November 1977 WRRI Report No. 091

STUDIES ON RAINFALL-RUNOFF MODELING

8. Comparison of Models

Partial Technical Completion Report
Project No. 3109-206

STUDIES ON RAINFALL-RUNOFF MODELING

8. Comparison of Models

Vijay P. Singh
Assistant Professor of Hydrology
New Mexico Institute of Mining and Technology
Socorro, New Mexico 87801

The work upon which this report is based was supported in part by funds provided through the New Mexico Water Resources Research Institute by the U. S. Department of the Interior, Office of Water Research and Technology, as authorized under the Water Resources Research Act of 1964, Public Law 88-379 as amended, under project number 3109-206.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Figure | Page

ABSTRACT

Mathematical models of (a) rainfall-excess, and (b) runoff are examined in light of appropriate objective functions. A comparison of models reflects on the goodness of one model relative to another. The regions of superiority of one model to another are established which can hopefully serve as a guide in choosing between them.

This report is the last in the sequence of eight on Rainfall-Runoff Modeling. The previous seven reports are:

1. Estimation of mean areal rainfall.
2. A distributed kinematic wave model of watershed surface runoff.
3. Converging overland flow.
4. Estimation of parameters of two mathematical models of surface runoff.
5. A uniformly nonlinear hydrologic cascade model.
6. A statistical analysis of rainfall-runoff relationship.
7. A nonlinear hydrologic cascade.

Chapter 1

INTRODUCTION

1.1 GENERAL REMARKS

There has been a proliferation in the development of mathematical models of hydrologic and water resource systems during the past two decades. However, there has been little effort directed toward what may be termed 'model validation'. Comparison of models, specification of the objective criteria of comparison and identification of the regions of superiority of one model to another constitute some of the important aspects of model validation that deserve serious study. Such a study will not only aid in studying the models and devising ways to improve them but will also be greatly useful to the user whose sole aim is to apply the model he is familiar with. His choice of a particular model and its subsequent application will be more judicious and less biased than would otherwise be.

An objective comparison of models of a given hydrologic phenomenon is very important in choosing among them. It seldom happens that one model is uniformly better than the other under all circumstances. What happens ordinarily and frequently is that one model is better than the other but only under certain circumstances. When the circumstances change, the superiority of one model to another may also change. This points out that the region of superiority of one model to another must be clearly spelled out. The measure of goodness of a model will, however, be in terms of an objective function. The choice of an objective function often depends on the problem at hand and may be dictated by factors other than hydrologic, physical or mathematical, such as economic, sociological, political, or environmental. Consideration of such factors is beyond the scope of the present study. Keeping in perspective the issues raised above we formulate our objectives.

1.2 OBJECTIVES

The objectives of the present study are:

1. To study the effect of rainfall-excess determination on runoff computation.
2. To study the sensitivity of runoff models to errors in rainfall-excess.
3. To compare and contrast two mathematical models of runoff.

Chapter 2

MODELS OF INFILTRATION AND RUNOFF

In this chapter we briefly describe those models of infiltration and surface runoff that will be used in the latter chapters.

2.1 MODELS OF INFILTRATION

Four methods of determining infiltration are considered:

a. ϕ-index

b. Horton equation (Horton, 1940)

c. Kostyakov equation (Rode, 1965)

d. Philip equation (Philip, 1957, 1968)

ϕ-index assumes a time-invariant rate of infiltration, varies from one rainfall episode to another and requires runoff records for its computation. Although far from accurate, its extreme simplicity has led to its widespread use in hydrologic modeling.

The Horton equation, probably the best known, can be written as:

$$f = f_c + (f_o - f_c)e^{-kt} \tag{2.1}$$

where f is infiltration capacity at time t, f_o initial infiltration capacity, f_c final infiltration capacity, and k parameter dependent on soil characteristics and initial soil moisture conditions.

The Kostyakov equation can be written as:

$$F = a\ t^b \tag{2.2}$$

where F is total infiltration volume, t time from the onset of infiltration, a parameter, and b parameter, $(0 < b < 1)$, dependent on soil characteristics.

The Philip equation can be written as:

$$F = At + St^{0.5} \tag{2.3}$$

where F is volume of infiltration to time t, A parameter dependent on soil characteristics, and S parameter dependent on initial soil moisture content.

2.2 MODELS OF SURFACE RUNOFF

Five mathematical models of surface runoff are briefly discussed, as deemed relevant in the context of the present study. Of them two are nonlinear and three linear.

2.2.1 CONVERGING OVERLAND FLOW MODEL (CONV)

The converging overland flow model (Woolhiser, 1969; Singh, 1975a, 1975b, 1975c, 1976b, 1976d; Sherman and Singh, 1976a, 1976b) considers surface runoff as gradually varied, unsteady, nonuniform, free surface flow, and approximates its dynamic behavior by kinematic wave theory (Lighthill and Whitham, 1955; Singh, 1976a). As its name suggests, it transforms the geometry of a natural watershed into a simpler, linearly converging section of a cone geometry as shown in Fig. 2.1. This transformation is based on the premise that the simplified geometry will have a hydrologic response similar to that of the natural geometry and is, hence, equivalent in that sense. To accomplish this transformation for a natural watershed, its topographic map provides all the necessary information (Singh, 1976b; Shelburne and Singh, 1976).

From Fig. 2.1 it is clear that the converging section has four geometric parameters including L_o, r, θ and S_o, where L_o is the length of the flow region, S_o slope, r a parameter related to the degree of convergence and θ the interior angle. Because of radial symmetry θ does not affect the relative response characteristics, and depends on L_o and r only, since the watershed area must be preserved.

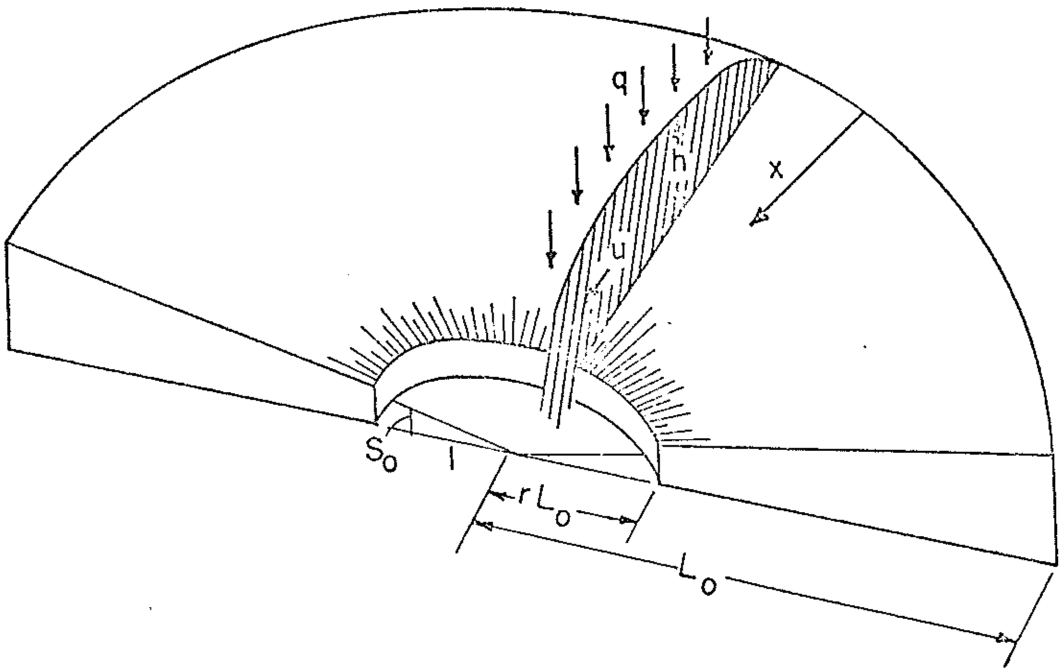

Fig. 2.1. Converging overland flow model.

The mathematical representation of the model consists of a continuity equation and a kinematic-momentum equation. These equations (Singh, 1976b) can be written respectively as:

$$\frac{\partial h}{\partial t} + \frac{\partial Q}{\partial x} = q(x,t) + \frac{Q}{(L_o - x)} \tag{2.4}$$

$$Q = \alpha h^n \tag{2.5}$$

where h is average local depth of flow, Q rate of outflow per unit width, q(x,t) lateral inflow (rainfall-excess) varying in time and space, L_o length of the converging section, x space coordinate, t time coordinate, α kinematic wave parameter of fraction relationship, and n parameter, an index of nonlinearity.

For many hydrologic problems of interest n can be fixed at 1.5 (Singh, 1975a, 1975d, 1975e, 1976b). Thus, with the converging geometry defined by the topographic map, CONV reduces to a one-parameter model. For complex q(x,t) analytical solutions are not wieldly because of the nonlinear nature of Eqs. (2.4) and (2.5). However, numerical and hybrid solutions (Singh, 1975b, 1975c, 1976a, 1976b) are relatively simple to develop and were utilized in the present investigation. For space-time invariant q, analytical solutions have been given by Singh (1976b). Due to a pulse rainfall-excess q the runoff hydrograph peak Q_p and its time t_p can be expressed (Singh, 1976b) respectively for the partial equilibrium situation as:

$$Q_p(x,x^*) = \frac{q}{2}\left[\frac{L_o^2 - (L_o - x^*)^2}{L_o - x}\right] \tag{2.6}$$

$$t_p(x,x^*) = T + \left(\frac{2}{q}\right)^{\frac{n-1}{n}} \left(\frac{1}{\alpha}\right)^{\frac{1}{n}} \frac{1}{(2n-1)} \left[\frac{(L_o-x^*)^{\frac{2n-1}{n}} - (L_o-x)^{\frac{2n-1}{n}}}{\left(L_o^2 - (L_o-x^*)^2\right)^{\frac{n-1}{n}}}\right] \quad (2.7)$$

where T is the duration of rainfall-excess and x^* is the solution of

$$T = \frac{1}{n}\left(\frac{2}{q}\right)^{\frac{n-1}{n}} \left(\frac{1}{\alpha}\right)^{\frac{1}{n}} \int_o^x \left\{\frac{L_o - \xi}{L_o^2 - (L_o - \xi)^2}\right\}^{\frac{n-1}{n}} d\xi \quad (2.8)$$

Equation (2.8) can be simplified and written as:

$$T = \frac{1}{2n}\left(\frac{2}{q}\right)^{\frac{n-1}{n}} \left(\frac{1}{\alpha}\right)^{\frac{1}{n}} [\beta(a,b) - \beta_\phi(a,b)] \quad (2.9)$$

where

$$a = 1 - \frac{1}{2n}$$

$$b = \frac{1}{n}$$

$$\phi = \left(\frac{L_o-x}{L_o}\right)^2$$

$$\beta(a,b) = \frac{\Gamma(a)\ \Gamma(b)}{\Gamma(a+b)}$$

and

$$\beta_\phi(a,b) = \frac{\phi^a\ (1-\phi)^b}{a}\left[1 + \sum_{j=o}^{\infty} \frac{\beta(a+1;\ j+1)}{\beta(a+b;\ j+1)}\ \phi^{j+1}\right]$$

Note that in Eqs. (2.6) and (2.7) we are interested in Q_p and t_p at the outlet; that is, $x = L_o(1-r)$.

2.2.2 KINEMATIC PLANE MODEL (PLANE)

This is also a nonlinear kinematic wave model (Wooding, 1965a; Kibler and Woolhiser, 1970; Singh, 1975d, 1976a, 1976c) of watershed surface runoff. As implied by its name, the geometry of a natural watershed, regardless of its complexity, is transformed into a simple rectangular plane as shown in Fig. 2.2, with the same assumption as made in CONV.

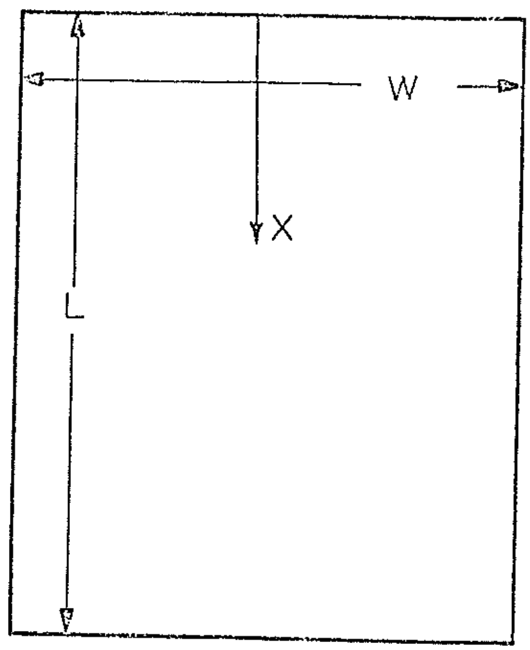

Fig. 2.2. Kinematic plane model.

Because the watershed area is to be preserved, the transformation requires the knowledge of only one parameter; that is, the width of the plan W or the length L. The kinematic equations for a plane are the same as for CONV except for a slight difference in the continuity equation which can be simply written as:

$$\frac{\partial h}{\partial t} + \frac{\partial Q}{\partial x} = q(x,t) \qquad (2.10)$$

Because of their nonlinear nature explicit analytical solutions for Eqs. (2.5) and (2.10) are not tractable (Wooding, 1965b; Kibler and Woolhiser, 1970; Singh, 1975b, 1976a) for space-time varying $q(x,t)$. For space-time invariant q, however, analytical solutions have been obtained by Wooding (1965a), Kibler and Woolhiser (1970) and Singh (1975d, 1976a). Thus, due to pulse rainfall-excess q, the hydrograph peak and its time can be expressed respectively for the partial equilibrium situation as:

$$Q_p(x, x_o) = q(x-x_o) \qquad (2.11)$$

$$t_p(x,x_o) = (q)^{\frac{1-n}{n}} \left(\frac{x-x_o}{\alpha}\right)^{\frac{1}{n}} \qquad (2.12)$$

For the plane kinematic case $t_p(x,x_o) = T$, and, of course, we want these quantities at $x = L$. With n fixed at 1.5 and the plane geometry defined by the topographic map, PLANE reduces a one-parameter model.

2.2.3. NASH MODEL (NASH)

The Nash model (Nash, 1957) represents a watershed by a cascade of linear reservoirs as shown in Fig. 2.3. It is noteworthy that the model does not explicitly account for the geometric configuration of a given watershed. The runoff dynamics is represented by a spatially

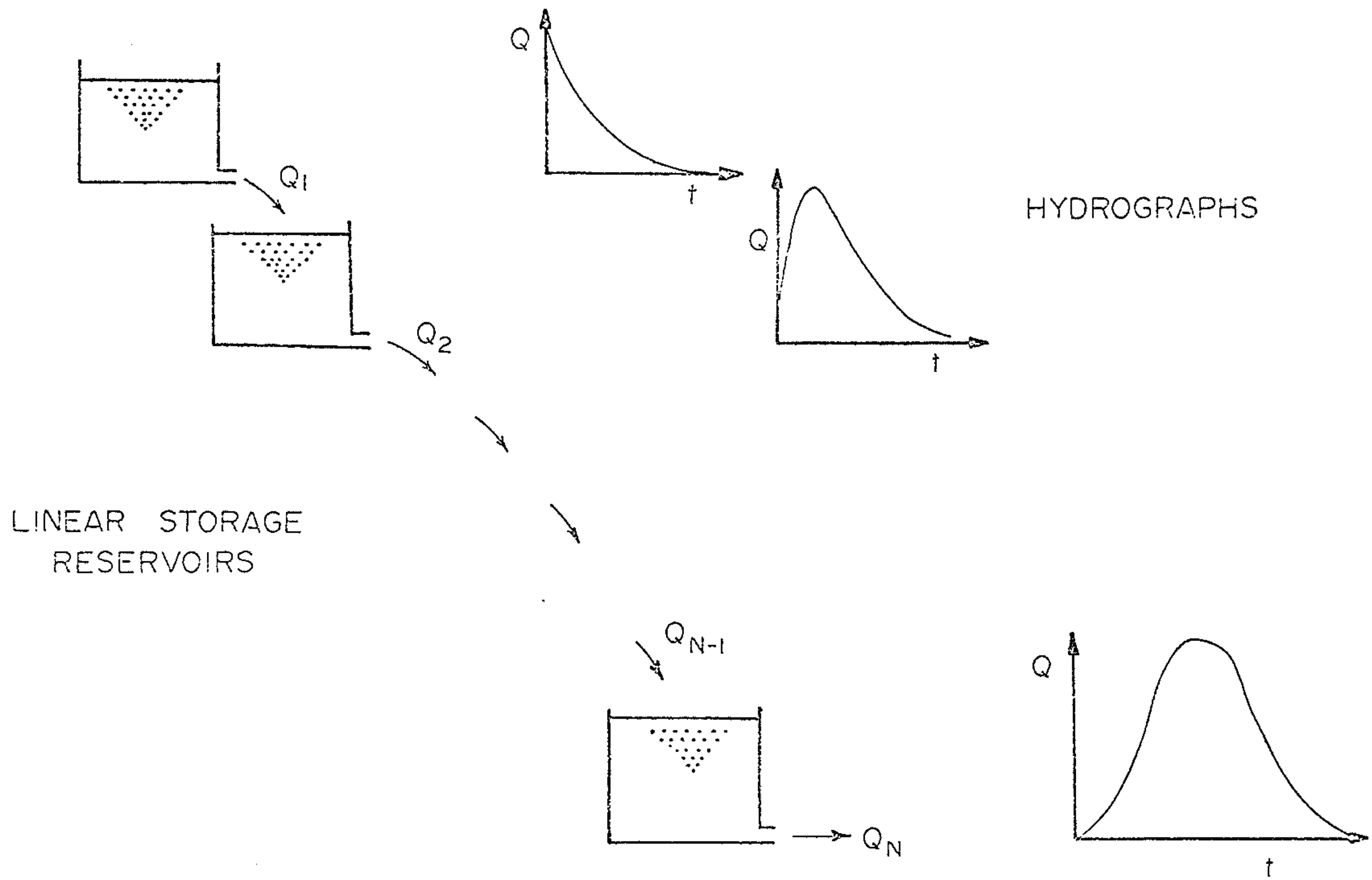

Fig. 2.3. Nash Model

lumped continuity equation and a linear storage law; for time interval Δt, these equations can be written as:

$$q(t) = Q(t) + \frac{ds(t)}{dt} \tag{2.13}$$

$$s = KQ \tag{2.14}$$

where s is storage, and K storage coefficient, signifying watershed characteristic lag time.

The basis of Nash model emerges principally from the notion of linearity inherent in Eq. (2.14). By routing an instantaneous inflow through a cascade of N equal linear reservoirs Nash (1957) derived an expression for the instantaneous unit hydrograph (IUH) $\omega(t)$ as:

$$\omega(t) = \frac{1}{K\Gamma(N)} \left(\frac{t}{K}\right)^{N-1} e^{-t/K} \tag{2.15}$$

For an inflow q(t), outflow Q(t) at time t can be computed by the convolution integral:

$$Q(t) = \int_0^t \omega(t-\zeta)q(\zeta)d\zeta \tag{2.16}$$

Thus, due to a pulse rainfall-excess q occurring for time T, the hydrograph peak and its time can be expressed (Dooge, 1959) as:

$$Q_p = \frac{V}{K} (N-1)^{N-1} \frac{1}{\Gamma(N)} e^{-(N-1)} \tag{2.17}$$

$$t_p = K(N-1) \tag{2.18}$$

where V = qT, volume of inflow. For further details see the references by Nash (1957, 1958, 1959, 1960).

2.2.4 O'KELLY MODEL (KELLY)

O'Kelly (1955) introduced the concept of triangular inflow and did away with the tediously derived time-area curve of a given watershed. By taking the triangle to be an isosceles triangle and fixing its volume at unity, it was completely described by its base length, T. The parameter T is the translation time. Equation (2.14) was used for routing this inflow. Consequently, O'Kelly reduced a unit hydrograph of any duration to the equivalent instantaneous unit hydrograph and described the latter in terms of only two parameters, the translation time T and delay time K. Later, Dooge (1959), using this model, derived expressions for hydrograph peak and its time as:

$$Q_p = \frac{4V}{T} \left(1 - \frac{t_p}{T}\right) \tag{2.19}$$

$$t_p = K \ln (2 e^{T/2K} - 1) \tag{2.20}$$

where V is the volume of triangular inflow. For a more complete discussion see the references by O'Kelly (1955) and Dooge (1973).

2.2.5 CLARK MODEL

The Clark model (Clark, 1945) is based on the routing of a time-area curve for a watershed through a single linear reservoir given by Eq. (2.14) to obtain the instantaneous unit hydrograph. The time area curve is a graph showing the distribution of the areas in the watershed in terms of travel time to the outlet in the absence of storage effects. It thus represents the instantaneous unit hydrograph which would occur if storage did not affect the hydrograph. Clark assumed that all the flow can be translated to the outlet and then the correction made for storage by a single reservoir routing. For a single watershed Dooge

(1959), using the Clark model, derived expressions for hydrograph peak and its time as:

$$Q_p = \frac{V}{T} \left(1 - e^{-T/K}\right) \qquad (2.21)$$

$$t_p = T$$

where T is base of the rectangular inflow, and V volume of the inflow.

Chapter 3

RAINFALL-EXCESS DETERMINATION AND RUNOFF COMPUTATION

3.1 GENERAL REMARKS

Rainfall-excess, which forms input to most models of watershed surface runoff, is determined by subtracting infiltration from rainfall such that the volume of rainfall-excess is equal to the volume of surface runoff. There are several methods of estimating infiltration (Green and Ampt, 1911; Horton, 1940; Philip, 1957, 1968; Holtan, 1961; Rode, 1965; Childs, 1969; Smith and Woolhiser, 1971), all yielding different estimates. Furthermore, the infiltration estimates by a method will differ from the true estimates for the reasons which no method seems to satisfactorily account for: (1) heterogeneous watershed characteristics in time and space, (2) heterogeneous antecedent soil moisture conditions over the watershed prior to the occurrence of a rainfall episode, (3) space-time variability of rainfall over the watershed. Thus it is obvious that the distribution of rainfall-excess in time and space will depend on the method of infiltration. For example, for a rainfall-runoff event on watershed 2-H, Hastings, Nebraska, Fig. 3.1 shows four methods of determining rainfall-excess, each method leading to a different distribution and each distribution differing from the true distribution. Therefore, it will not be an exaggeration to say that the true distribution of rainfall-excess in time and space is really never known.

