
246           DO 700  J = 2, M1
247           EM = J-1
250           FACT1 =  COS(EM*6.2832*EI/TT)
251           FACT4  =  SIN(EM*6.2832*EI/TT)
252           GG  =  (TT/2.)* (A(J)*ALPH(J)  -  B(J)*BETA(J))
```

APPENDIX A (CONTINUED)

```
ISN        SOURCE STATEMENT

253             HH = (TT/2.) * (A(J)*BETA(J)  +  B(J)*ALPH(J))
254        700  QQ  =  QQ + GG*FACT1 + HH*FACT4
256        600  Q(I)  =  TT*A(1)*ALPH(1)  +  QQ
260             WRITE (6,81)
261         81  FORMAT (38H0 PREDICTED VALUES OF SURFACE RUNOFF Q)
262             WRITE (6,16) (Q(I), I = 1, N2)
267             GO TO 1
270        999  STOP
271             END
```

APPENDIX B

```
ISN         SOURCE STATEMENT

  0 $IBFTC HOURLY
  1          INTEGER TI, TER
  2          DIMENSION A(100), B(100), ALPH(100), BETA(100), FNT(100)
  3          DIMENSION FNT3(100), Q(130), QPRM(30,130), FNT2(100)
  4        1 WRITE(6,61)
  5       61 FORMAT(12H1 INPUT DATA)
  6          DO 891  I = 1,100
  7          A(I) = 0.
 10          B(I) = 0.
 11          ALPH(I) = 0.
 12      891 BETA(I) = 0.
 14        3 READ(5,10) N, M
 17       10 FORMAT(2I5)
 20          WRITE(6,10) N,M
 21          N2 = 2*N + 1
 22          M1 = M + 1
 23          TT = N2
 24          READ(5,11) (ALPH(I),BETA(I), I = 1,M1)
 31       11 FORMAT(2E15.8)
 32          WRITE(6,62)
 33       62 FORMAT(38H0 FOURIER VECTOR, CONVOLUTION, COSINES)
 34          WRITE(6,63) (ALPH(I), I = 1,M1)
 41       63 FORMAT(5E12.6)
 42          WRITE(6,64)
 43       64 FORMAT(36H0 FOURIER VECTOR, CONVOLUTION, SINES)
 44          WRITE(6,63) (BETA(I), I = 1,M1)
 51        2 READ(5,12) AR, DA, NO
 53       12 FORMAT(2A6,I3)
 54          WRITE(6,70)
 55       70 FORMAT(1H1 )
 56          WRITE(6,65) AR, DA, NO
 57       65 FORMAT(1X,2A6,I5)
 60          DO 111 I = 1,30
 61          DO 111 J =1,130
 62      111 QPRM(I,J) = 0.
 65          DO 121 I = 1,130
 66      121 Q(I) = 0.
 70          NN = 24*NO
 71          DO 101 I = 1,100
 72          FNT(I)  = 0.
 73      101 FNT2(I) = 0.
 75          READ(5,13) (FNT(I), I = 1,NN)
102       13 FORMAT(8X,12F6.2)
103          WRITE(6,66) (FNT(I), I = 1,NN)
110       66 FORMAT(1X,6F6.2)
111          READ(5,14) TI, TER
114       14 FORMAT(1X,2I5)
115          WRITE(6,14) TI, TER
116          KE = TER/TI + 1
117          NN = NN + 1
120          DO 131 I = NN,100
121      131 FNT(I) = 0.
123          IL = 100/TI
124          DO 141 I = 1,IL
125          FNT2(I) = 0.
```

APPENDIX B (CONTINUED)

```
ISN          SOURCE STATEMENT

126          DO 151 J = 1,TI
127          KK = (I-1)*TI + J
130     151  FNT2(I) = FNT(KK) + FNT2(I)
132     141  CONTINUE
134          WRITE(6,67)
135      67  FORMAT(16H0 VALUES OF FNT2)
136          WRITE(6,63) (FNT2(I), I = 1,IL)
143          MM = 1
144          N3 = N2
145          DO 161 I = 1,KE
146          DO 171 K = 1,100
147          A(K)     = 0.
150          B(K)     = 0.
151     171  FNT3(K) = 0.
153          FNT3(1) = FNT2(I)
154          DO 191 L = 2,N2
155     191  FNT3(L) = 0.
157          CALL FORIT(FNT3,N,M,A,B,IER)
160          DO 181 J = MM,N3
161          EI = J-I
162          QQ = 0.
163          DO 201 JJ = 2,M1
164          EM = JJ - 1
165          FACT1 = COS(EM*6.2832*EI/TT)
166          FACT2 = SIN(EM*6.2832*EI/TT)
167          GG = (TT/2.)*(A(JJ)*ALPH(JJ) - B(JJ)*BETA(JJ))
170          HH = (TT/2.)*(A(JJ)*BETA(JJ) + B(JJ)*ALPH(JJ))
171     201  QQ = QQ + GG*FACT1 + HH*FACT2
173     181  QPRM(I,J) = TT*A(1)*ALPH(1) + QQ
175          MM = MM + 1
176     161  N3 = N3 + 1
200          KN = KE + N2
201          DO 211 J = 1,KN
202          Q(J) = 0.
203          DO 211 I = 1,KE
204          Q(J) = Q(J) + QPRM(I,J)
205     211  IF(Q(J).LT.0.)Q(J) = 0.
212          WRITE(6,69)
213      69  FORMAT(26H0 OUTPUT DATA, PREDICTED Q)
214          WRITE(6,63) (Q(I), I = 1,KN)
221          GO TO 2
222     999  STOP
223          END
```

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