In watershed runoff modeling use is frequently made of simple methods of estimating infiltration (Nash, 1957; Dooge, 1973; Eagleson, 1972; Wooding, 1966; Woolhiser, Hanson and Kuhlman, 1970; Singh, 1975a, 1976a, 1976b). Further, it has been recognized (Singh and Woolhiser, 1976) that errors in rainfall-excess may be large and might constitute a major source of errors in runoff prediction. A question arises: How do various methods of infiltration compare in runoff prediction? An answer to this question will

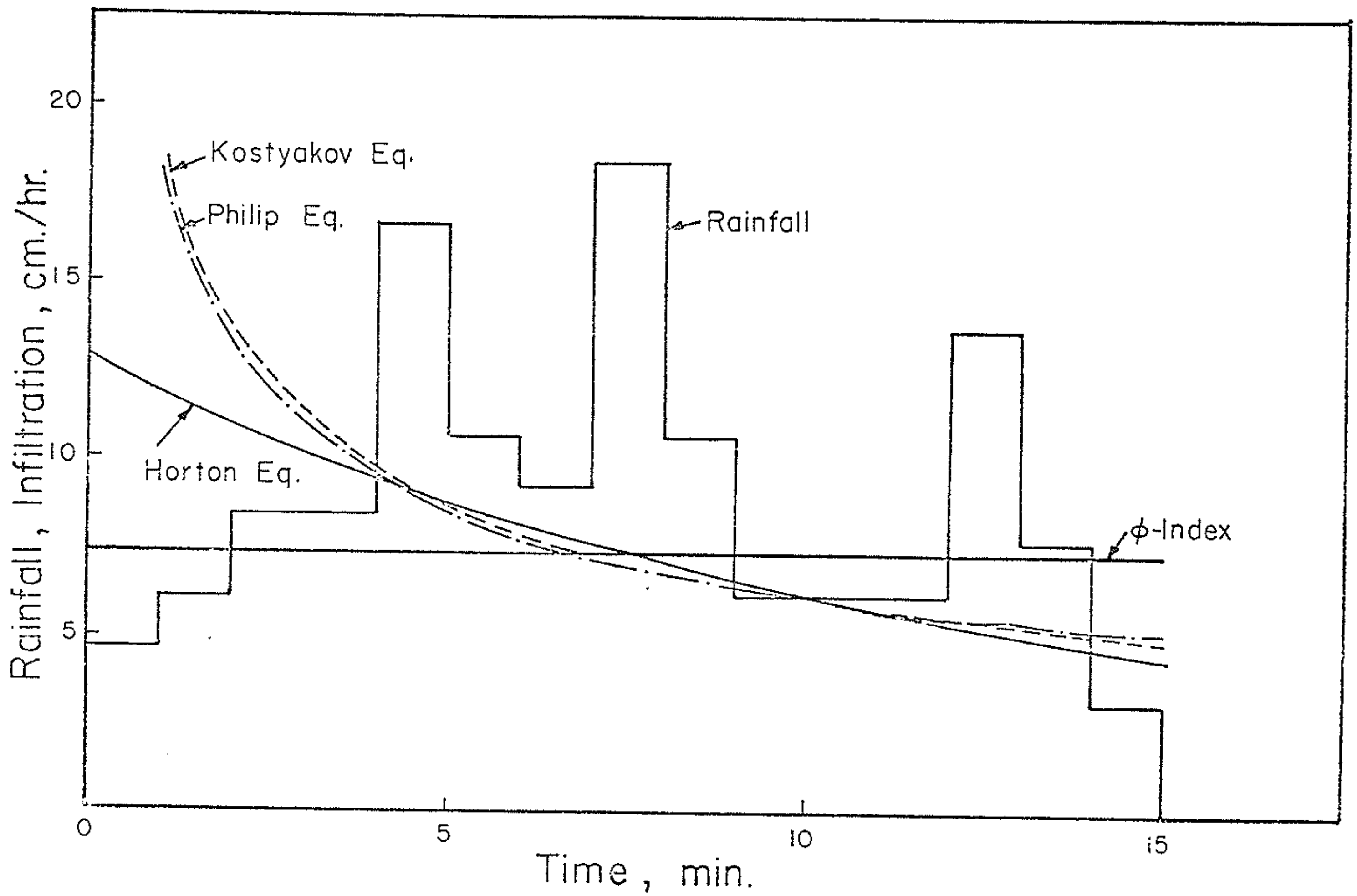

Fig. 3.1. Determination of rainfall-excess by four methods for rainfall event of 9-4-1942 on watershed 2-H, Hastings, Nebraska.

point the way toward (1) quantification of the effect of rainfall-excess determination on runoff computation, (2) choosing amongst the methods of infiltration and (3) qualitative assessment of errors in runoff prediction due to errors in rainfall-excess. We attempt to answer the above question by considering four simple methods of infiltration, as described in the previous chapter, in prediction of surface runoff by a nonlinear converging overland flow model from two natural agricultural watersheds.

3.2 EFFECT OF RAINFALL-EXCESS ON RUNOFF

Two small natural agricultural watersheds, 2-H and 4-H, were selected near Hastings, Nebraska. The watershed 2-H is 1.21 ha in area and watershed 4-H 1.47 ha. These watersheds have Loessial soils. The top soil is normally a mixture of silt loam and silt clay. The internal drainage is medium. The permeability of subsoil is moderately slow. Surface drainage is good. The watersheds develop arterial flow toward a central drainageway. Channel meandering is noticeable and leads to impounding some water. These watersheds have agricultural cover on the surface most of the time. A considerable portion of rainwater seeps into the ground; thus infiltration is significant. For a more complete discussion of these watersheds see the USDA publications on hydrologic data (USDA, 1963).

Rainfall-runoff data on these watersheds were obtained from the USDA Hydrologic Data Center, Beltsville, Maryland. Twenty events were available on watershed 2-H and 18 on watershed 4-H. These events were divided into two sets: (1) the optimization set consisting of 10 events on watershed 2-H and 10 on watershed 4-H, (2) the prediction set consisting of 10 events on watershed 2-H and 8 on watershed 4-H. For each event rainfall-excess was determined by the aforementioned four methods.

For Horton equation f_c was 0.51 cm/hr and 0.025 cm/hr and f_o was 12.5 cm/hr and 10.0 cm/hr for watersheds 2-H and 4-H respectively. The parameter k was allowed to vary with the rainfall event, thus accounting for antecedent soil moisture conditions. For Kostyakov equation b was chosen to be 0.5 and a was allowed to vary from one rainfall event to another to account for moisture conditions existing prior to the occurrence of rainfall. For Philip equation A was 0.51 cm/hr and 0.025 cm/hr for watersheds 2-H and 4-H respectively. S was allowed to vary from event to event to account for antecedent soil moisture conditions.

The parameters of converging overland flow model were then specified (Singh, 1976b; Shelburne and Singh, 1976). n was taken as 1.5 and r as 0.01. L_o was considered equal to the horizontal projection of the distance from the most remote point of the watershed to its outlet. It was 185 m for watershed 2-H and 155 m for watershed 4-H. The only parameter that remains to be specified now is α. The modified Rosenbrock algorithm (Rosenbrock, 1960; Palmer, 1970; Himmelblau, 1972) was utilited to optimize α over the optimization set of events on each watershed. The objective function for optimization was (Singh, 1976a):

$$E = \sum_{j=1}^{M} \left\{ Q_{p_o}(j) - Q_{p_e}(j) \right\}^2 \Rightarrow \min \tag{3.1}$$

where E is objective function or error, $Q_{p_o}(j)$ observed hydrograph peak for jth event, $Q_{p_e}(j)$ estimated hydrograph peak for jth event and M number of events in the optimization set. The optimized α values are:

Watershed Hastings, Nebraska	optimized α			
	ϕ-index	Horton Equation	Kostyakov Equation	Philip Equation
2-H	9.1008	18.2	14.63	14.3
4-H	100.0	58.875	55.0	55.0

The objective function corresponding to each method of infiltration is:

Watershed	M	Objective Function, E			
		ϕ-index	Horton Equation	Kostyakov Equation	Philip Equation
2-H	10	28.881	5.220	9.678	11.018
4-H	10	43.155	44.183	38.076	37.588

It is clear that both α and E are quite sensitive to the method of infiltration. From the values of E, it is seen that ϕ-index leads to the highest error. Its use should be avoided whenever possible, or it may be replaced by either of the three equations.

3.2.1 HYDROGRAPH PREDICTION

Using optimized values of α hydrographs were predicted by the converging overland flow model for events in the prediction set of each watershed. A comparison of observed and predicted hydrograph peak characteristics is shown in Tables 3.1-3.4. From these tables it is clear that hydrograph peak characteristics are very sensitive to the method of infiltration used to determine rainfall-excess. Understandably, ϕ-index leads to the highest errors in runoff peak predictions. Generally, Horton equation performs the best of all. Philip equation and Kostyakov equation are comparable. Thus the error in prediction of peak and its time can be mimimized considerably by the use of more refined methods of infiltration.

On comparing the relative error due to one method of infiltration with another it is apparent that the principal source of error in runoff prediction is the error in rainfall-excess. For example, for rainfall event of 9-5-1946 on watershed 2-H the relative error in peak prediction due to ϕ-index is 80.2%; it can, however, be minimized to 21.84% by Philip

Table 3.1. A comparison of observed and predicted hydrograph peaks.

Watershed	Date of Events	Observed Peak (cm/hr)	ϕ-index		Philip Equation		Kostyakov Equation		Horton Equation	
			Predicted Peak (cm/hr)	Relative Error* (%)	Predicted Peak (cm/hr)	Relative Error (%)	Predicted Peak (cm/hr)	Relative Error (%)	Predicted Peak (cm/hr)	Relative Error (%)
Hastings, Nebraska	8-11-1939	2.8194	0.6627	76.49	1.0397	63.12	1.0604	62.39	1.3274	+52.92
2-H	5-20-1949	0.7036	0.0876	87.55	0.1374	80.47	0.1410	79.96	0.1764	74.93
	6-12-1965	8.8138	7.0742	19.74	8.5226	3.3	8.6643	1.7	9.3584	- 6.18
	6-12-1965	2.1565	1.6322	24.31	2.0611	4.42	1.9472	9.71	1.9304	10.48
	6-29-1965	2.0676	1.2235	40.82	1.8595	10.05	1.8929	8.45	2.1513	- 4.05
	8-17-1946	3.7592	0.9324	75.20	1.4504	61.42	1.4854	60.49	1.8292	51.34
	8- 7-1942	2.5197	0.9752	61.30	1.4635	41.92	1.5120	39.99	1.9646	22.03
	6-16-1950	0.1735	0.0029	98.33	0.0073	95.77	0.0077	95.56	0.0120	93.10
	9- 7-1942	3.5306	1.3042	63.06	1.9656	44.33	2.004	43.24	2.3964	32.12
	9- 5-1946	2.2885	0.4535	80.20	1.7887	21.84	1.8231	20.34	2.1253	7.13

$$* \text{ Relative Error} = \frac{\text{Observed Quantity} - \text{Predicted Quantity}}{\text{Observed Quantity}}$$

Table 3.2. A comparison of observed and predicted hydrograph peaks.

Watershed	Date of Events	Observed Peak (cm/hr)	ϕ-index		Philip Equation		Kostyakov Equation		Horton Equation	
			Predicted Peak (cm/hr)	Relative Error (%)	Predicted Peak (cm/hr)	Relative Error (%)	Predicted Peak (cm/hr)	Relative Error (%)	Predicted Peak (cm/hr)	Relative Error (%)
Hastings, Nebraska	8-11-1939	4.5212	8.3918	-85.61	6.8409	-51.31	6.8147	-50.73	7.0639	-56.24
4-H	6-20-1942	5.8166	6.6806	-14.85	6.3668	- 9.46	6.3502	- 9.17	5.3913	7.31
	9- 5-1946	3.8862	2.8168	27.52	4.3812	-12.74	4.3790	-12.68	4.4299	-13.99
	6- 1-1951	6.7564	9.7074	-43.68	8.2870	-22.65	8.2656	-22.34	8.4311	-24.79
	7-13-1952	9.1948	10.0487	- 9.29	10.0433	- 9.23	9.3968	- 2.2	8.1860	10.97
	6-12-1958	0.9195	1.5718	-70.95	1.1972	-30.20	1.1789	-28.22	1.2441	-35.31
	6-12-1965	9.7028	9.1490	5.71	9.1733	5.46	9.1593	5.6	8.9717	7.53
	6-12-1965	6.1468	6.5100	- 5.91	6.4889	- 5.57	6.4661	- 5.2	6.4288	- 4.59

Table 3.3. A comparison of observed and predicted hydrograph peaks.

Watershed	Date of Events	Observed Peak Time (min)	ϕ-index		Philip Equation		Kostyakov Equation		Horton Equation	
			Predicted Peak Time (min)	Relative Error (%)	Predicted Peak Time (min)	Relative Error (%)	Predicted Peak Time (min)	Relative Error (%)	Predicted Peak Time (min)	Relative Error (%)
Hastings, Nebraska 2-H	8-11-1939	10.00	27.2	- 172.00	18.8	- 88.00	18.8	- 88.00	15.3	- 53.00
	5-20-1949	3.00	51.8	-1626.67	35.5	-1083.33	35.3	-1076.67	30.4	- 913.33
	6-12-1965	9.00	28.7	- 218.99	25.8	-186.67	25.3	- 181.11	20.2	- 124.44
	6-12-1965	13.00	26.3	- 102.31	20.4	- 56.92	20.5	- 57.69	17.2	- 32.31
	6-29-1965	9.00	23.8	- 164.44	16.8	- 86.67	15.9	- 76.67	15.9	- 76.67
	8- 7-1946	7.00	26.5	- 278.56	17.8	-154.29	17.8	- 154.29	14.9	- 112.86
	8- 7-1942	10.00	30.6	- 206.00	28.1	-181.00	27.9	- 179.00	16.6	- 66.00
	6-16-1950	2.00	40.0	-1900.00	40.0	-1900.00	40.0	-1900.00	40.0	-1900.00
	9- 7-1942	15.00	24.6	- 64.00	16.7	- 11.33	16.6	- 10.67	14.3	4.67
	9- 5-1946	9.00	133.3	-1381.11	18.6	-106.67	17.8	- 97.78	15.6	- 73.33

Table 3.4. A comparison of observed and predicted hydrograph peak times.

Watershed	Date of Events	Observed Peak Time (min)	ϕ-index		Philip Equation		Kostyakov Equation		Horton Equation	
			Predicted Peak Time (min)	Relative Error (%)	Predicted Peak Time (min)	Relative Error (%)	Predicted Peak Time (min)	Relative Error (%)	Predicted Peak Time (min)	Relative Error (%)
Hastings, Nebraska	8-11-1939	5.0	3.8	24.00	5.5	-10.00	5.5	-10.00	5.3	- 6.00
	6-20-1942	8.0	4.1	48.75	6.2	22.50	6.2	22.50	10.0	-25.0
4-H	9- 5-1946	12.0	4.7	60.83	6.4	46.67	6.4	46.67	6.2	48.33
	6- 1-1951	123.0	146.0	-18.70	124.6	- 1.30	124.6	- 1.30	79.3	35.53
	7-13-1952	21.0	14.8	29.52	5.4	74.28	5.5	73.81	18.0	14.29
	6-12-1958	14.0	7.6	45.71	10.6	24.29	10.6	24.29	9.2	34.28
	6-12-1965	19.0	17.1	10.00	10.1	46.84	10.1	46.84	9.8	48.42
	6-12-1965	7.0	9.2	-31.43	11.0	-57.14	11.0	-57.14	11.0	-57.14

equation, to 20.34% by Kostyakov equation and to 7.13% by Horton equation. This was observed in an earlier study by Singh and Woolhiser (1976). It then goes without saying that accurate determination of rainfall-excess is crucial to runoff prediction.

Now a question arises: How is the entire hydrograph affected by the choice of a particular infiltration method. For two sample events observed and predicted hydrographs are shown in Figs. 3.2-3.9. It is evident from these figures that hydrograph shape is quite sensitive to the method of infiltration. For example, for rainfall event of 9-7-1942 on watershed 2-H ϕ-index leads to a hydrograph shape which is very different from the observed hydrograph shape. On the other hand, Philip equation, Kostyakov equation and Horton equation lead to more realistic hydrograph shapes as is clear from Figs. 3.3-3.5. Thus it can be concluded that more accurate the method of infiltration, more realistic the hydrograph shape.

We must remark here that in no way we are saying that rainfall-excess is the only factor affecting hydrograph characteristics. Of course, there are other factors too such as space-time distribution of rainfall, watershed physiography and their interacting influences. As stated previously, rainfall-excess is probably one of the most important factors in simulation of runoff hydrograph. We have attempted, in a small measure, to identify its role in runoff modeling.

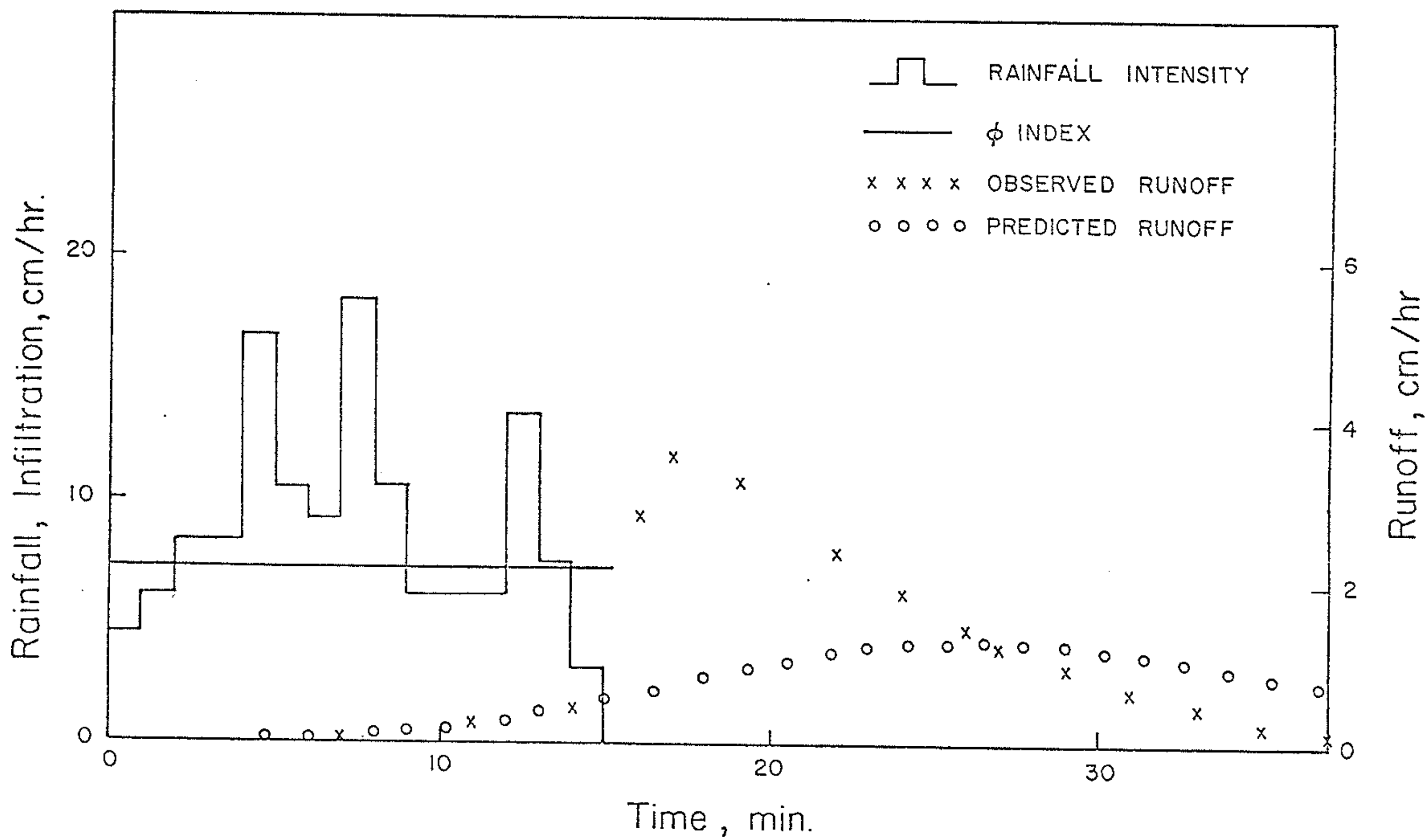

Fig. 3.2. Hydrograph prediction by the model, using ϕ-index for infiltration, for rainfall event of 9-7-1942 on watershed 2-H, Hastings, Nebraska.

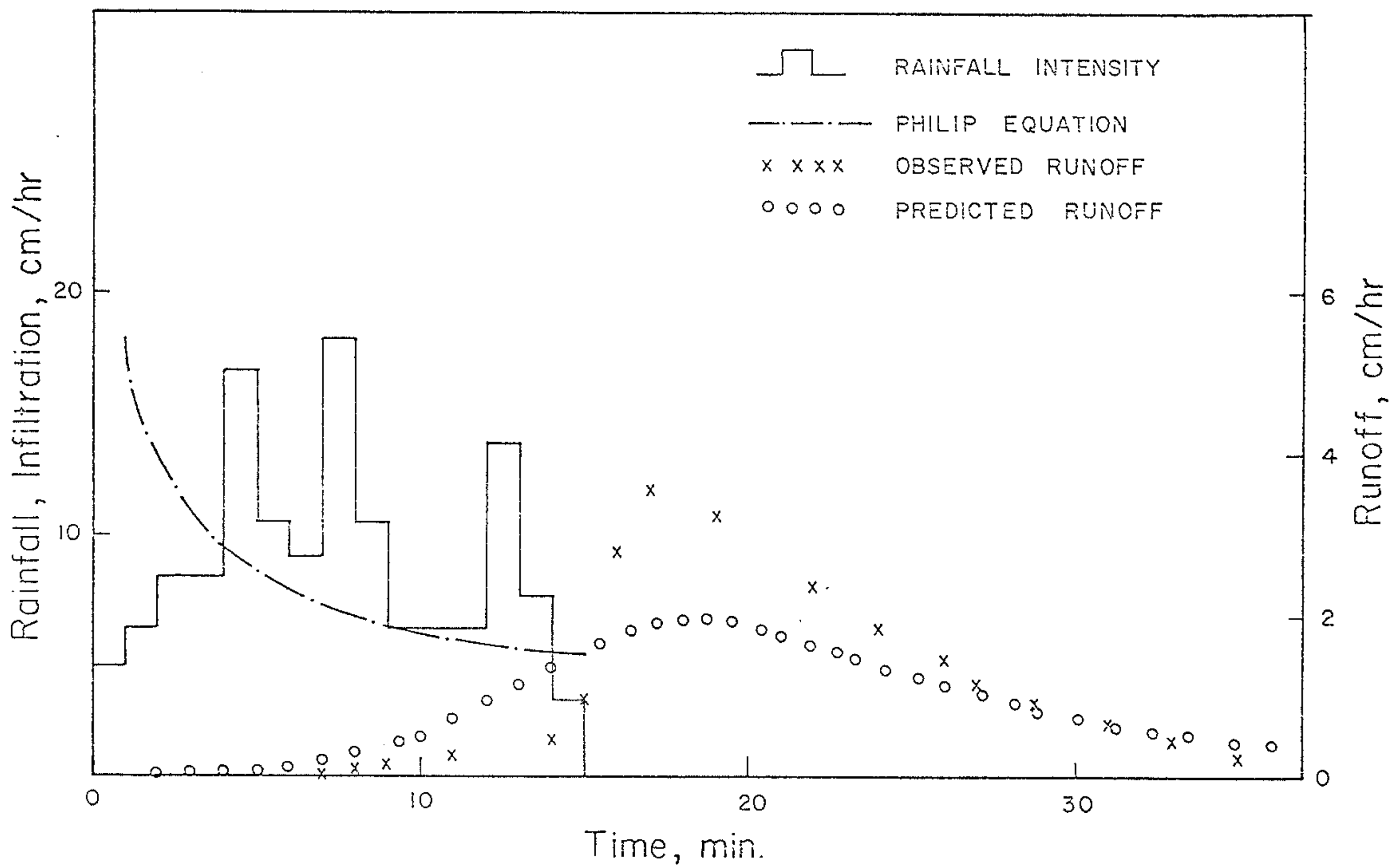

Fig. 3.3. Hydrograph prediction by the model, using Philip equation for infiltration, for rainfall event of 9-7-1942 on watershed 2-H, Hastings, Nebraska.

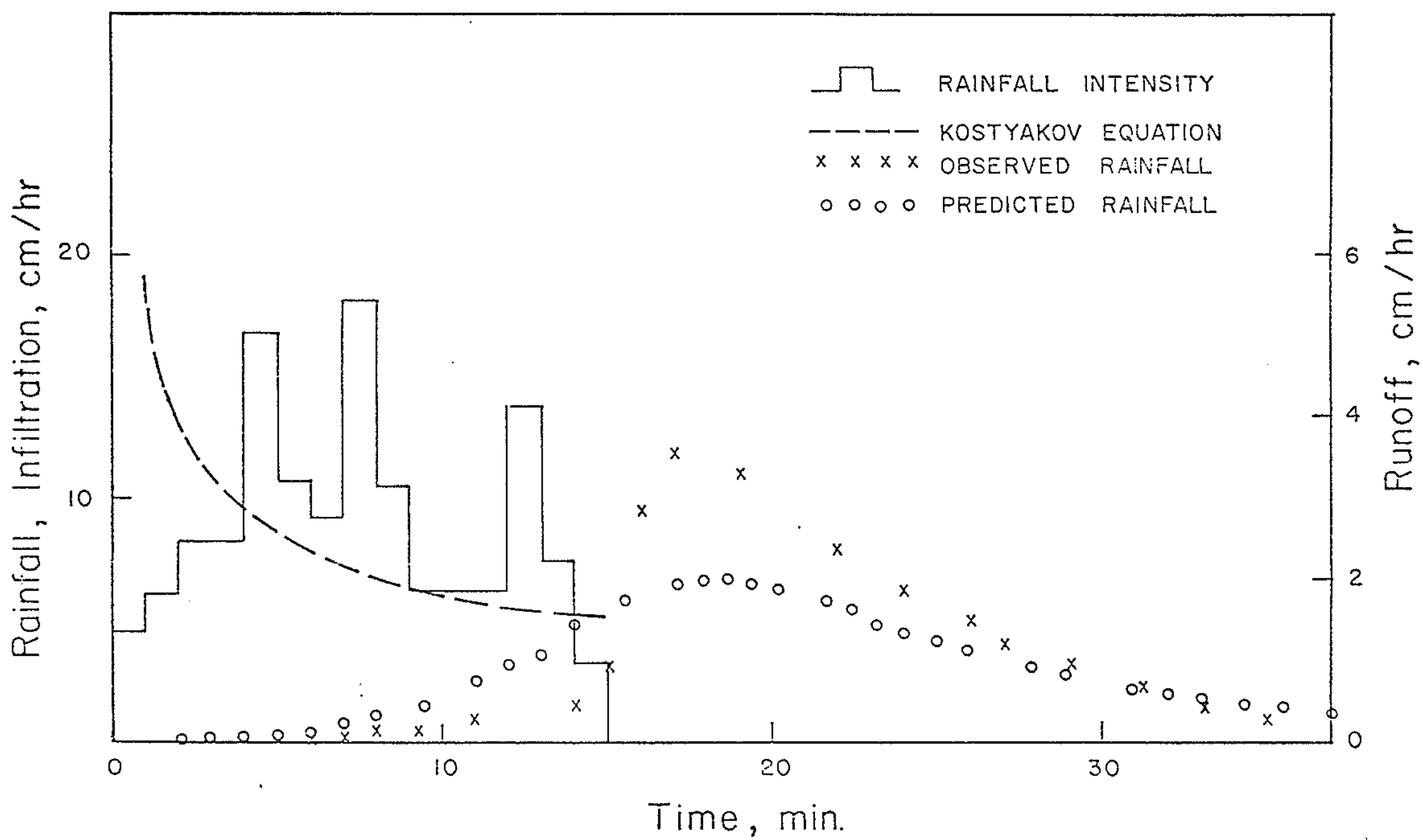

Fig. 3.4. Hydrograph prediction by the model, using Kostyakov equation for infiltration, for rainfall event of 9-7-1942 on watershed 2-H, Hastings, Nebraska.

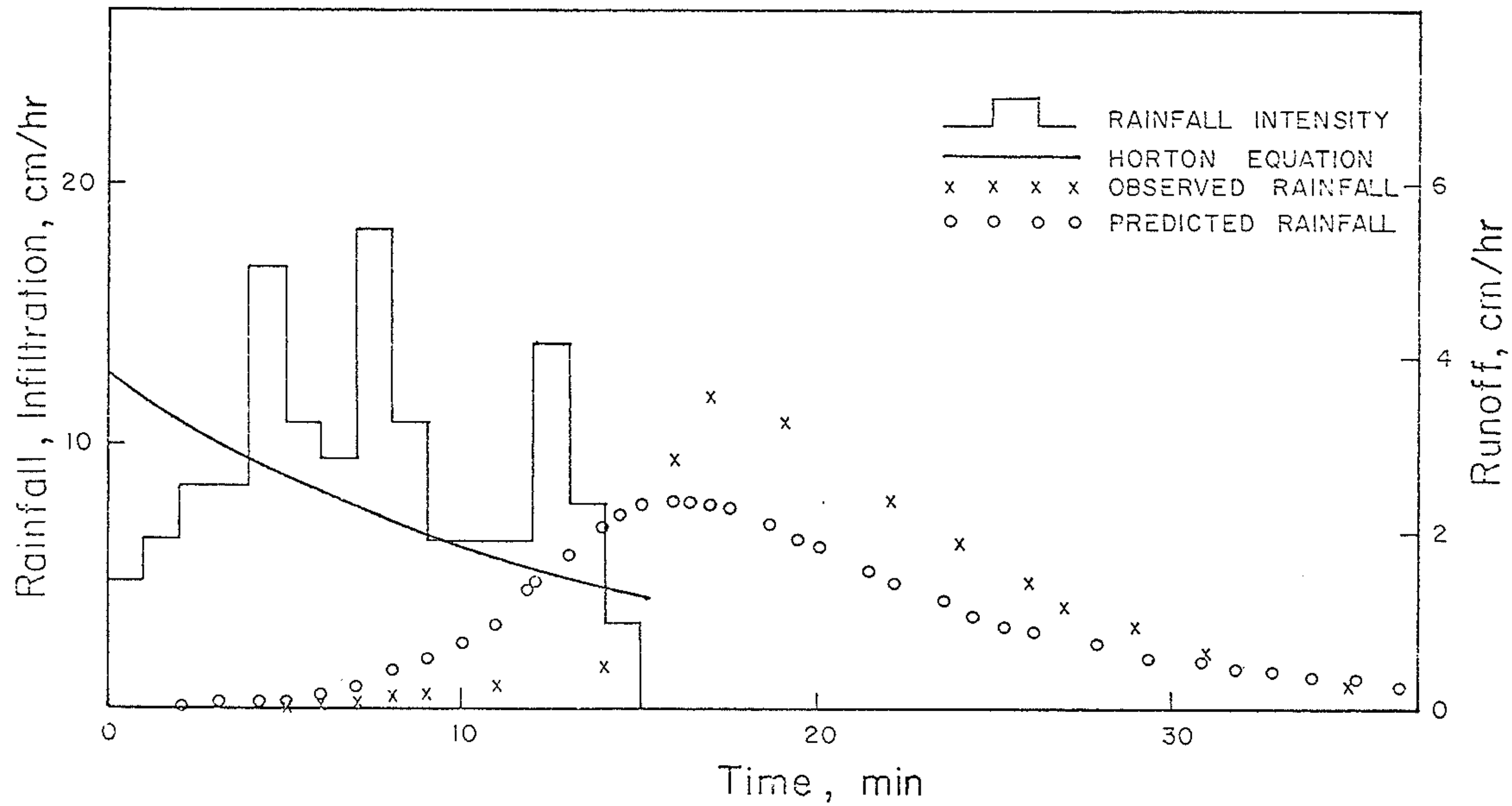

Fig. 3.5. Hydrograph prediction by the model, using Horton equation, for rainfall event of 9-7-1942 on watershed 2-H, Hastings, Nebraska.

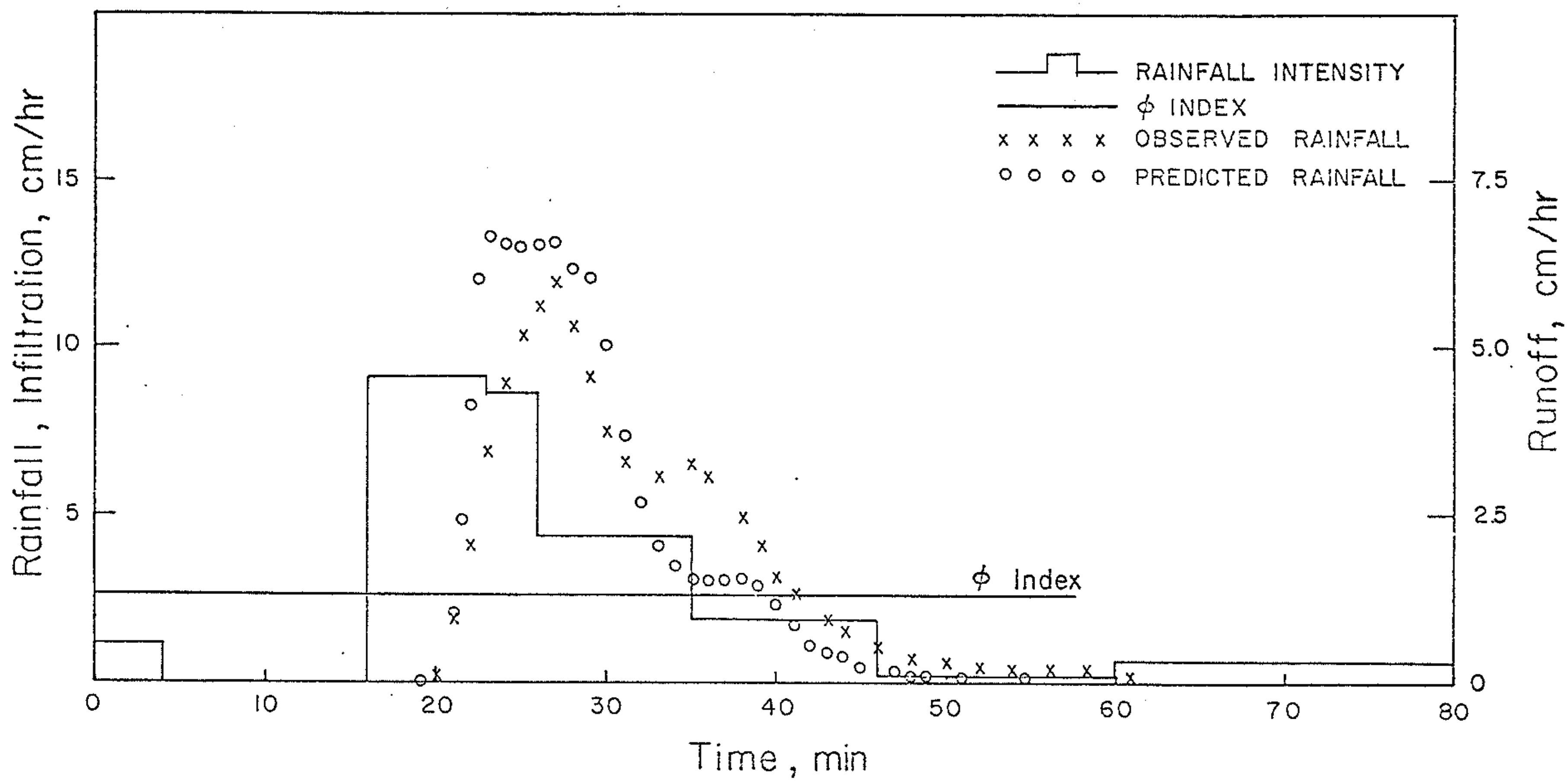

Fig. 3.6. Hydrograph prediction by the model, using ϕ-index for infiltration, for rainfall event of 6-20-1942 on watershed 4-H, Hastings, Nebraska.

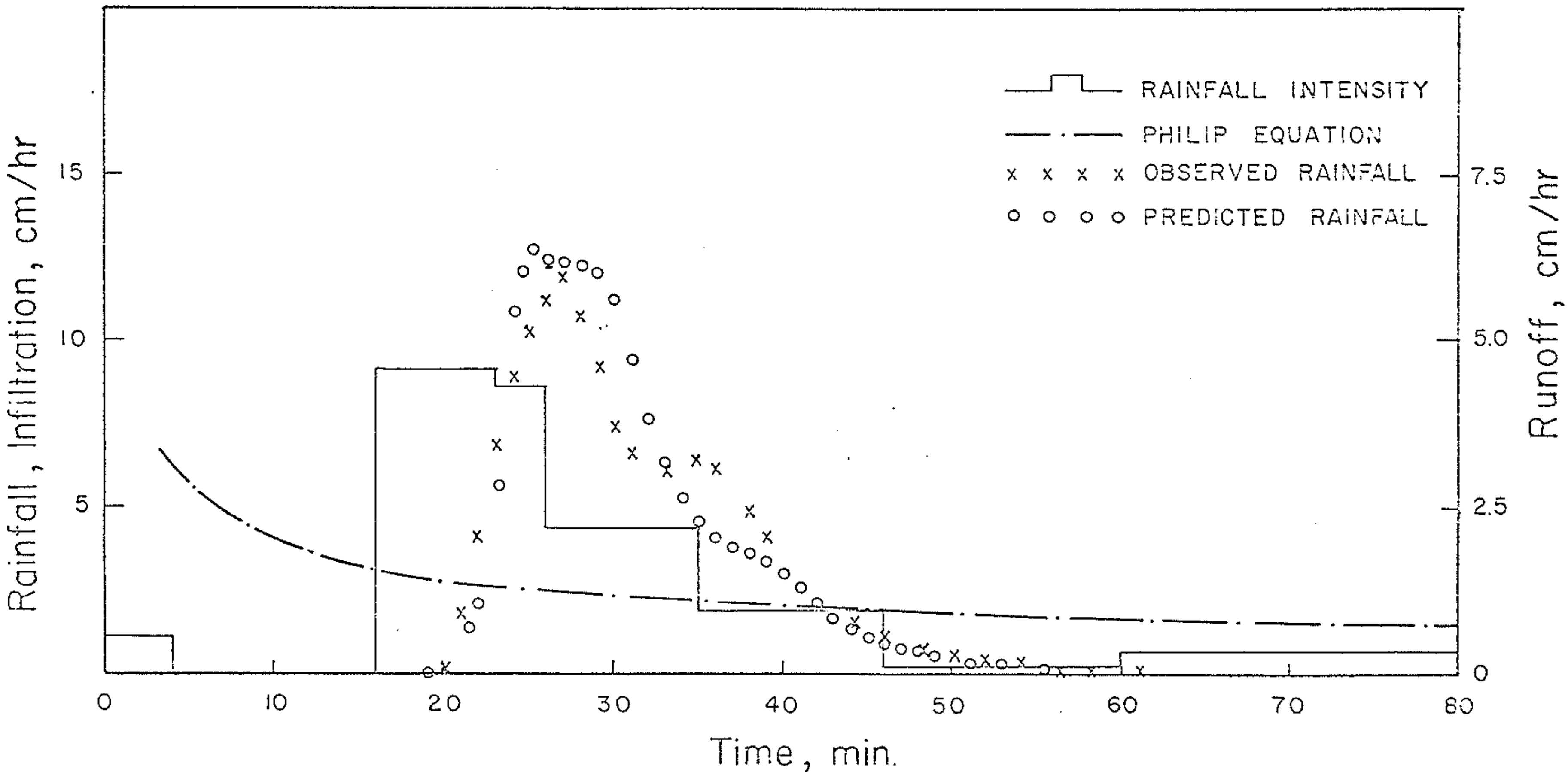

Fig. 3.7. Hydrograph prediction by the model, using Philip equation for infiltration, for rainfall event of 6-20-1942 on watershed 4-H, Hastings, Nebraska.

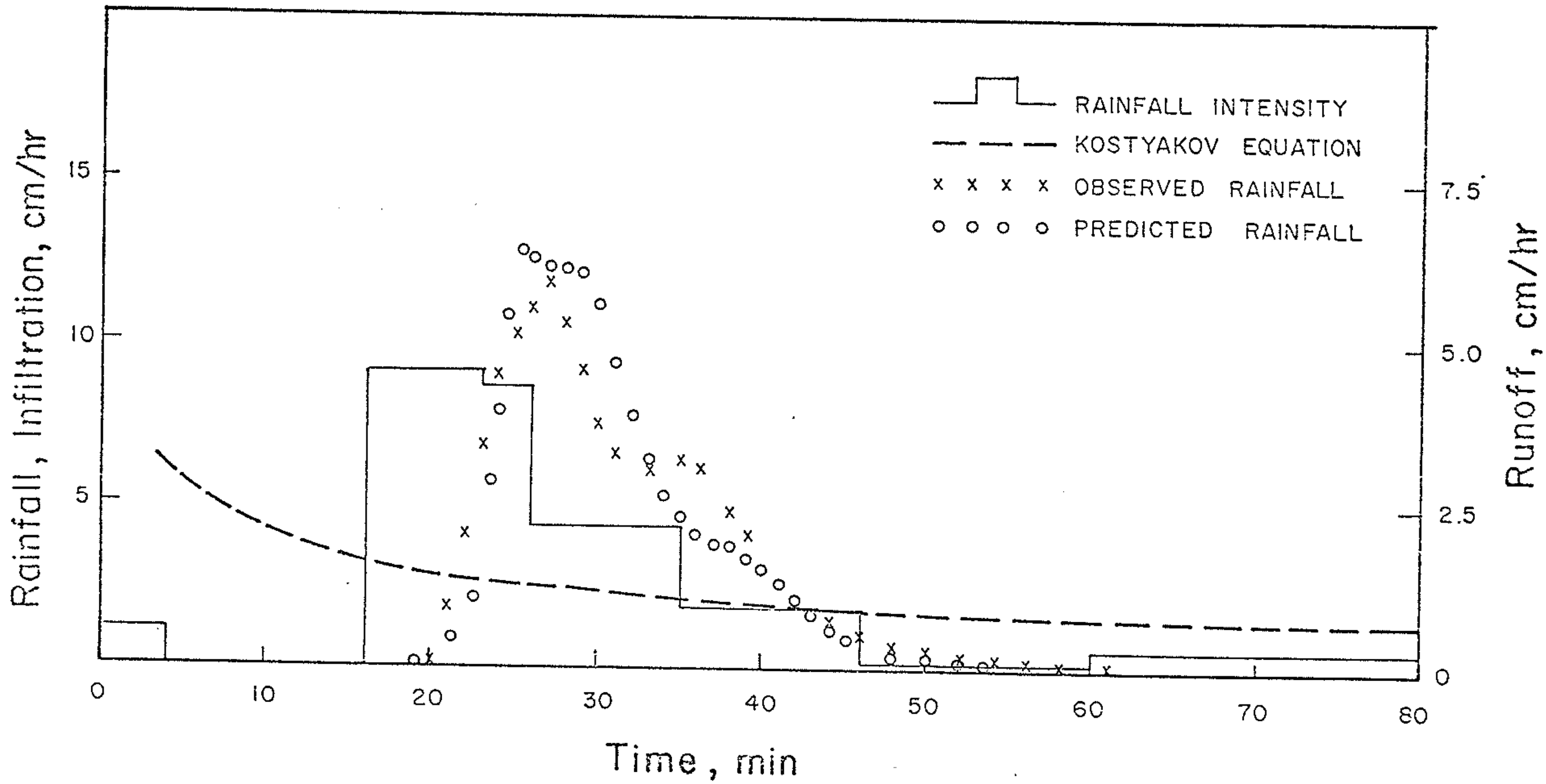

Fig. 3.8. Hydrograph prediction by the model, using Kostyakov equation for infiltration, for rainfall event of 6-20-1942 on watershed 4-H, Hastings, Nebraska.

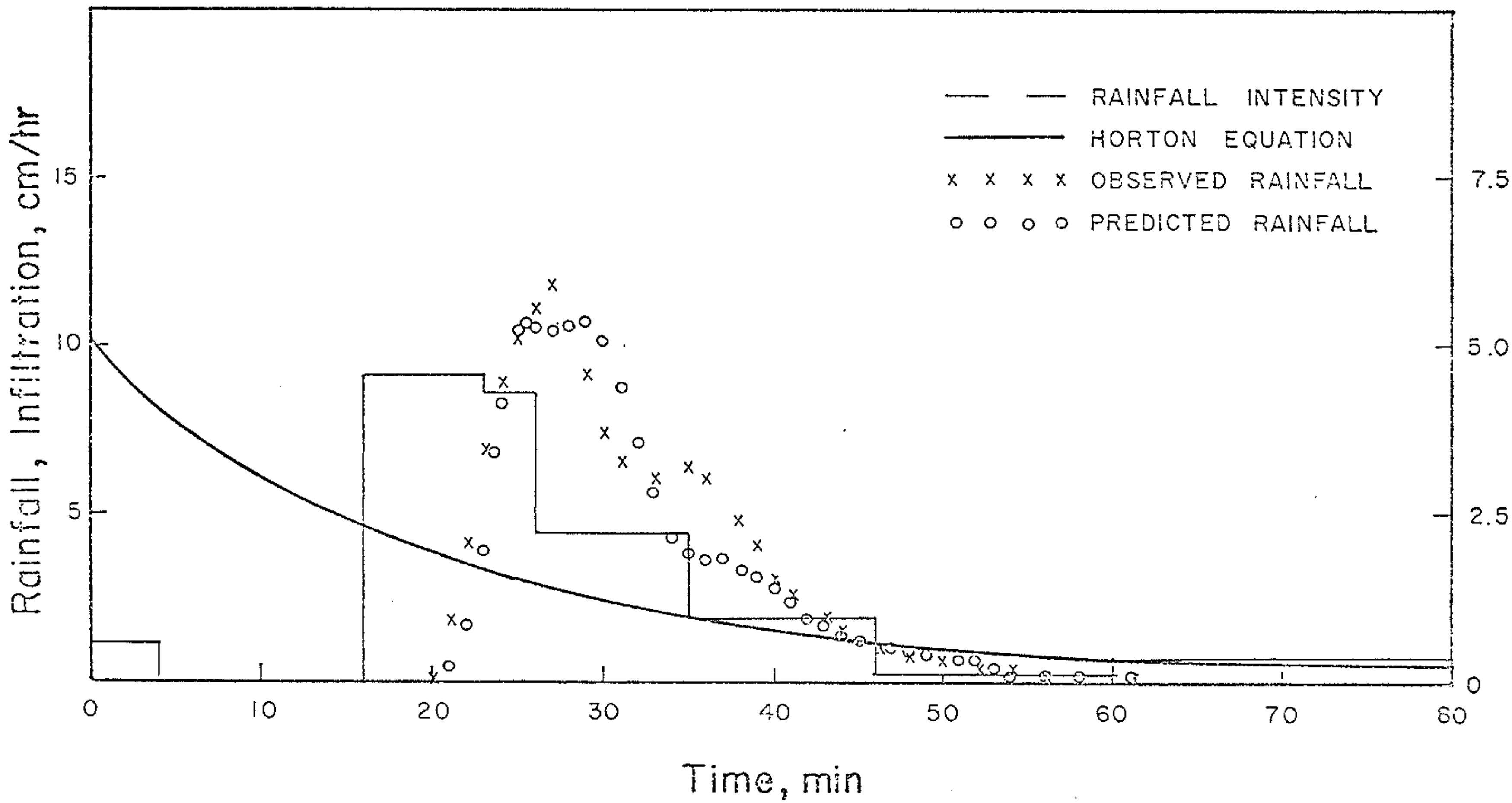

Fig. 3.9. Hydrograph prediction by the model, using Horton equation for infiltration, for rainfall event of 6-20-1942 on watershed 4-H, Hastings, Nebraska.

Chapter 4

RESPONSE OF RUNOFF MODELS TO ERRORS IN RAINFALL-EXCESS

4.1 GENERAL REMARKS

Surface runoff is generally recognized as a nonlinear process, a recognition which has spurred criticism of using linear models in describing the surface runoff response of a watershed (Amorocho and Orlob, 1961). Nevertheless, linear models are continually used in routine hydrologic applications.

Several nonlinear surface runoff models are known today in hydrology (Amorocho and Orlob, 1961; Chow, 1964; Singh, 1964; Kibler and Woolhiser, 1970; Wooding, 1965a, 1965b, 1966). Some attempts have been made to evaluate the departure from the linearity hypothesis conventionally used for characterizing the watershed surface runoff response (Amorocho and Orlob, 1961). Based on these investigations arguments have been presented favoring nonlinear models.

However a basic question either ignored or only qualitatively approached is: How do linear and nonlinear models of surface runoff compare if there are errors in the input? This is the question that is addressed here.

It is recognized that of various sources of discrepancies in the model results, the one due to errors in the input is perhaps the most important. Usually, rainfall-excess forms input to surface runoff models.

4.2 SOURCES OF INPUT ERRORS

There seem to be two principal sources of errors in rainfall-excess. We briefly discuss them here.

4.2.1 DETERMINATION OF MEAN AREAL RAINFALL

Of practical necessity, rainfall is measured at a number of sample points in the watershed, and the amounts recorded at these points must be utilized to form an estimate of mean areal rainfall for the storm of interest. This estimate may, however, differ substantially from the true mean areal rainfall for three reasons (Singh and Birsoy, 1975):

a. The sample points may be unrepresentative of the watershed in that no gauge may lie in the sector of watershed having extreme rainfall (Rodda, 1970).

b. The record may be consistently higher or lower than the true rainfall at a sample point (Rodda, 1967).

c. These factors may combine to cause the rainfall amounts recorded by guages to differ from their true values in an unsystematic manner (Sutcliffe, 1966; Herbst and Shaw, 1969).

It is, therefore, not surprising that little is known about the accuracy of the mean areal rainfall estimates (McGuiness, 1963); the ease of making accurate point measurements of rainfall and the simplicity of determining the mean areal rainfall are both deceptive indeed (Rodda, 1970). Nevertheless, a number of techniques have been developed; some of them are simple and well-tried and frequently used without sufficient appreciation of their limitations.

4.2.2 DETERMINATION OF RAINFALL-EXCESS

Not all rainfall that falls on the ground becomes surface runoff; indeed only a fraction of it does. A widely used approach to determine rainfall-excess is the one based on infiltration. The process of infiltration is extremely complex. Nevertheless, there has been a proliferation in development of infiltration models in the recent years (Philip, 1957; Holtan, 1961; Smith and Woolhiser, 1971). The infiltration estimated by a model will differ from the true infiltration for the following reasons which no model seems to account for:

a. Heterogeneous watershed characteristics in space and time.

b. Heterogeneous antecedent soil moisture conditions over the watershed prior to occurrence of a rainfall episode.

c. Hetergeneous, complex nature of rainfall event occurring over the watershed.

d. Interaction of some or all of the above factors.

Therefore, the amount and distribution of rainfall-excess in time and space is really never known. Errors in the distribution of rainfall-excess estimates may be large even when the runoff volume is known. For example, Fig. 4.1 shows three methods of determining rainfall-excess for a hypothetical case, each maintaining the same volume of rainfall-excess. Each method will lead to a different distribution of rainfall-excess, and each distribution will differ from the true distribution. Frequently, rainfall-excess is either assumed or computed rather arbitrarily even though this is a major source of error in the model results.

4.3 EVALUATION OF THE EFFECTS OF INPUT ERRORS ON RUNOFF PEAK PREDICTIONS

4.3.1 CHOICE OF MODELS

The five models chosen for this study are more or less representative of two groups of surface runoff models: 1. kinematic wave models, 2. storage models. These models have been extensively studied and widely applied to natural watersheds with varying degrees of success. Although these models are relatively simple, it is hoped that they will serve to illustrate the objective of this study. Simplicity and known explicit analytical solutions were two main considerations for choosing them.

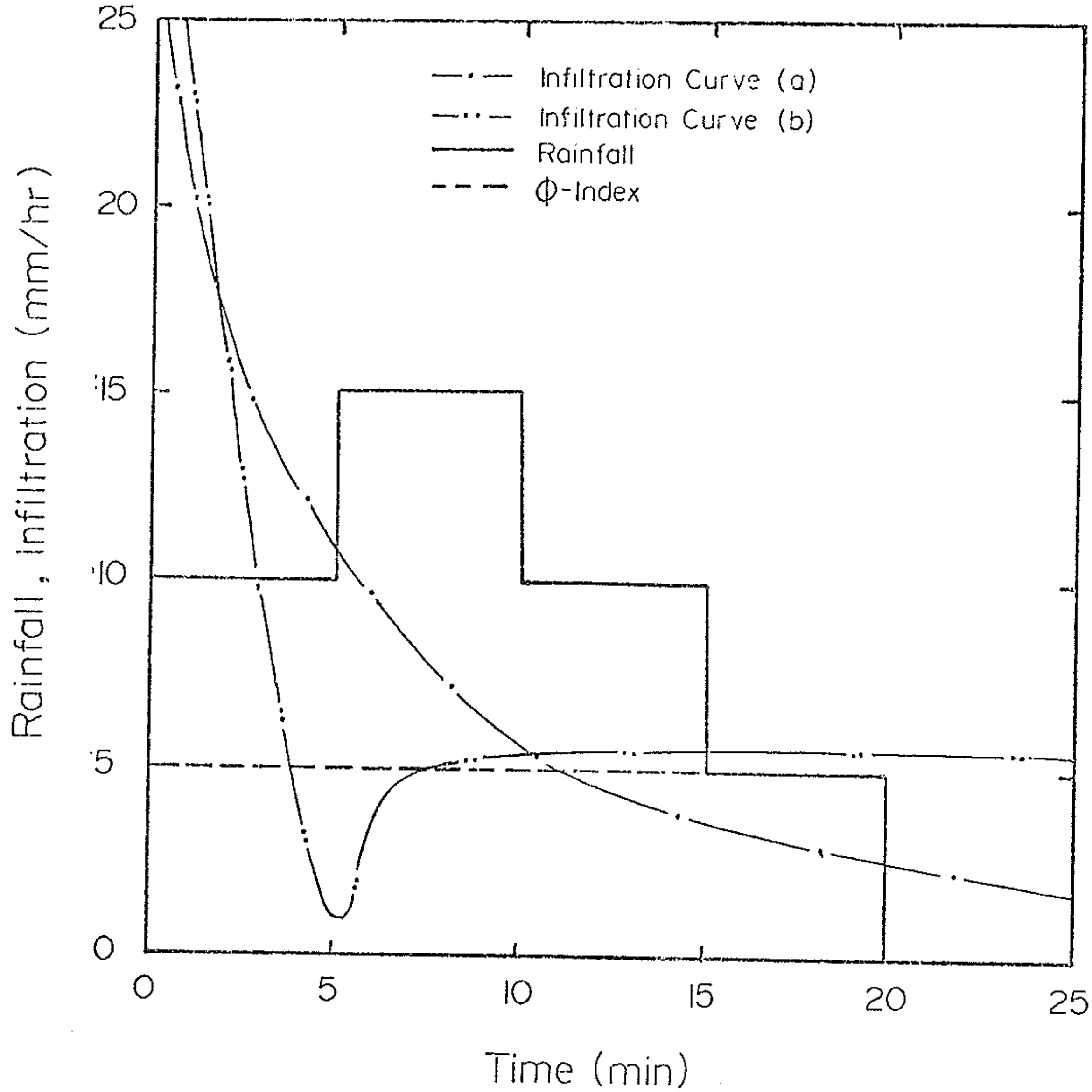

Fig. 4.1. Determination of rainfall-excess by three methods for a hypothetical rainstorm.

4.3.2 ESTIMATION OF MODEL PARAMETERS

It is assumed that the system is truly nonlinear, that it can be perfectly represented by the converging overland flow model, and that the parameters of CONV are perfectly identifiable. On this assumption the parameters of CONV were chosen to be: $n = 1.5$, $\alpha = 10$, $r = 0.01$, $L_o(1 - r) = 300$ m, $\theta = 60°$. The choice of these values is entirely arbitrary, but is such that for a given set of rainfall events the resulting hydrographs will be partial equilibrium hydrographs. Then the parameters of the remaining four models were optimized by a least squares technique utilizing the objective function:

$$\min \sum_{j=1}^{M} \left\{ Q_{p_o}(j) - Q_{p_e}(j) \right\}^2 \tag{4.1}$$

where Q_{p_o} and Q_{p_e} denote observed (from CONV) and estimated (from a model) hydrograph peaks for the jth event, and M is the number of events in the optimization set. It is interesting to note that this procedure leads to explicit solutions for the model parameters.

For PLANE obviously $n = 1.5$, $L = 300$ m and α is given by:

$$\alpha = \frac{\sum_{j=1}^{M} Q_{p_o}(j)\, V^n(j)}{\sum_{j=o}^{M} V^{2n}(j)} \tag{4.2}$$

where V is volume of the jth event. W for PLANE is computed by dividing the area of CONV by L. By considering a set of q rainfall events of intensities, 3 cm/hr to 10 cm/hr, all lasting for 1000 seconds, the parameter α was optimized and obtained to be 15.5. Thus PLANE is now completely specified.

For NASH we fixed N at 3 (Shelburne and Singh, 1976) and K was obtained from:

$$K = \frac{C \sum_{j=1}^{M} V^2(j)}{\sum_{j=1}^{M} Q_p(j)\ V(j)} \tag{4.3}$$

where $C = \frac{(N-1)^{N-1}}{\Gamma(N)} \bar{e}^{\ (N-1)}$

For the aforementioned rainfall events K was found to be 708.62 seconds.

For O'Kelly model K is the same as in NASH and T was taken to be equal to K (Dooge, 1973). So was done for CLARK. Thus, parameters of these models were estimated explicitly in terms of the parameters of CONV.

4.3.3 SENSTIVITY OF MODELS TO INPUT ERRORS

Define ε_q as the error in the input q, ε_Q as the error in the hydrograph peak and ε_t as the error in the hydrograph peak time. First, we consider the linear models.

For NASH it is easy to show from Eqs. (2.17) and (2.18) that:

$$\varepsilon_Q = \frac{CT}{K}\ \varepsilon_q \tag{4.4}$$

$$\varepsilon_t = o \tag{4.5}$$

where $C = (N-1)^{N-1} \frac{1}{\Gamma(N)} \bar{e}^{\ (N-1)}$

It is thus clear that ε_Q is a linear function of ε_q and that t_p is independent of q so long as its duration T remains the same.

For KELLY we can show from Eqs. (2.19) and (2.20) that:

$$\varepsilon_Q = \frac{4}{T}\ D\left(1 - \frac{t_p}{T}\right)\ \varepsilon_q \tag{4.6}$$

$$\varepsilon_t = o \tag{4.7}$$

where D = duration of q. Here again we arrive at the same conclusion as we did in NASH. And so is true for CLARK, as seen from the following (using Eqs. (2.21) and (2.22)

$$\varepsilon_Q = \frac{D}{T}\left(1 - \bar{e}^{T/K}\right)\varepsilon_q \tag{4.8}$$

$$\varepsilon_t = o \tag{4.9}$$

Now we turn to the nonlinear models. For PLANE we can combine Eqs. (2.11) and (2.12) and obtain:

$$Q_p = \alpha\ (qD)^n \tag{4.10}$$

Thus we can write:

$$\varepsilon_Q = \alpha\ (Dq)^n \left[\left(1 + \frac{\varepsilon_q}{q}\right)^n - 1\right] \tag{4.11}$$

Expanding the bracketed term,

$$\left[\left(1 + \frac{\varepsilon_q}{q}\right)^n - 1\right] = 1 + \left(\frac{\varepsilon_q}{q}\right)\binom{n}{1} + \left(\frac{\varepsilon_q}{q}\right)^2\binom{n}{2} + \cdot\cdot\cdot\cdot\cdot\cdot\cdot - 1$$

Equation (4.11) can then be written as:

$$\varepsilon_Q = \alpha D^n\ q^n \sum_{j=1}^{\infty} \left(\frac{\varepsilon_q}{q}\right)^j \binom{n}{j} \tag{4.12}$$

$$\varepsilon_t = o \tag{4.13}$$

Thus for PLANE, although ε_Q is a nonlinear function of q and ε_q, ε_t is independent of q so long as D remains the same.

However, for CONV an explicit analytic solution for ε_Q does not seem tractable but it is obvious from Eqs. (2.6) and (2.7) that both ε_Q and ε_t will depend nonlinearly on q even though its duration is kept fixed.

4.3.4 EVALUATION OF ERRORS IN RUNOFF PEAK PREDICTIONS BY THE MODELS

In this study only pulse rainfall events of intensities 3 cm/hr to 10 cm/hr lasting for 1000 seconds were considered. The relative error was defined as (observed quantity - estimated quantity) / observed quantity. The absolute error ε was defined as (observed quantity - estimated quantity). The relative error introduced in rainfall-excess ranged from 0 to ±50%.

For an error-free input it was assumed that CONV would produce an error-free hydrograph peak and its time. For an erroneous input this, however, will not be the case. A sample comparison of errors in runoff peak predictions by the models due to errors in input of 8 cm/hr lasting for 1000 seconds is shown in Figs. 4.2 and 4.3. It is clear from Fig. 4.2 that there are substantial input error ranges where CONV is inferior to other models. From Fig. 4.3, however, CONV is superior to every other model than NASH.

Figures 4.4-4.7 show error surface for runoff peak predicted by the models for a range of input intensities and relative input errors. Evidently, there are substantial regions where CONV is inferior to the model in comparison.

Figures 4.8-4.11 show error surfaces for runoff peak time predicted by the models for the same range of input intensities and relative errors. Obviously, CONV is superior to each of the models except NASH throughout the domain of comparison.

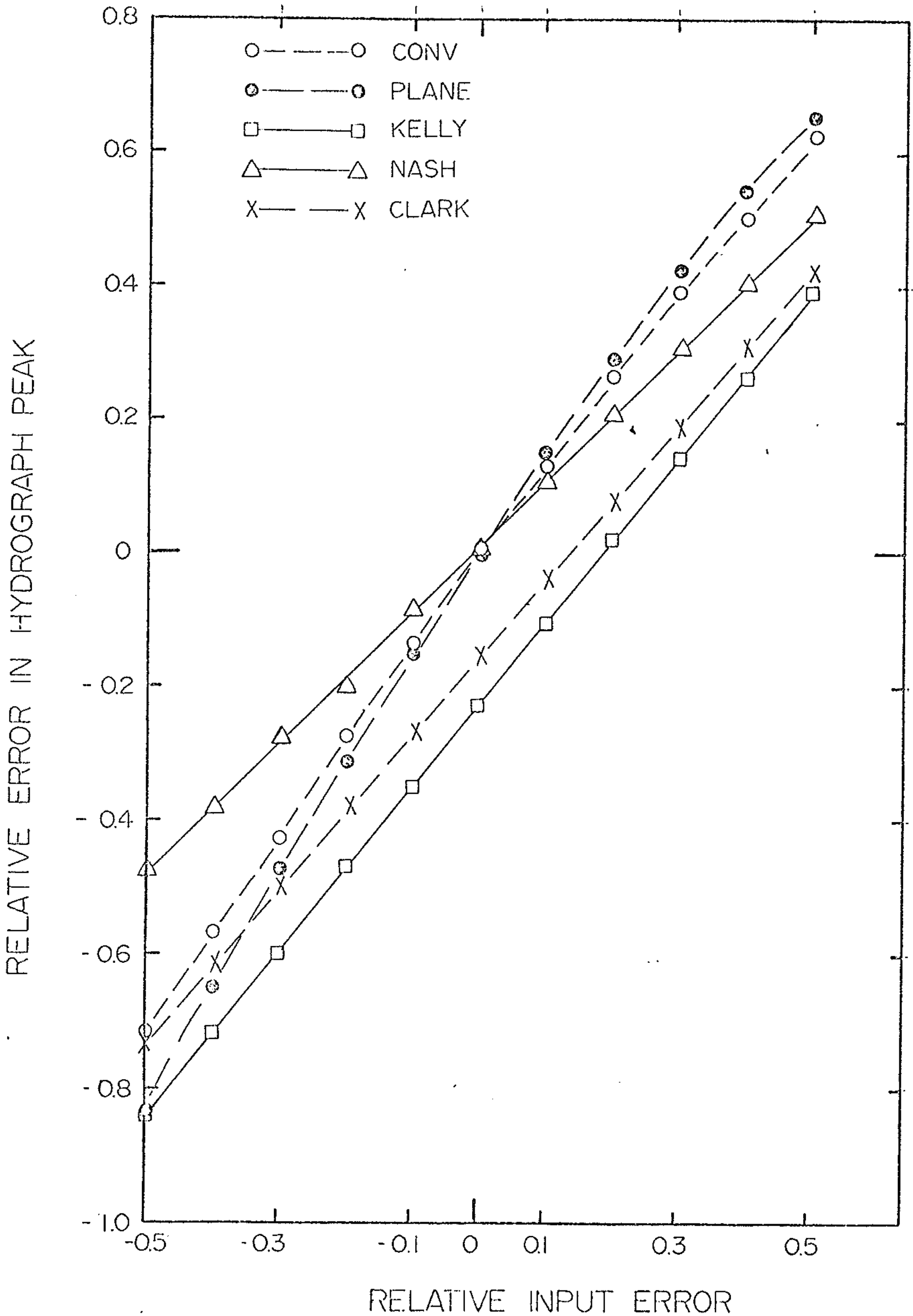

Fig. 4.2. Relative error in hydrograph peak predictions due to relative error in input.

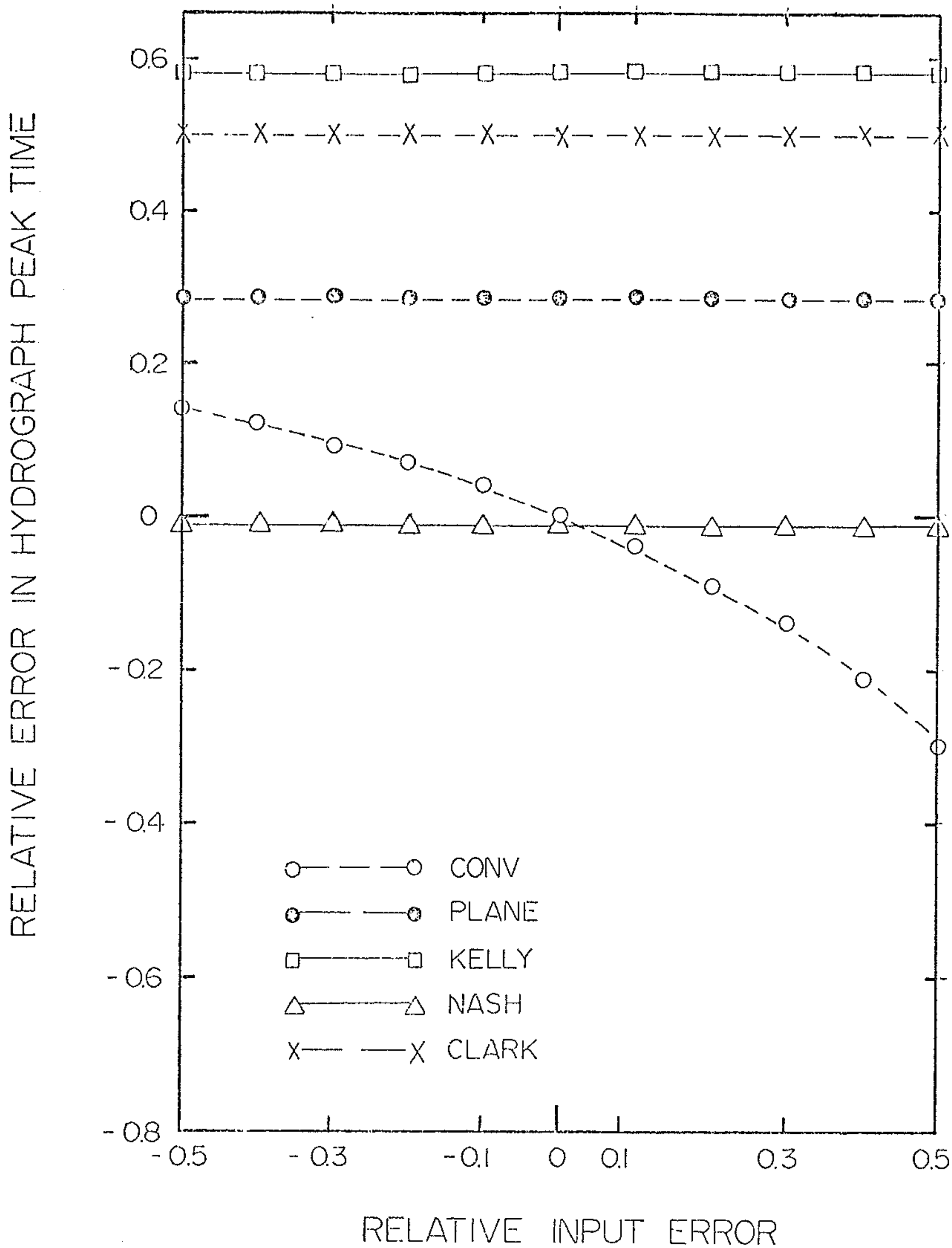

Fig. 4.3. Relative error in hydrograph peak time predictions due to relative error in input.

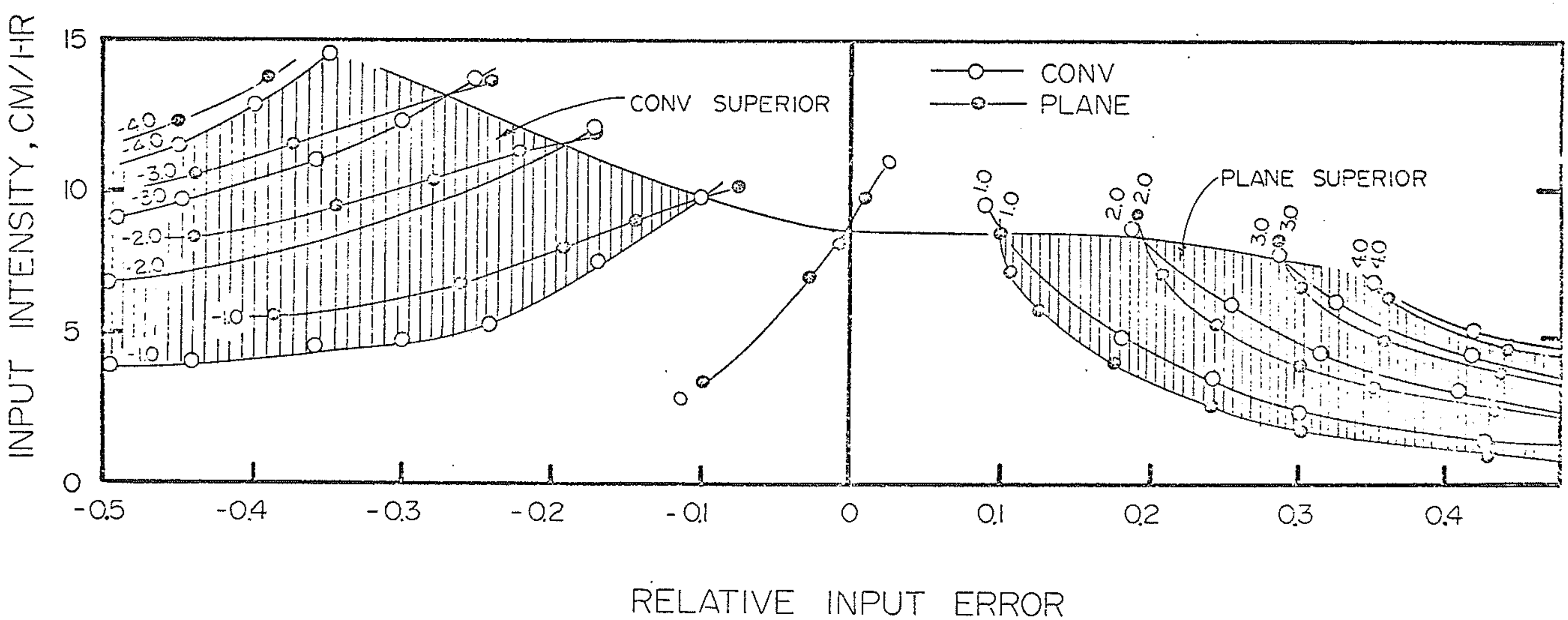

Fig. 4.4. Absolute error (cm/hour) surfaces for hydrograph peak predicted by CONV and PLANE models.

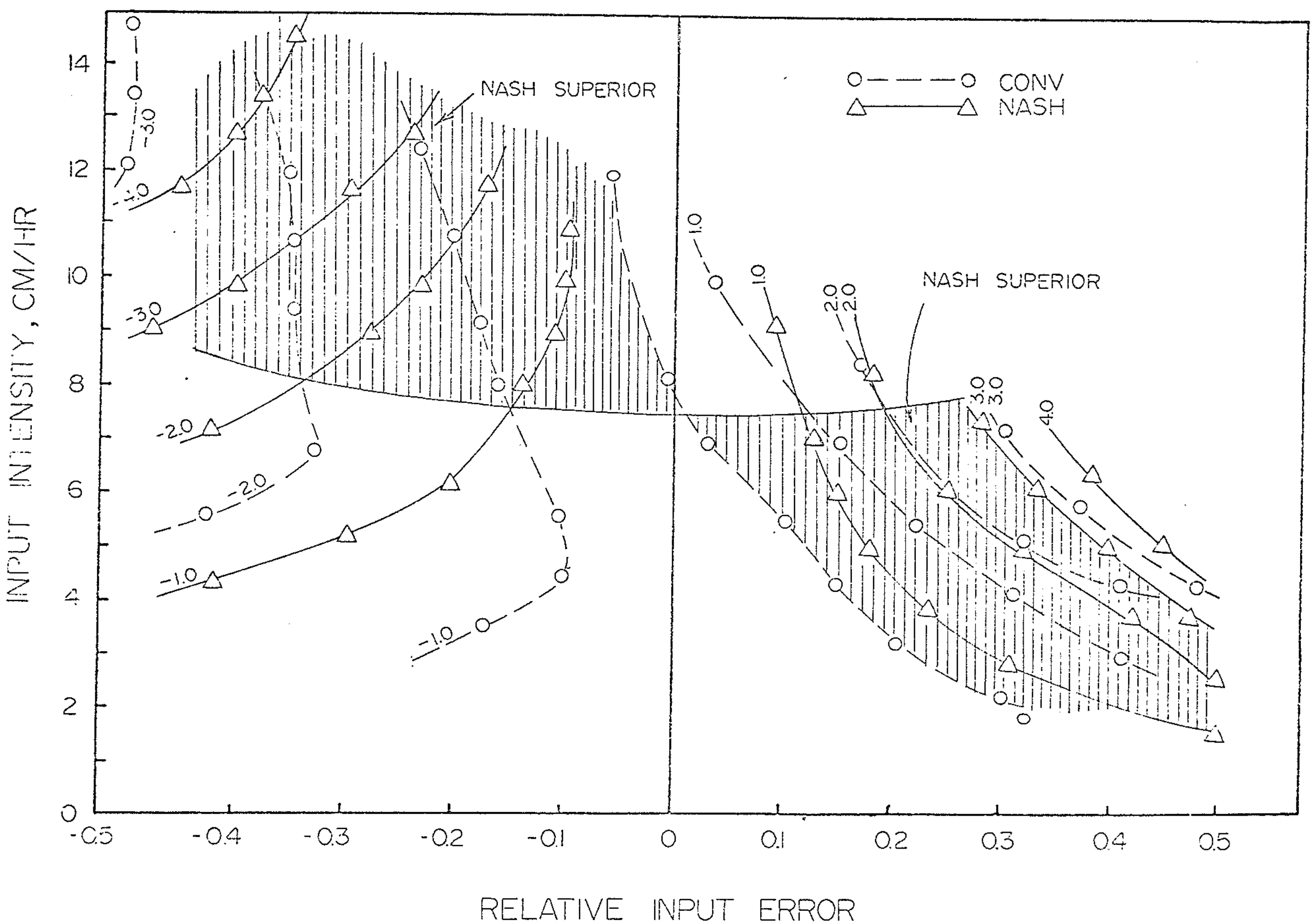

Fig. 4.5. Absolute error (cm/hour) surfaces for hydrograph peak predicted by CONV and NASH models.

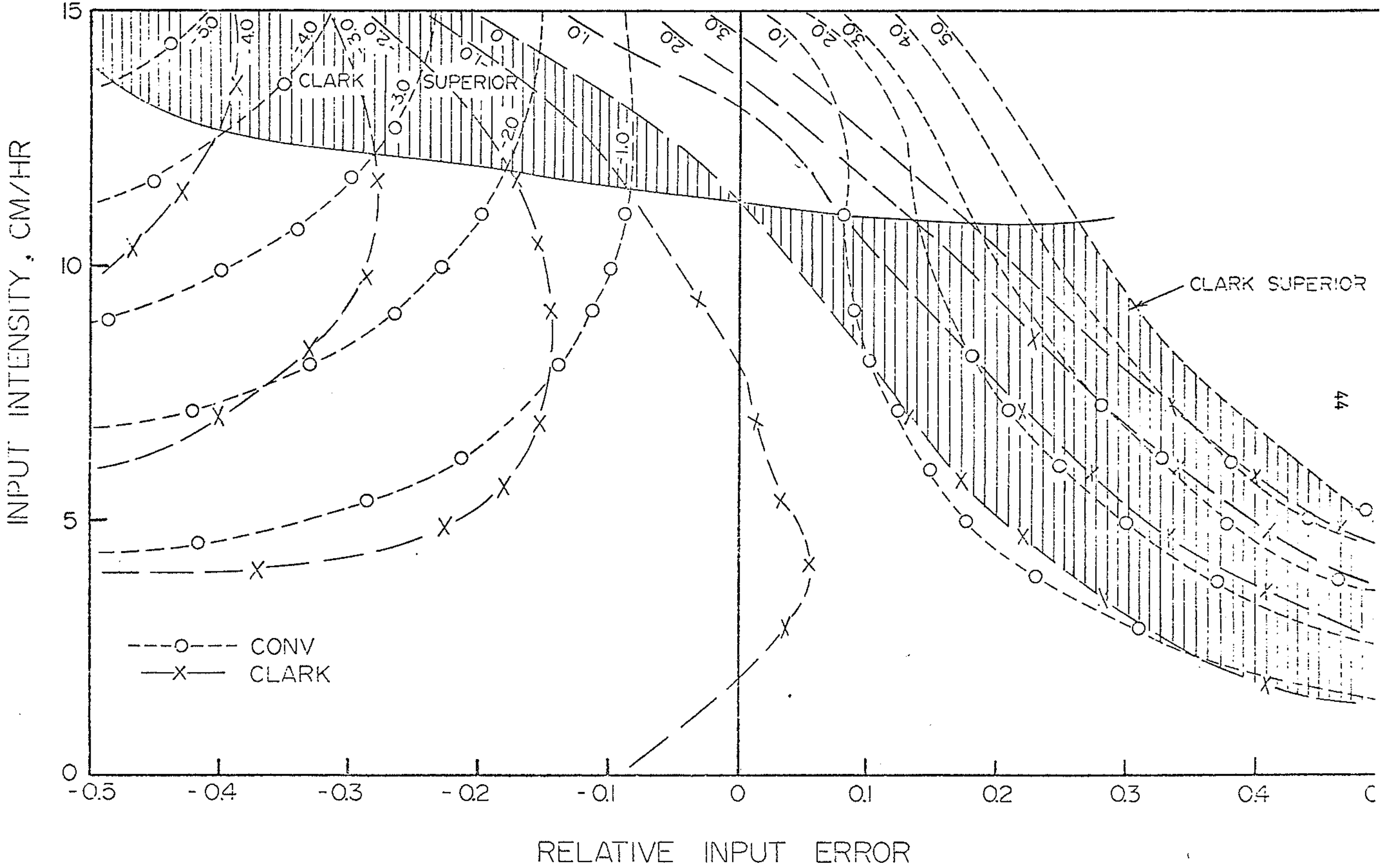

Fig. 4.6. Absolute error (cm/hour) surfaces for hydrograph peak predicted by CONV and CLARK models.

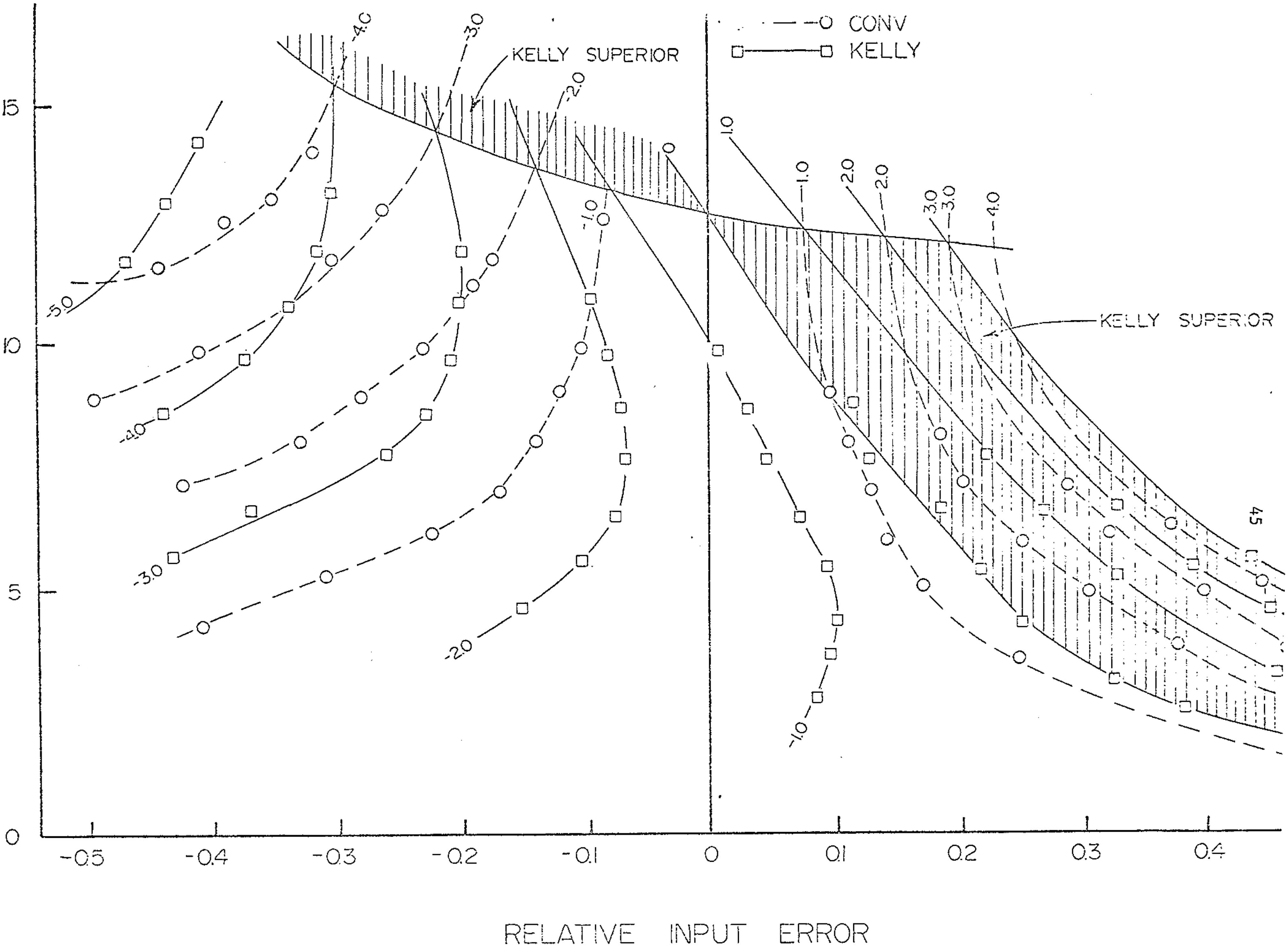

Fig. 4.7. Absolute error (cm/hour) surfaces for hydrograph peak predicted by CONV and KELLY models.

It is clear that the distribution function of the input errors will determine which model is superior for a given set of events. To further examine the errors produced by the models, a random sample of 100 rainfall-excess events ranging from 3 to 10 cm/hr was generated from a rectangular probability distribution. Then four samples of 100 events were generated from the uniform distribution over the interval 3 to 10 cm/hr and random relative error terms from the distribution A through D as shown in Fig. 4.12 were added to the input data. The mean squared error (MSE) in runoff peak and its time predicted by each model using this input data was evaluated in the following manner:

$$MSE(Q_p) = \frac{1}{100} \sum_{j=1}^{100} \left\{ Q_{p_o}(j) - Q_{p_e}(j) \right\}^2 \tag{4.14}$$

$$MSE(t_p) = \frac{1}{100} \sum_{j=\phi}^{100} \left\{ t_{p_o}(j) - t_{p_e}(j) \right\}^2 \tag{4.15}$$

where $Q_{p_o}(j)$ and $t_{p_o}(j)$ are the correct (CONV) peak and its time with error free input, and $Q_{p_e}(j)$ and $t_{p_e}(j)$ are peak and its time predicted by CONV or the other model when the input had an error term added. Tables 4.1 and 4.2 show the mean squared error values. The value of mean squared error function for Q_p is smaller for NASH than CONV and larger for other models. The value of MSE for t_p is smaller for CONV than any other model. The difference between the MSE values is the smallest for the error distribution B which is the triangle with mean zero (Fig. 4.12). From Figs. 4.4-4.11 it can be seen that the joint distribution of input intensity and relative error will control the goodness-of-fit of a given model. If the prediction set of events were small, an incorrect ranking of models is possible even though the input errors were small.

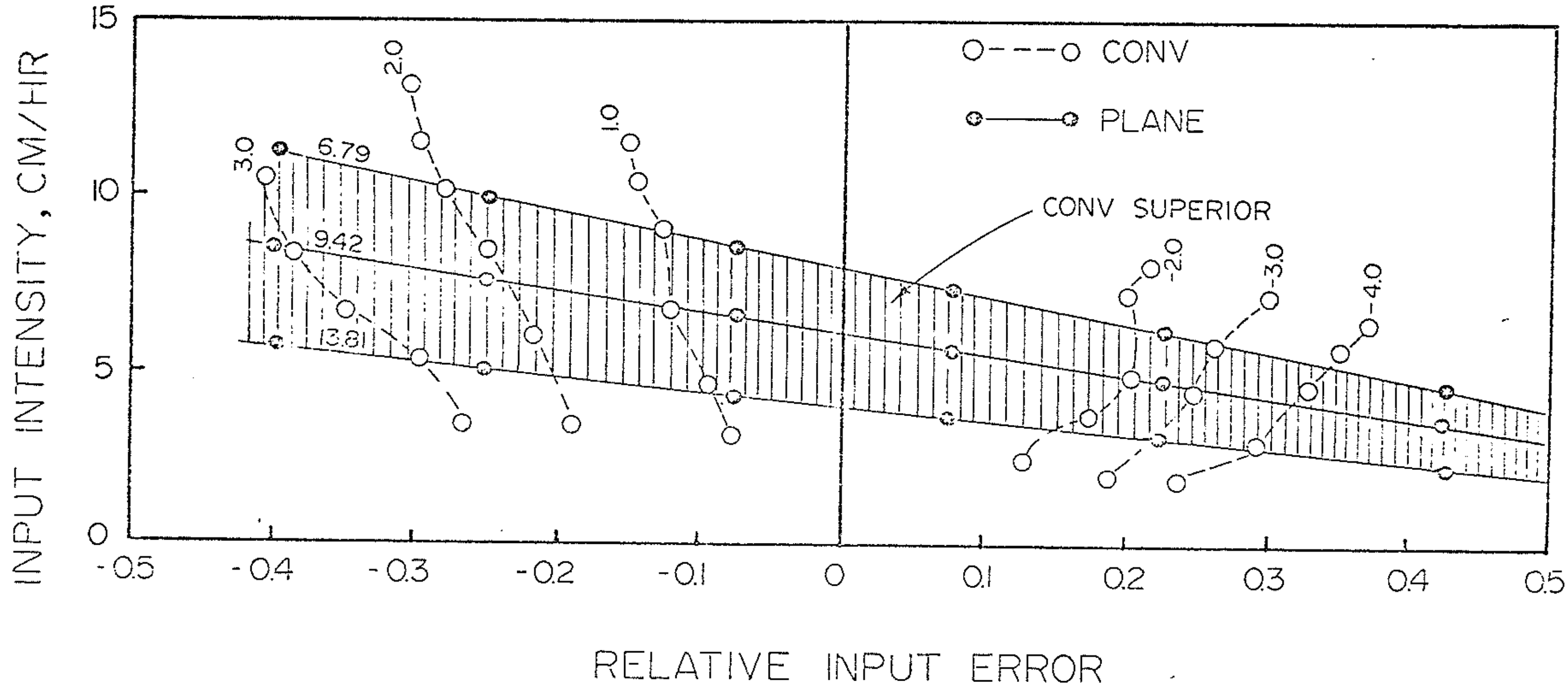

Fig. 4.8. Absolute error (minutes) surfaces for hydrograph peak time predicted by CONV and PLANE models.

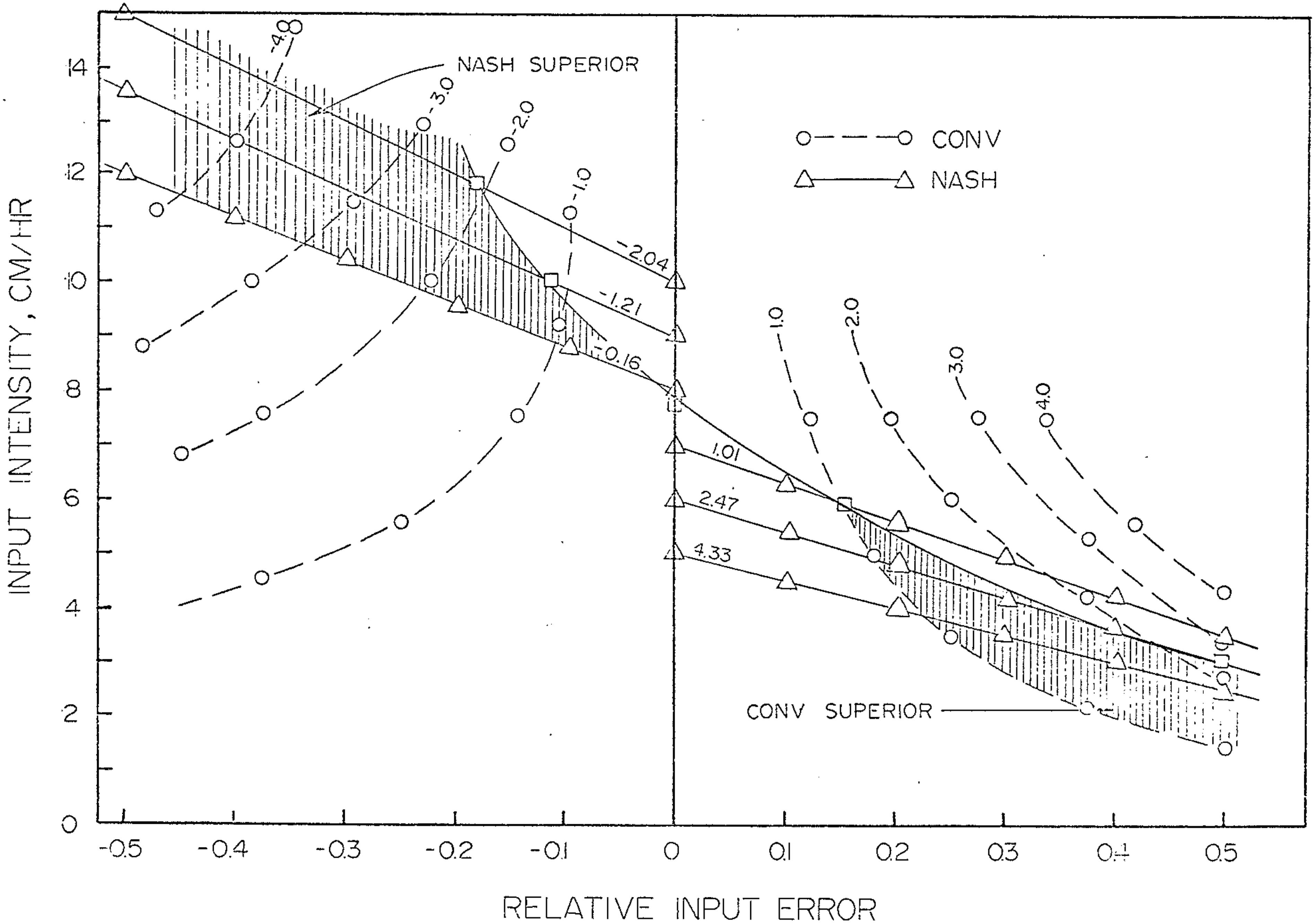

Fig. 4.9. Absolute error (minutes) surfaces for hydrograph peak time predicted by CONV and NASH models.

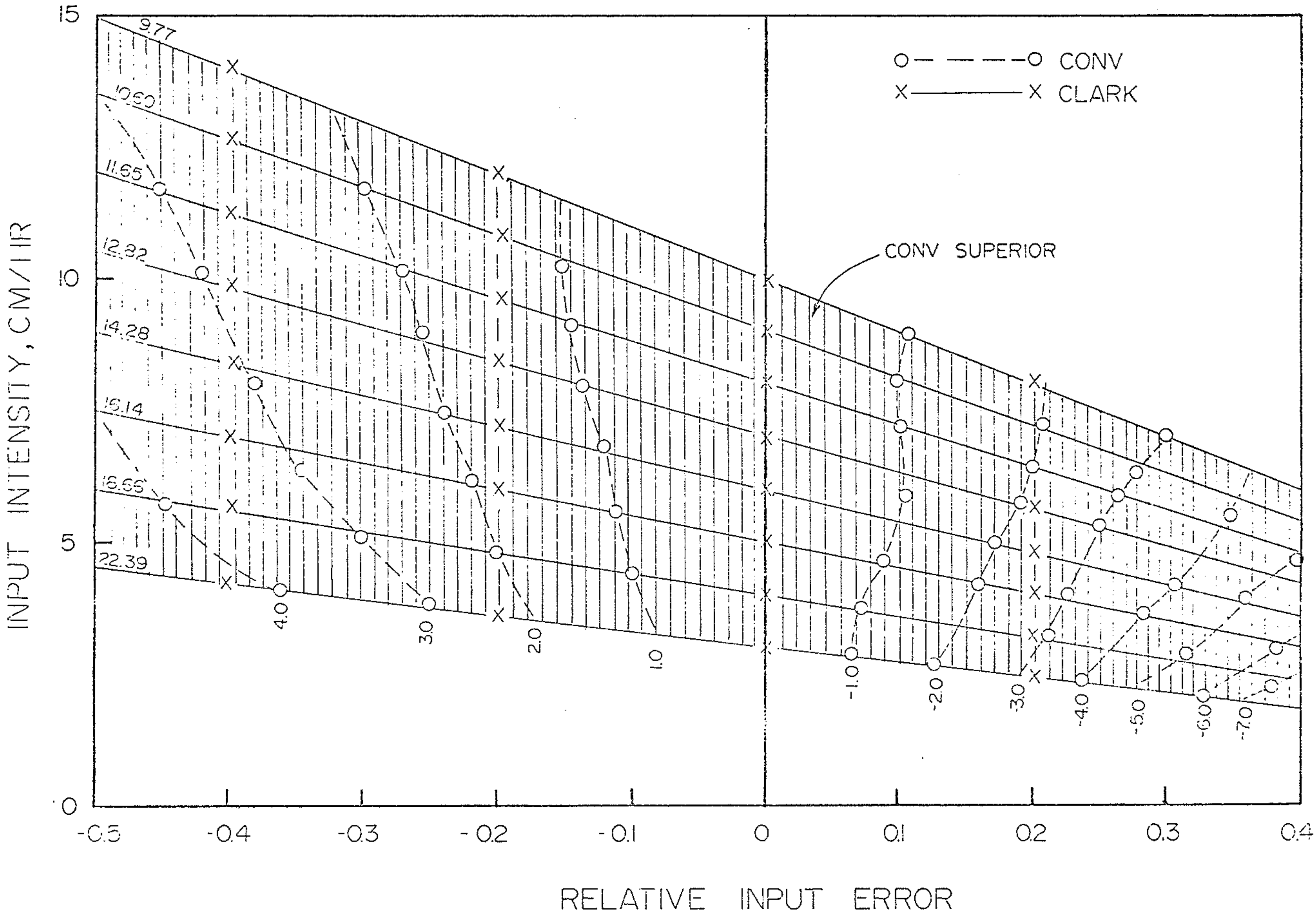

Fig. 4.10. Absolute error (minutes) surfaces for hydrograph peak time predicted by CONV and CLARK models.

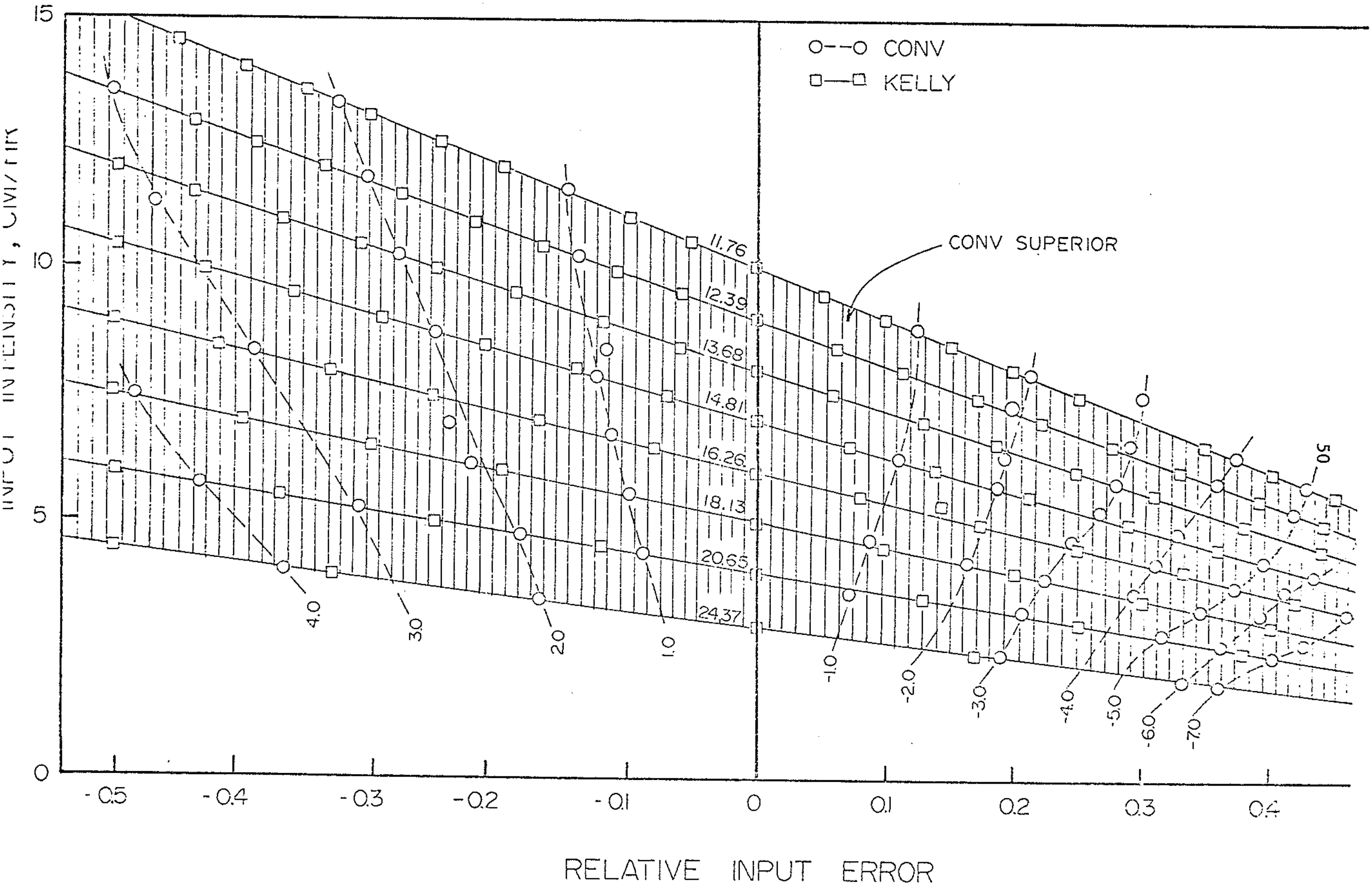

Fig. 4.11. Absolute error (minutes) surfaces for hydrograph peak time predicted by CONV and KELLY models.

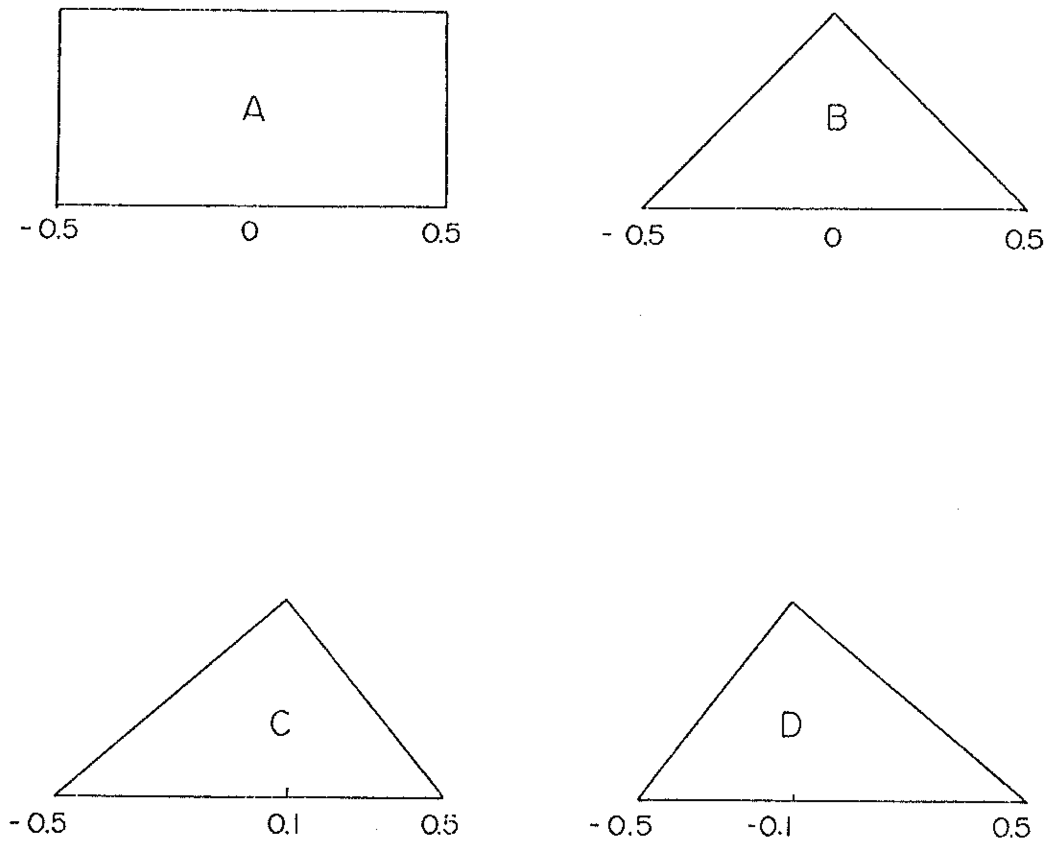

Fig. 4.12. Relative input error density functions.

Table 4.1. Mean squared error for predicted peak utilizing five surface runoff models.

Error Distribution	MSE (Qp) in $(cm/hr)^2$				
	CONV	PLANE	NASH	KELLY	CLARK
A	3.51	4.25	2.39	5.73	4.34
B	1.80	2.16	1.39	4.10	2.32
C	1.82	2.19	1.37	3.59	2.59
D	1.92	2.31	1.49	4.74	3.37

Table 4.2. Mean squared error for predicted peak time utilizing five surface runoff models.

Error Distribution	MSE (tp) in $(Min)^2$				
	CONV	PLANE	NASH	KELLY	CLARK
A	8.57	116.10	23.82	301.33	237.91
B	4.05	116.10	23.82	301.33	237.91
C	4.44	116.10	23.82	301.33	237.91
D	4.05	116.10	23.82	301.33	237.31

The above analysis shows that according to a reasonable objective function even a perfectly identified nonlinear model cannot be uniformly better than an optimally identified linear model. Under certain circumstances a linear model may, in fact, be preferable to a nonlinear one although the system is truly nonlinear. These circumstances arise when the input errors overpower the nonlinearity of the runoff process. This raises a fundamental question regarding the choice of a model. This choice will obviously depend upon the degree of nonlinearity of the real system, the distribution of input errors, the method of system identification and the objective function.

Chapter 5

COMPARISON OF TWO MATHEMATICAL MODELS OF RUNOFF

5.1 GENERAL REMARKS

A comparative assessment of two mathematical models of surface runoff is made by applying them to 21 terraced and unterraced natural agricultural watersheds located in two geographically different regions. The models include CONV and NASH. The former may be considered to represent the group of models based on watershed dynamics, while the latter to represent the group of what might be termed as conceptual models. Objective criteria are formulated to compare the performance of two models.

4.2 APPLICATION OF MODELS TO NATURAL WATERSHEDS

Twenty-one natural agricultural watersheds were selected from two geographically different regions; 16 watersheds were from Riesel (Waco), Texas and 5 from Hastings, Nebraska. These watersheds were divided into two groups; a terraced group and an unterraced group. If the amount of terracing at any time was greater than or equal to 10% of the total watershed area, the watershed was classified as terraced, otherwise unterraced. Thus terraced group consisted of 12 watersheds and unterraced group 11 watersheds. Note that depending upon the amount of terracing some watersheds were terraced for some time and unterraced for the other. These watersheds are small ranging in areal extent from about 1.2 hectares to 1720 hectares.

5.2.1 DETERMINATION OF MEAN AREAL RAINFALL

Rainfall-runoff data are available for these watersheds in the USDA publications on hydrologic data (for example, USDA, 1965). These publications are released almost every year, and consist of about one rainfall-runoff event per year for each watershed.

Although a watershed may have more than one raingage, data are normally available in the USDA publications for a centrally located raingage only, indicating that this has been taken to represent the mean areal rainfall. For consistency, this practice was followed on each watershed.

5.2.2 DETERMINATION OF RAINFALL-EXCESS

Rainfall-excess forms the lateral inflow $q(t)$, and must be obtained by subtracting infiltration from precipitation. Philip's equation (Philip, 1957) was utilized to estimate infiltration loss. In the equation the parameter A was considered roughly identical to the saturated hydraulic conductivity, thus it was amenable to determination from physical characteristics of the soil. In the absence of information on saturated hydraulic conductivity it was taken roughly equal to somewhere between 50% and 80% of the lowest ϕ-index (for infiltration) estimated for several storms for a watershed under consideration.

The parameter S was allowed to vary with each rainfall episode, thus accounting for antecedent soil moisture conditions. It was estimated for each storm by Newton's algorithm (Conte, 1965) subject to preserving the mass continuity. The parameter S was found to be sensitive to antecedent soil moisture conditions and relatively less sensitive to the parameter A .

5.2.3 PARAMETER ESTIMATION

When the slope S_o is incorporated into the parameter α , the converging section has 2 geometric parameters L_o and r . If the area of the watershed is to be maintained, θ is fixed when L_o and r are known.

In an earlier study, Singh (1976b) performed on optimization study in which it was shown that there was a strong interaction between the geometric parameter r and the resistance parameters. He found that a r value of 0.01 led to a minimum variance of α for several storms on the same watershed. He also found that the maximum straight line distance from the most remote point of the watershed to its outlet was a good estimator of the length of flow $L_o(1-r)$. Thus a single measurement from the topographic map of a watershed is sufficient to transform the natural geometry into a simpler converging section geometry.

5.2.4 CHOICE OF OBJECTIVE FUNCTION

The concept of determining optimal model parameters requires that the objective function be compatible with the intended use. However, there is difficulty in defining an error criterion that, upon minimization, will produce optimum parameter values without an undesirable bias. The following objective function was used in this study:

$$F = \sum_{j=1}^{M} \left\{ Q_{p_o}(j) - Q_{p_e}(j) \right\}^2 \Rightarrow \min \tag{5.1}$$

where F is objective function, $Q_{p_o}(j)$ observed hydrograph peak for the jth event, $Q_{p_e}(j)$ estimated hydrograph peak for the jth event, and M number of runoff events in the optimization set. This objective function is particularly suitable for flood studies. Greater weight is obviously placed on higher peaks. The choice of this objective function is based on a study by Singh (1976a, 1976b).

5.2.5 PARAMETER OPTIMIZATION

In a laboratory study of the converging overland flow (Singh, 1975a) it was shown that it would be reasonable to keep the kinematic wave parameter n fixed at 1.5, and that this value of n would lead to less variance in the parameter α . With geometric parameters known from watershed characteristics, the converging overland flow model becomes a one-parameter model. Nash's model remains a 2-parameter model.

Two sets of events were randomly selected: an optimization set and a prediction set for each of the terraced and unterraced watersheds. However, published data consist of large events and are not randomly selected from all events. The two sets of events were mutually exclusive. An optimization set consisted of only those events that were exclusively utilized in the optimization of model parameters, a prediction set consisted of those events for which hydrograph predictions were made by the models utilizing the optimized parameters. The number of events in the two sets ranged from 3 to 5 on an average.

Utilizing the objective function of Eq.(5.1) the parameter α of CONV, and N and K of NASH were optimized for each watershed by the modified Rosenbrock optimization algorithm (Himmelblau, 1972). To check the suitability of the objective function, hydrographs were reproduced by the models for some events in the optimization sets, using the optimized parameters. The reproductions were remarkably good both with respect to hydrograph peak and timing. This indicated that the objective function based on only hydrograph peak was sufficiently adequate for parameter optimization.

5.2.6 HYDROGRAPH PREDICTION

Utilizing the optimized model parameters, hydrograph predictions were made by the models for the events in the prediction set for each watershed in the terraced and unterraced groups. Then the models were compared in regard to their predictive performance. For clarity, comparison of models is described according to the criterion of comparison.

5.3 COMPARISON OF MODELS

Two aspects must be incorporated in a comparison criterion; one should pertain to accuracy and the other to computational efficiency. The former implies that the model should encompass in some way those physical details that affect the dynamics of surface runoff, and hence be able to reproduce it as closely as possible. The latter must include: (a) efficiency of computation, (b) ease of programming, and (c) storage (computer) requirements. It is quite likely that one model may be more accurate than the other but may be more costly to operate or require more data. In such circumstances, some kind of trade-off may be suggested. Accuracy probably forms a necessary requirement for prediction without prior data. The requirement of sufficiency may be met by satisfying the computational efficiency requirement. While comparing the models these two aspects will be kept in mind.

5.3.1 PREDICTION OF THE ENTIRE HYDROGRAPH

The shape, ordinates and timing characteristics are important in the prediction of the entire hydrograph. The entire hydrograph was better predicted by CONV than NASH. A sample of the entire hydrograph prediction by the two models is shown in Figs. 5.1 and 5.2. It is apparent from these figures that time and shape characteristics are better predicted by CONV than NASH. We note here that although hydrograph timing was not explicitly involved in the procedure for optimizing model parameters, the timing was well predicted by the models. On the basis of the entire hydrograph prediction, CONV is a better predictor than NASH.

5.3.2 PREDICTION OF HYDROGRAPH PEAK

Both models performed reasonably well in predicting hydrograph peak. Sample predictions of hydrograph peak for two watersheds from the

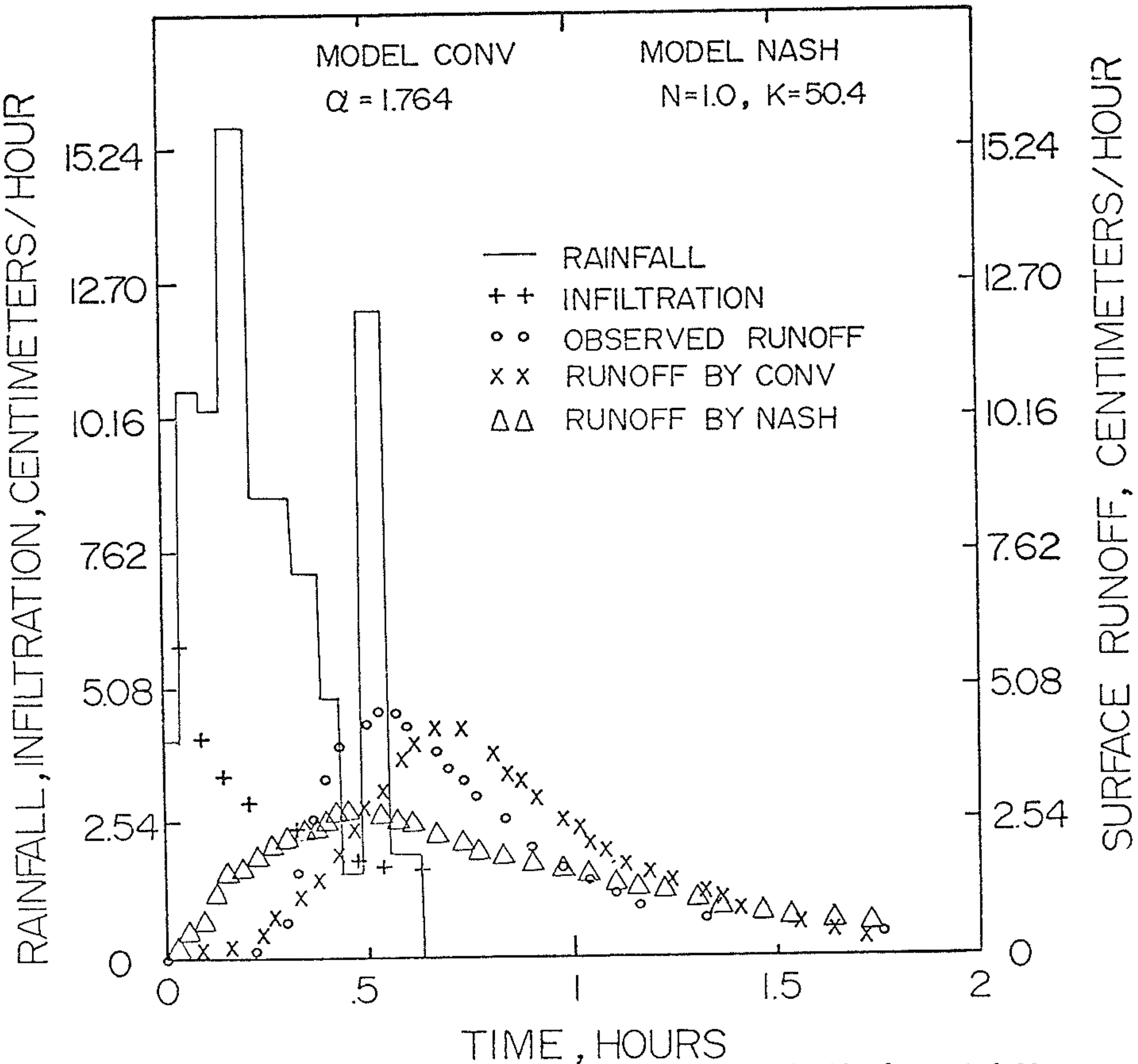

Fig. 5.1. Prediction of surface runoff hydrograph by CONV and NASH for rainfall event of 6-4-1957 on terraced watershed Y-2, Riesel (Waco), Texas.

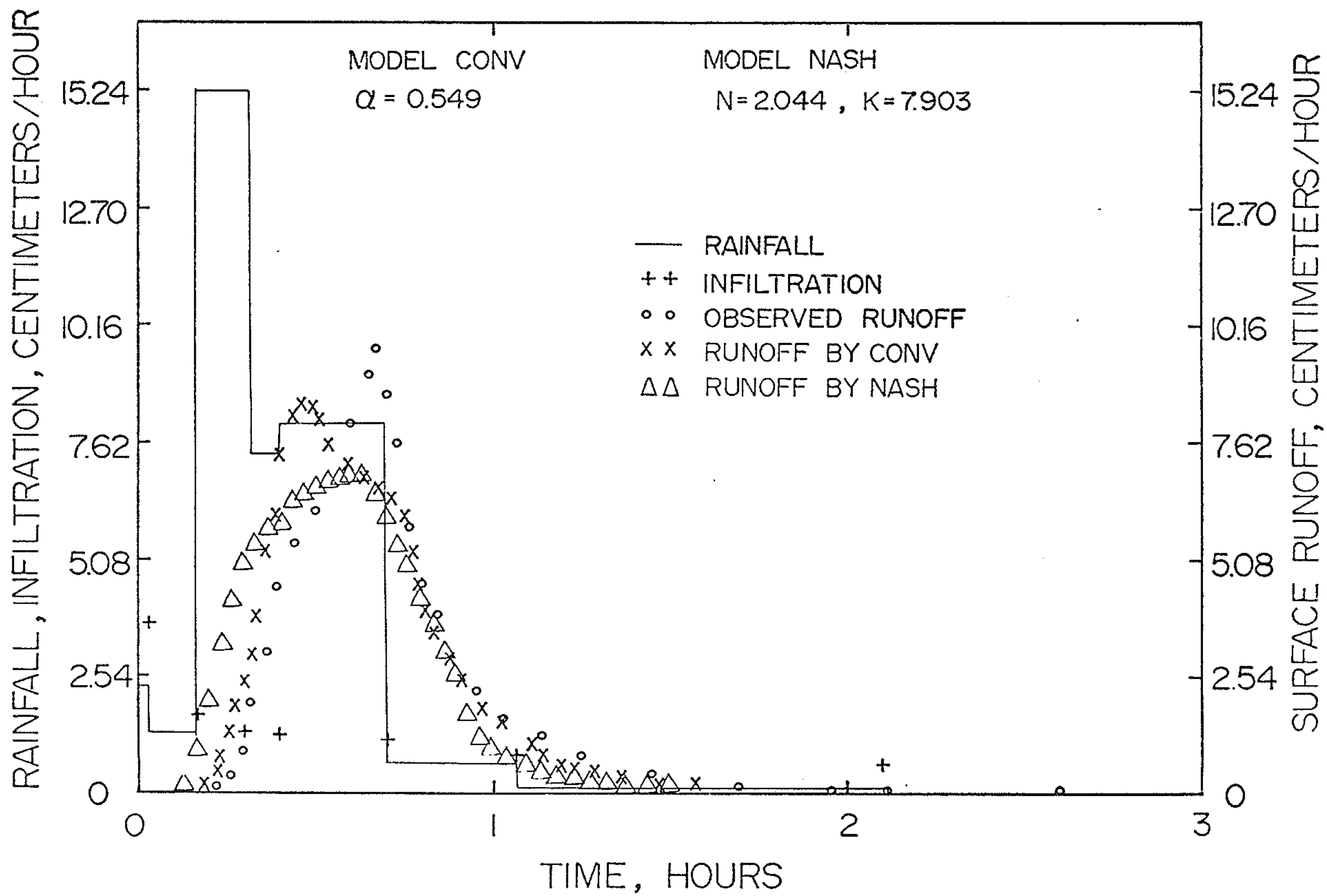

Fig. 5.2. Prediction of surface runoff hydrograph by CONV and NASH for rainfall event of 6-9-1962 on unterraced watershed SW-17, Riesel (Waco), Texas.

terraced group and for two from the unterraced group are given in Table 5.1 and 5.2. Generally, CONV performed better than NASH, although NASH did quite well in predicting hydrograph peak. In most cases the prediction error, defined as (observed quantity-predicted quantity)/observed quantity, was below 30% for CONV and 40% for NASH. It was observed that CONV had a tendency to overpredict the peak; NASH had a tendency to underpredict it. This is also seen from Tables 5.1 and 5.2. To further examine the predictive campability of the two models, the cumulative density function of peak discharge was plotted separately for the terraced and unterraced groups of watersheds, as shown in Figs. 5.3 and 5.4 These groups had 39 and 48 events respectively. From these figures it is evident that CONV will predict a higher peak, and NASH a lower peak for a given probability of observed peak. This behavior is more noticeable on terraced watersheds than on unterraced ones. Also this observation is more true for higher peaks; for lower peaks the distinction between CONV and NASH fades out. An interesting observation is that the cumulative density function of peak discharge for both models has more or less the same general pattern. These observations are further substantiated by Figs. 5.5 and 5.6 which plot the cumulative density function of error by the models in prediction of peak discharge on the terraced and unterraced watersheds.

5.3.3 PREDICTION OF TIME TO HYDROGRAPH PEAK

Generally, NASH performed better than CONV in predicting time to peak. This is seen from sample predictions of time to peak, given in Tables 5.3 and 5.4, for two watersheds from the terraced group and for two from the unterraced group. The prediction error in time to peak can be very high for both models. This does not necessarily reflect on model structure; rather it has more to do with improper synchronization between rainfall-runoff observations and other data errors.

Table 5.1. Prediction of hydrograph peak by CONV and NASH for two terraced watersheds, Riesel (Waco), Texas.

Watershed identification	Date of event	Observed hydrograph peak	CONV		NASH	
			Predicted hydrograph peak	Error* in predicted hydrograph peak	Predicted hydrograph peak	Error in predicted hydrograph peak
		Cm/hr	Cm/hr		Cm/hr	
Watershed Y-4 Area=32.3 hectares	4-24-1957	4.09	5.25	-0.284	3.72	0.090
L(1-r)=731.52 meters r=0.01	5-13-1957	2.90	3.13	-0.019	2.88	0.006
α=1.42 N=4.63	6-4-1957	4.04	4.19	-0.036	3.13	0.225
K=9.77	3-29-1965	6.35	6.43	-0.013	5.37	0.153
Watershed Y-7 Area=16.2 hectares	4-24-1957	6.00	7.09	-0.181	4.36	0.274
L(1-r)=548.64 meters r=0.01	5-13-1957	5.14	5.56	-0.078	3.32	0.356
α=1.802 N=0.5	6-23-1959	4.47	5.68	-0.177	2.89	0.355
K=51.5	3-29-1965	5.17	7.12	-0.336	5.27	0.088

$$* \text{ Error} = \frac{\text{Observed quantity-predicted quantity}}{\text{observed quantity}}$$

Table 5.2. Prediction of hydrograph peak by CONV and NASH for two unterraced watersheds near Hastings, Nebraska.

Watershed identification	Date of event	Observed hydrograph peak	CONV		NASH	
			Predicted hydrograph peak	Error in predicted hydrograph peak	Predicted hydrograph peak	Error in predicted hydrograph peak
		Cm/hr	Cm/hr		Cm/hr	
Watershed W-3	9-5-1946	1.40	1.41	-0.085	1.77	-0.366
Area=194.67 hectares	6-15-1947	2.49	2.41	0.034	2.34	0.059
L(1-r)=2298.2 meters	6-5-1949	0.36	0.26	0.266	0.58	-0.627
r=0.01	6-25-1951	1.70	1.26	0.260	1.55	0.088
α=3.124	7-13-1952	3.38	3.99	-0.180	3.51	-0.038
N=4.63	6-16-1957	1.73	1.45	0.160	1.82	-0.052
K=9.77						
Watershed 4-H	8-11-1939	4.52	6.87	-0.525	4.55	-0.007
Area=1.47 hectares	6-20-1942	5.82	6.37	-0.095	5.60	0.037
L=(1-4)=152.4 meters	9-5-1946	3.89	4.42	-0.137	3.23	0.169
r=0.01	6-1-1951	6.16	8.34	-0.234	1.90	0.719
α=6.43	7-13-1953	9.19	10.04	-0.092	8.93	0.029
N=9.95	6-12-1958	0.92	1.22	-0.330	1.15	-0.255
K=1.00	6-12-1965	9.10	9.20	0.050	8.42	0.132
	6-12-1965	6.14	6.48	-0.056	5.54	0.100
	6-29-1965	8.10	10.48	-0.352	7.30	0.099

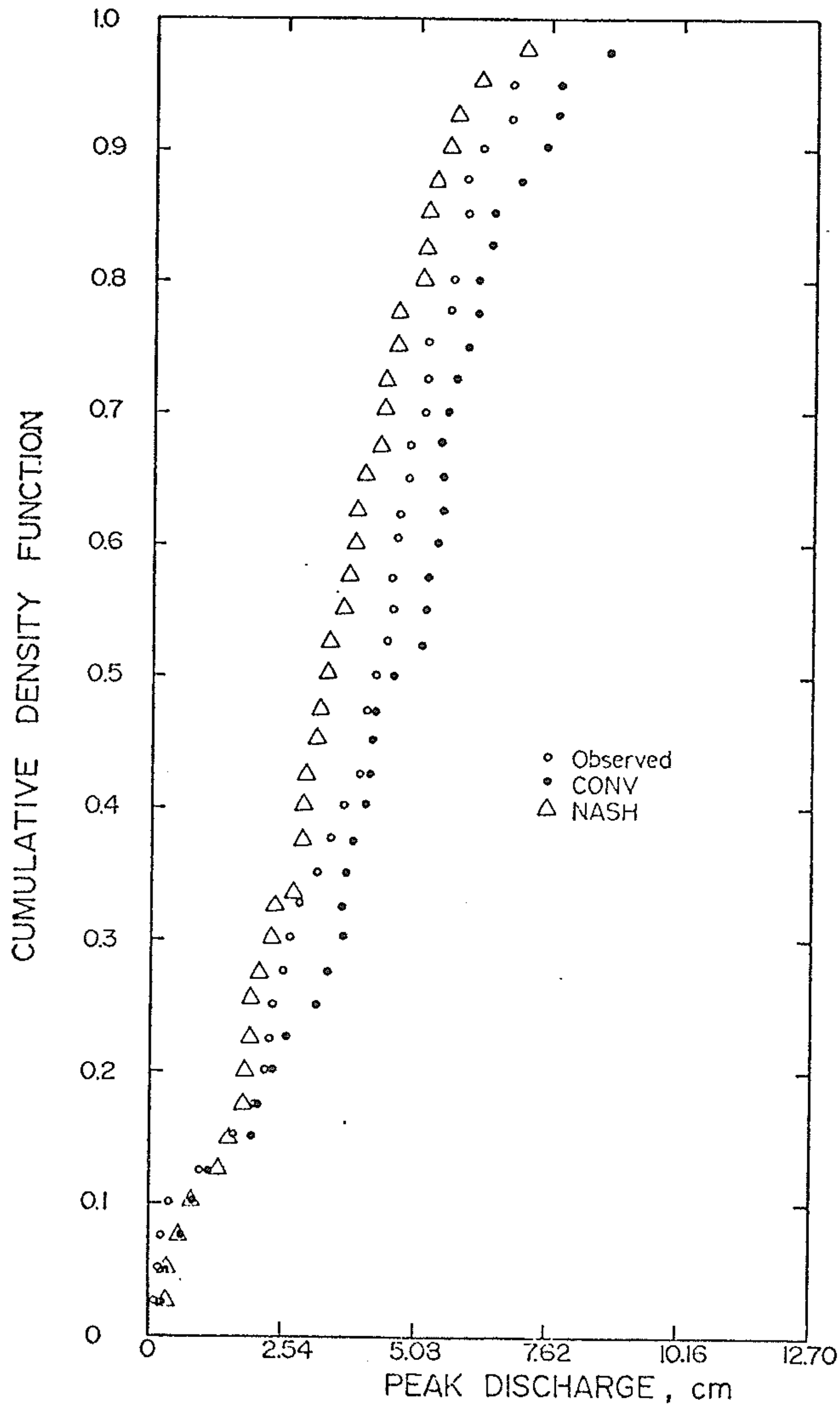

Fig. 5.3. Cumulative density function of peak discharge for terraced watersheds.

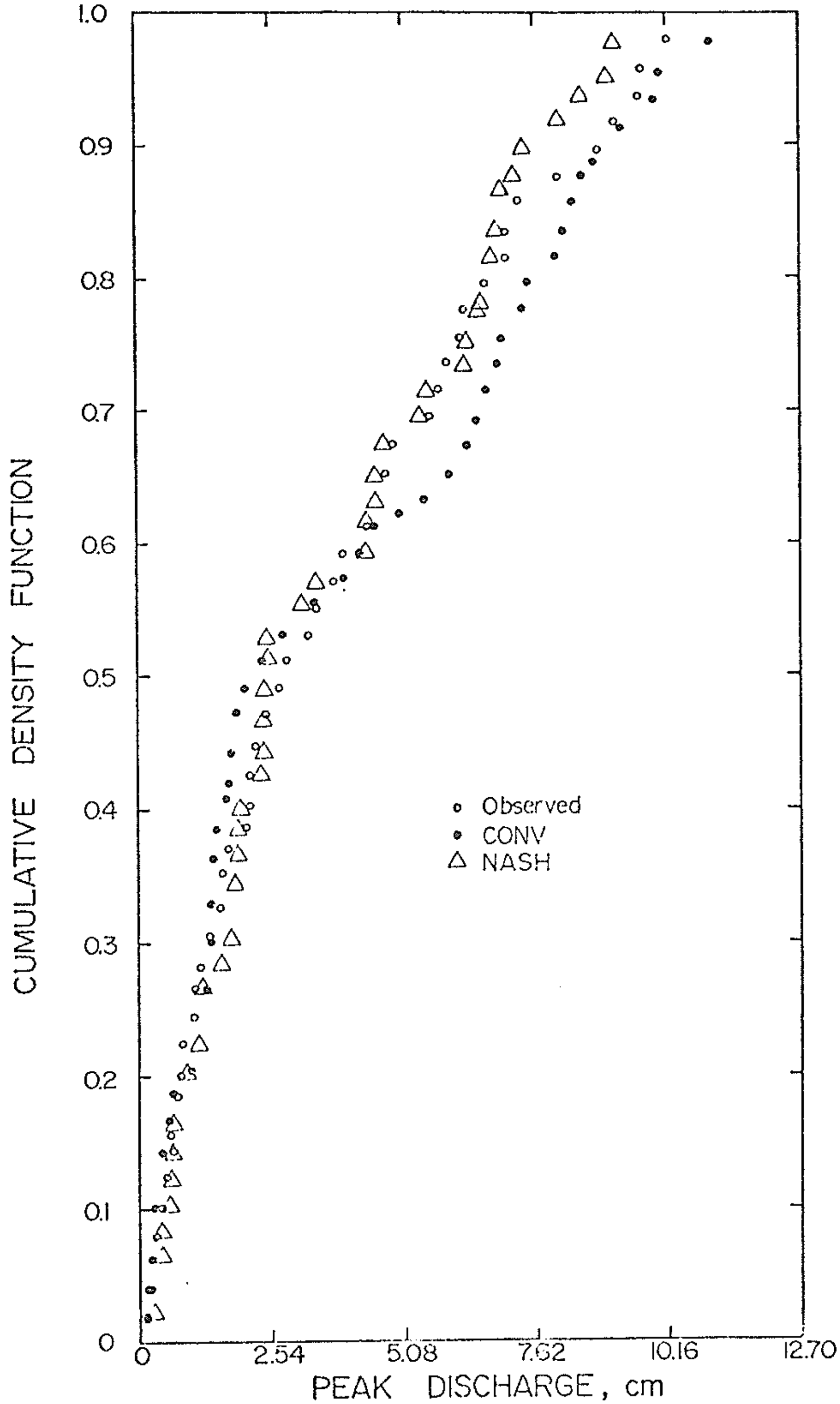

Fig. 5.4. Cumulative density function of peak discharge for unterraced watersheds.

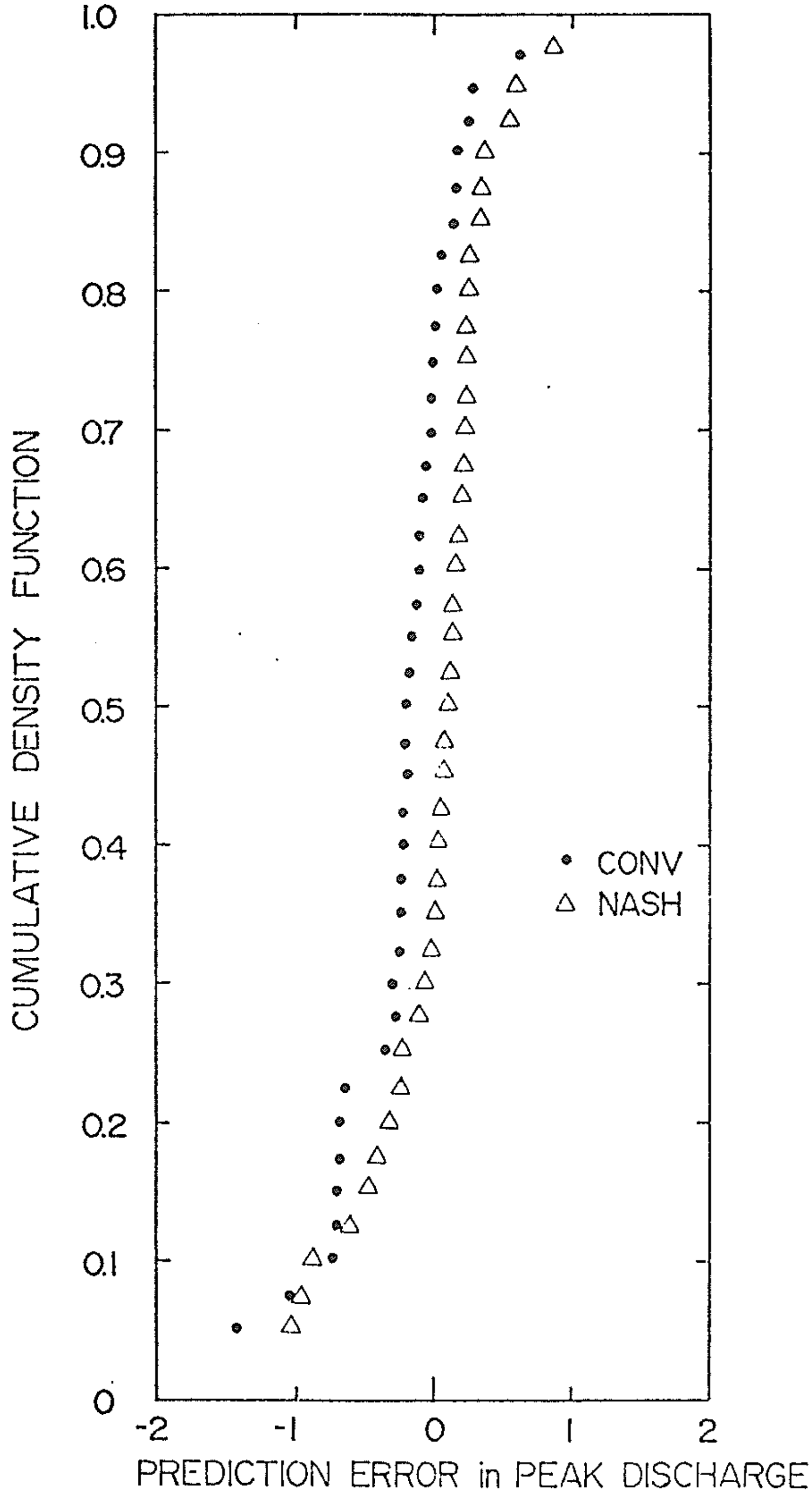

Fig. 5.5. Cumulative of density function of error by the models in prediction of peak discharge for terraced watersheds.

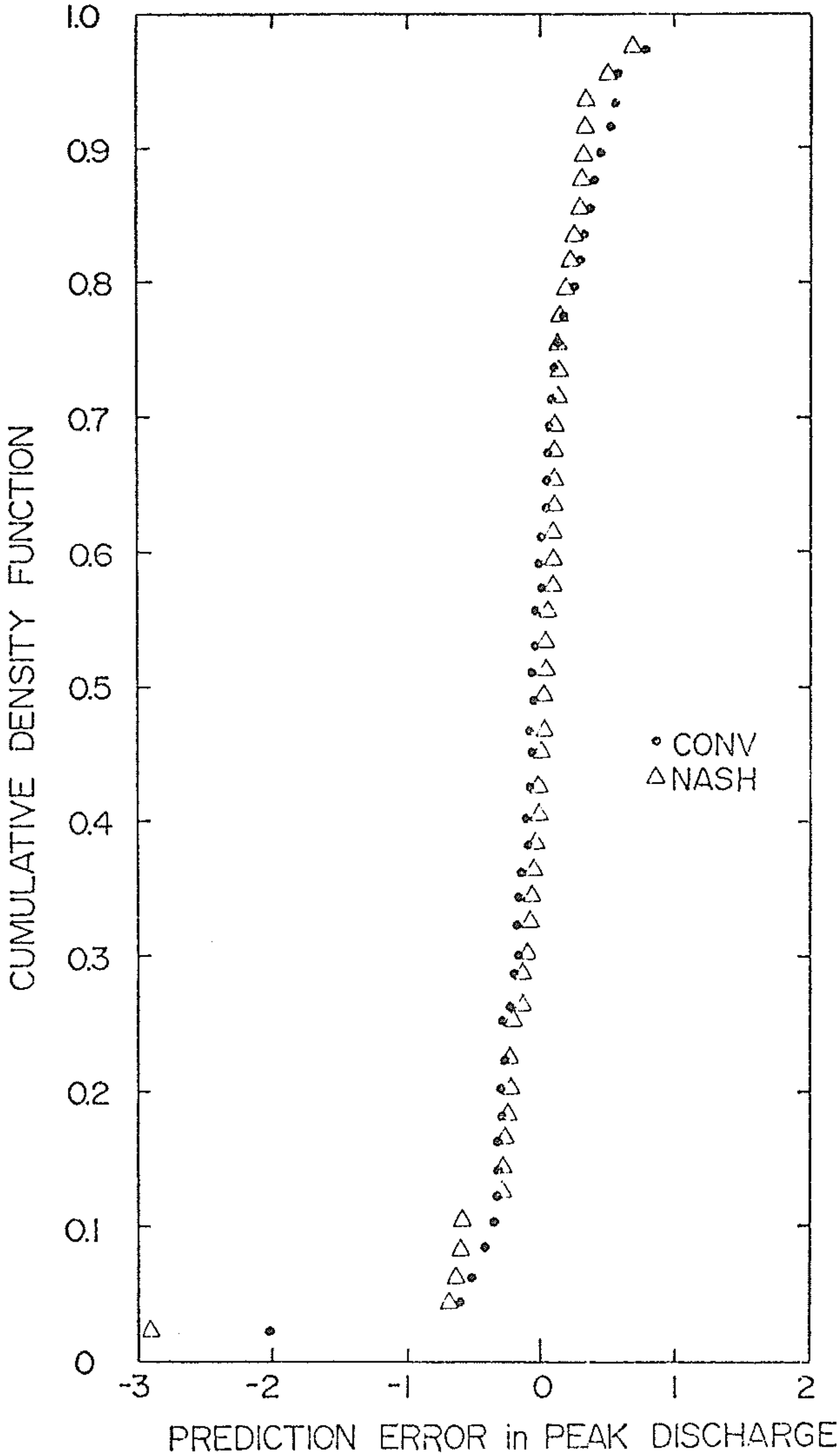

Fig. 5.6. Cumulative density function of error by the models in prediction of peak discharge for unterraced watersheds.

Table 5.3. Prediction of hydrograph peak time by CONV and NASH for two unterraced Watersheds near Hastings, Nebraska.

Watershed identification	Date of event	Observed hydrograph peak time	CONV		NASH	
			Predicted hydrograph peak time	Error in predicted hydrograph peak time	Predicted hydrograph peak time	Error in predicted hydrograph peak time
		min	min		min	
Watershed W-3	9-5-1946	146.0	163.6	-0.121	158.0	-0.082
	6-15-1947	64.0	66.7	-0.042	75.0	-0.172
	6-5-1949	74.0	97.0	-0.322	59.0	0.203
	6-25-1951	129.0	168.3	-0.304	170.0	-0.318
	7-13-1952	124.0	133.7	-0.079	140.0	-0.129
	6-16-1957	154.0	64.2	-0.067	162.0	-0.052
Watershed 4-H	8-11-1939	5.0	5.4	-0.071	12.0	-1.40
	6-20-1942	8.0	6.1	0.235	15.0	-0.875
	9-5-1946	12.0	6.4	0.466	12.0	0.000
	6-1-1951	123.0	124.5	0.013	11.0	0.910
	7-13-1953	21.0	5.3	0.750	16.0	0.236
	6-12-1958	14.0	10.5	0.252	14.0	0.000
	6-12-1965	19.0	10.2	0.474	19.0	0.000
	6-12-1965	7.0	11.0	-0.571	17.0	-1.429
	6-20-1965	3.0	5.0	-0.667	12.0	-3.000

Table 5.4. Prediction of hydrograph peak time by CONV and NASH for two terraced watersheds, Riesel (Waco), Texas.

Watershed identification	Date of event	Observed by hydrograph peak time	CONV		NASH	
			Predicted hydrograph peak time	Error in predicted hydrograph peak time	Predicted hydrograph peak time	Error in predicted peak time
		min	min		min	
Watershed Y-4	4-24-1957	48.0	70.0	-0.458	49.3	-0.037
	5-13-1957	47.0	66.0	-0.404	50.0	-0.064
	6-4-1957	37.0	61.0	-0.650	44.7	-0.208
	3-29-1965	60.0	130.0	-0.970	86.9	-0.311
Watershed Y-7	4-24-1957	31.0	22.0	0.290	30.9	0.002
	5-13-1957	33.0	26.0	0.212	33.0	0.000
	6-23-1957	57.0	104.0	-0.825	112.0	-0.964
	3-29-1965	79.0	65.0	0.177	74.0	0.055

Here also, CONV has a tendency to overpredict time to peak; NASH has a tendency to underpredict it. This is evident from Figs. 5.7 - 5.10 which show cumulative density functions of time to peak and prediction error in time to peak by the models for both groups of watersheds. Again, the distinction between the predictive performances of the two models is less obvious on unterraced watersheds than terraced ones as seen from Figs. 5.9 and 5.10. This is in conformity with what was observed earlier for hydrograph peak. An important consequence of this observation is that agricultural practices alter hydrologic behavior of a watershed and it is plausible to quantify the change attributable to these practices.

At this point it might be useful to point out the sources of error other than the model inadequacy that lead to prediction errors. There may be several reasons for model prediction errors; but of them all two appear to be most important:

(1) The size of the optimization set for each watershed is very small and, therefore, we cannot hope to obtain representative values of the model parameters. This is even more true when we see that the rainfall-runoff events for each watershed under consideration represent a long stretch of time, often 15 years or more. During this period of time several changes in land management and cropping pattern must have taken place on these watersheds. These changes can, in no way, be fully represented by such small samples as we have considered.

(2) There is difficulty in determining rainfall-excess which, in fact, generated observed runoff. The determination of rainfall-excess seems to be the major problem in most rainfall-runoff models. Philip's equation, utilized in this study, is too simple to accurately predict the time distribution of infiltration and then there is the difficulty of estimating its parameters. It was used primarily for its simplicity.

5.3.4 THE MEAN SQUARE ERROR

The error function (Eq. (5.1)) was computed by both models for all

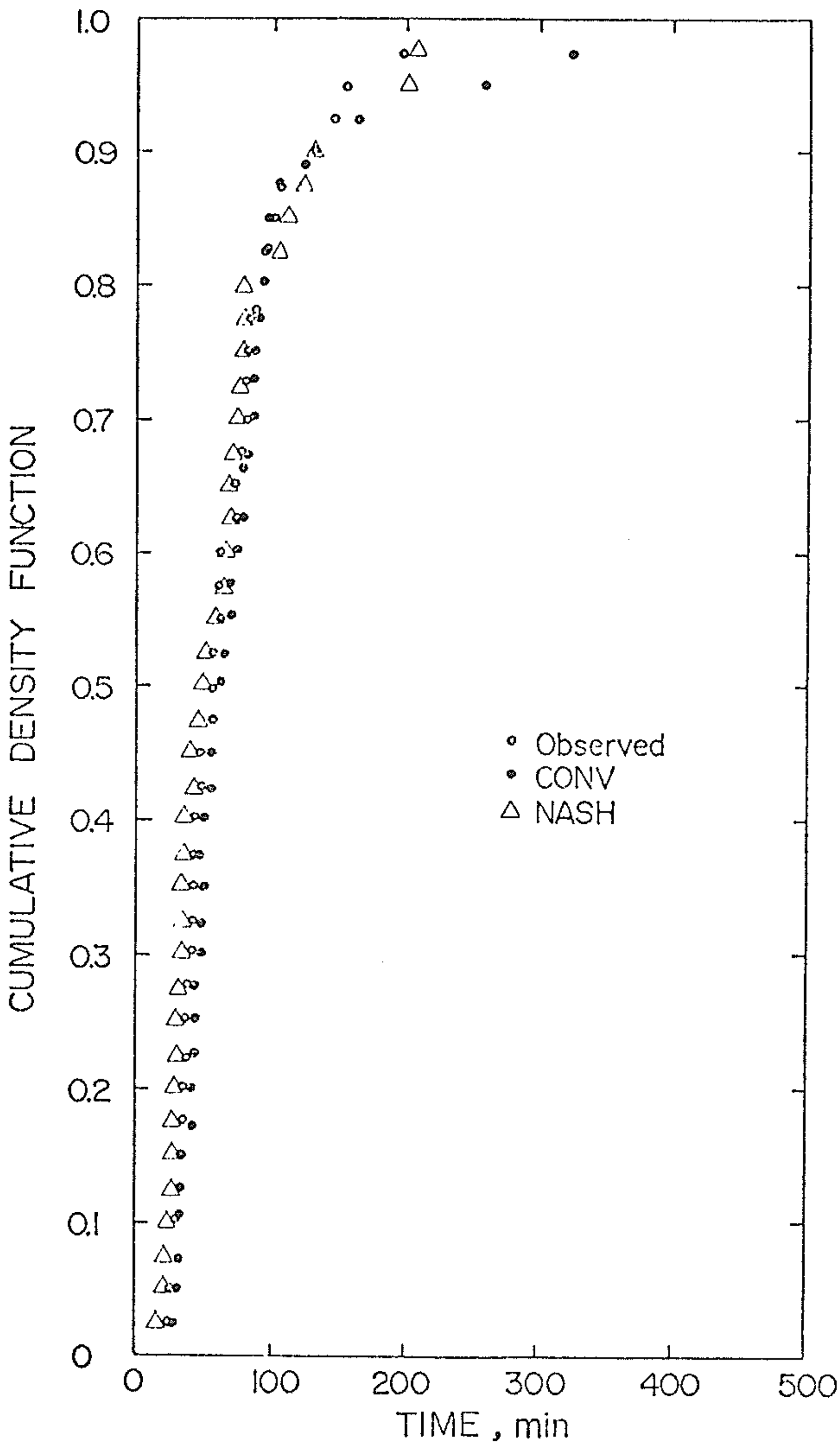

Fig. 5.7. Cumulative density function of time to peak for terraced watersheds.

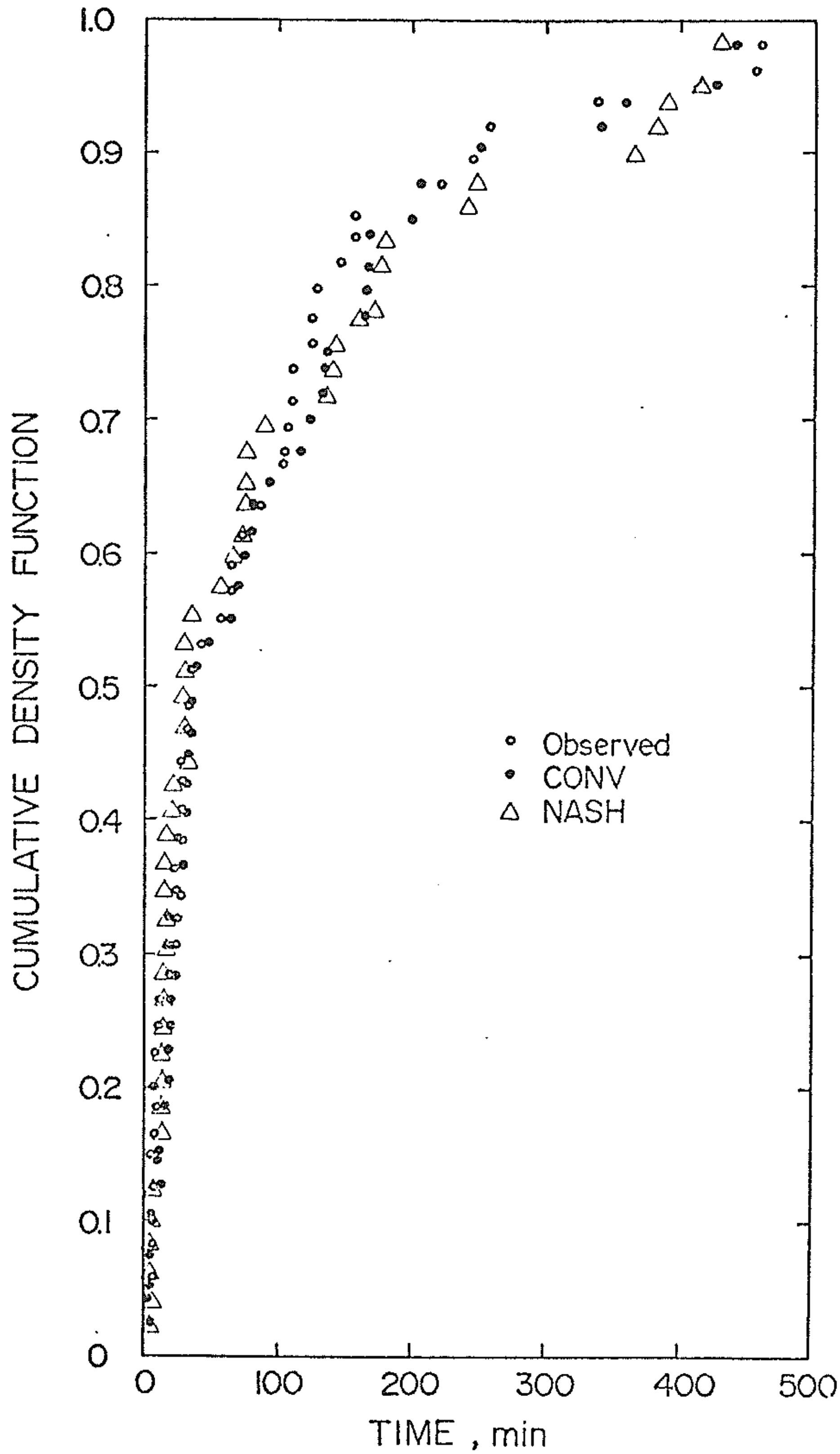

Fig. 5.8. Cumulative density function of time to peak for unterraced watersheds.

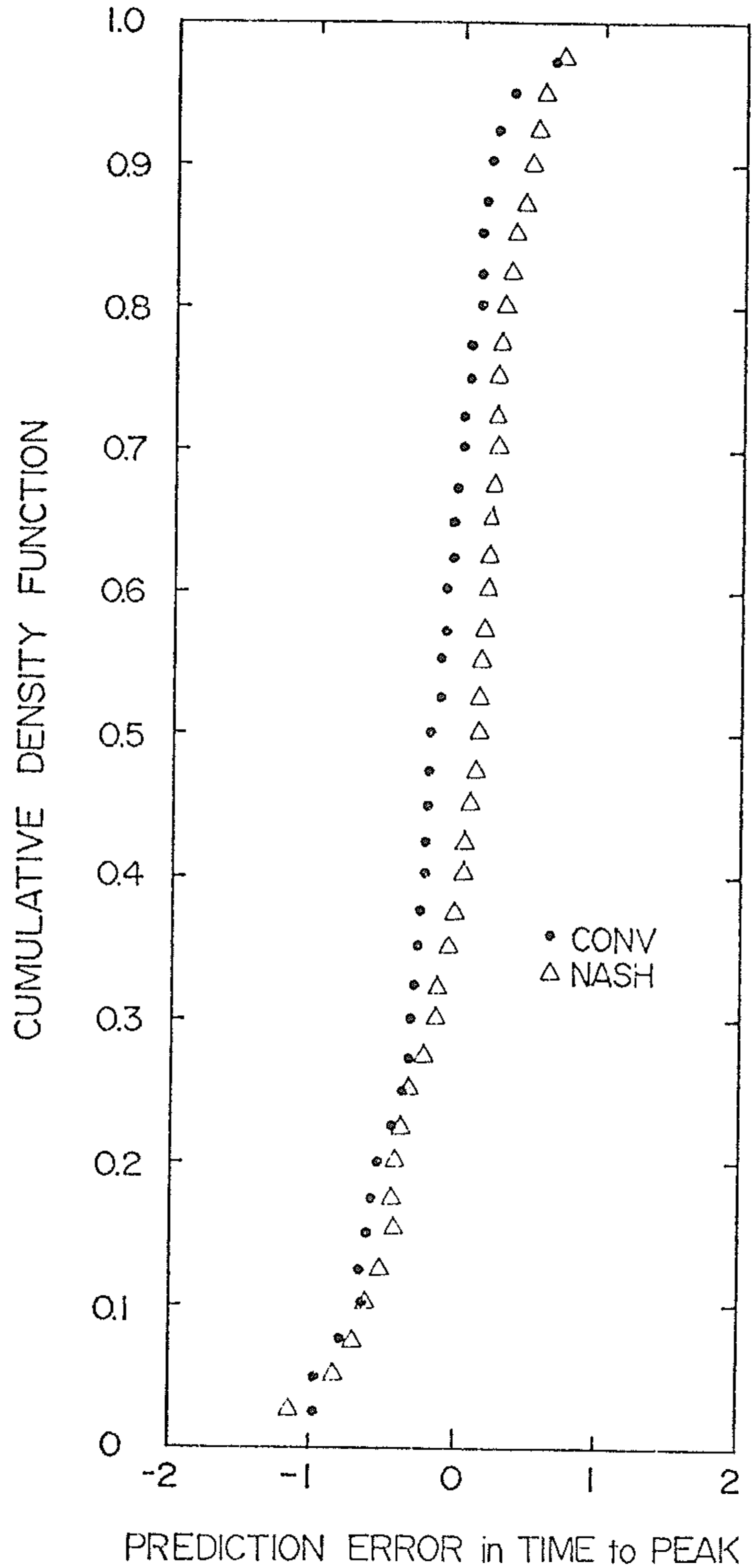

Fig. 5.9. Cumulative density function of error by the models in time to peak for terraced watersheds.

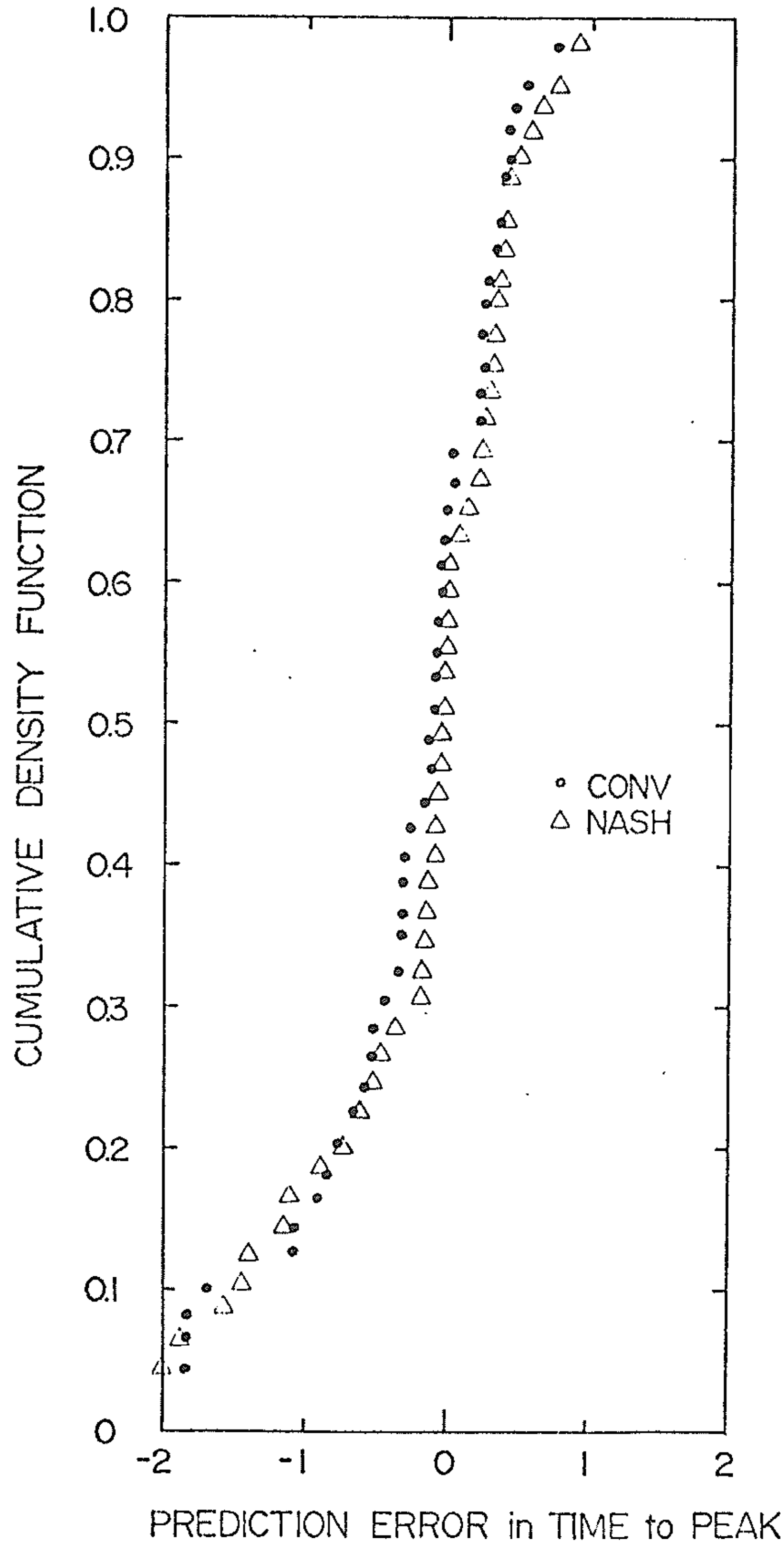

Fig. 5.10. Cumulative density function of error by the models in time to peak for unterraced watersheds.

available events on terraced and unterraced watersheds. The error function of CONV was 50.10 and of NASH 76.17 for 48 available events on unterraced watersheds, and the mean square error of CONV and NASH respectively was 1.045 and 1.587; for 39 available events on terraced watersheds it was 54.50 and 68.76 respectively for CONV and NASH, and their mean square error respectively was 1.393 and 1.761. This criterion indicates that CONV did better than NASH.

5.3.5 CORRELATION COEFFICIENT

The correlation coefficient between runoff predictions and observations was computed. It was 0.95 for CONV and 0.91 for NASH on unterraced watersheds; it was 0.78 for CONV and 0.77 for NASH on terraced watersheds. This says that only for unterraced watersheds CONV is a better predictor than NASH.

The correlation coefficient between observed time to peak and predicted time to peak was 0.85 for CONV and 0.76 for NASH on terraced watersheds; it was 0.86 for CONV and 0.89 for NASH for unterraced watersheds. This indicates that NASH is a better predictor of time to peak on unterraced watersheds but worse on terraced watersheds. This is in contrast to what was observed for hydrograph peak.

5.3.6 STANDARD ERROR OF ESTIMATE

To further examine the relative performance of the models, a regression analysis was performed between observations and predictions, and standard error of estimate was computed. For peak discharge it was 3.48 for CONV and 3.76 for NASH on unterraced watersheds; it was 3.63 for CONV and 3.22 for NASH on terraced watersheds. For time to peak it was 110.11 for CONV and 102.10 for NASH on unterraced watersheds; it was 40.44 for CONV and 49.52 for NASH on terraced watersheds. This further supports the contention based on the correlation coefficient.

5.3.7 SAMPLE STATISTICS

Statistical parameters of runoff observations (cm/hr) and corresponding predictions by the models were computed:

Terraced Group

Model	Observations $\bar{x}$	σ_x	C_v	Predictions $\bar{y}$	σ_y	C_v
CONV	3.89	1.96	1.27	4.50	2.18	1.24
NASH	3.89	1.96	1.27	3.38	1.70	1.27

Unterraced Group

Model	Observations $\bar{x}$	σ_x	C_v	Predictions $\bar{y}$	σ_y	C_v
CONV	3.73	3.00	2.03	3.34	3.53	2.16
NASH	3.73	3.00	2.03	3.56	2.74	1.96

These statistical parameters for time (minutes) to peak are:

Terraced Group

Model	Observations $\bar{x}$	σ_x	C_v	Predictions $\bar{y}$	σ_y	C_v
CONV	66.26	39.47	0.60	76.64	58.72	0.77
NASH	66.26	39.47	0.60	66.34	44.74	0.71

Unterraced Group

Model	Observation $\bar{x}$	σ_x	C_v	Predictions $\bar{y}$	σ_y	C_v
CONV	90.54	124.62	1.38	93.82	111.07	1.18
NASH	90.54	124.62	1.38	93.48	122.08	1.31

where $\bar{x}$ = mean of observed quantity (x) , $\bar{y}$ = mean of predicted quantity (y) , σ = unbiased standard deviation, and C_v = coefficient of variation. From these statistical parameters it is clear that on the whole both models performed quite well. CONV has a tendency to overpredict while NASH has a tendency to underpredict. Because of its nonlinear character CONV is more sensitive to input errors than NASH (Singh, (1975a). Because the rainfall-excess forms input to the models, only seldom can we accurately determine it.

Now the question arises: which model is better? As mentioned previously the answer to this question will depend on the criterion of comparison. The dynamical features of surface runoff are better represented by CONV than NASH. CONV can provide runoff hydrograph at any point in watershed while NASH cannot. CONV is more sensitive to input errors than NASH. If input errors are large, NASH is preferable to CONV. CONV is a one parameter model while NASH is a 2-parameter model. To optimize one parameter is much easier and faster than to optimize two parameters. A study by Singh (1976b) suggested that CONV parameter could be estimated from watershed physiography and the results from the study were encouraging. Although many attempts have been made to correlate NASH parameters with watershed characteristics, reliable relationships have yet to be developed. Computationally, NASH is more efficient than CONV for complex inputs. Based on these observations one model cannot be uniformly better than the other. Each model has its advantages and disadvantages. One model overpredicts; the other underpredicts; what is more serious will depend upon the nature of the problem at hand. The conclusion is that the choice of a model is seldom unique.

Chapter 6

CONCLUDING REMARKS

The following conclusions are drawn from this study:

(1) Accurate determination of rainfall-excess is crucial to runoff prediction. Of the four methods considered Horton equation is the best; equations of Philip and Kostyakov are comparable; and ϕ-index grossly misrepresents rainfall-excess. By using a more refined method of infiltration errors in hydrograph computation can be minimized.

(2) Based on the examination of five surface runoff models regarding their sensitivity to input errors it is shown that even a perfectly identified nonlinear model is not uniformly better than another optimally identified model. Furthermore, under certain circumstances an optimally identified linear model may perform as well as or even better than a perfectly identified nonlinear model.

(3) The choice of a model varies with the choice of criterion of comparison.

REFERENCES

Amorocho, J. and Orlob, G. T. (1961): Nonlinear analysis of hydrologic systems. Contribution No. 40, Water Resources Center, University of California, Berkeley, California.

Childs, E. C. (1969): An introduction to the physical basis of soil water phenomena. 493 p., J. Wiley and Sons, Ltd., New York.

Chow, V. T. (1964): Handbook of Applied Hydrology: A Compendium of Water Resource Technology. pp. 14-27 - 14-25, McGraw-Hill Book Company, New York.

Clark, C. O. (1945): Storage and the unit hydrograph. Trans. ASCE 110: 1419-1446.

Conte, S. D. (1965): Elementary Numerical Analysis: An Algorithm Approach. 278 p., McGraw-Hill Book Company, New York.

Dooge, J. C. I. (1959): A general theory of unit hydrograph. JGR 64(2): 241-256.

Dooge, J. C. I. (1973): Linear theory of hydrologic systems. Technical Bulletin No. 1468, 327 p., USDA-ARS, Washington, D. C.

Eagleson, P. S. (1972): Dynamics of flood frequency. WRR 8(4): 878-894.

Green, W. H. and Ampt, G. A. (1911): Studies on soil physics: 1. the flow of air and water through soils. J. Agricultural Sciences 4: 1-24.

Herbst, P. H. and Shaw, E. M. (1969): Determining raingage densities in England from limited date, given a required precision for monthly rainfall estimates. J. Water Engineers 23(4): 218-230.

Himmelblau, D. M. (1972): Applied Nonlinear Programming. pp. 58-167, McGraw-Hill Book Company, New York.

Holtan, H. N. (1961): A concept for infiltration estimates in watershed engineering. ARS Paper 41-51, 25 p., U. S. Department of Agriculture, Washington, D. C.

Horton, R. E. (1940): An approach toward a physical interpretation of infiltration capacity. Soil Science Society of America Proceedings 5: 399-417.

Kibler, D. F. and Woolhiser, D. A. (1970): The kinematic cascade as a hydrologic model. Hydrology Paper No. 39, 27 p., Colorado State University, Fort Collins, Colorado.

Lighthill, M. J. and Whitham, G. B. (1955): On kinematic waves: flood movement in long rivers. Proc. Royal Society (London), Series A22: 281-316.

McGuiness, J. L. (1963): Accuracy of estimating watershed mean rainfall. JGR 68(16): 4763-4768.

Nash, J. E. (1957): The form of the instantaneous unit hydrograph. IASH Pub. 3(45): 114-121.

Nash, J. E. (1958): Determining runoff from rainfall. Proc. Inst. Civil Engrs. 10: 163-184.

Nash, J. E. (1959a): Systematic determination of unit hydrograph parameters. JGR 64(1): 111-115.

Nash, J. E. (1959b): The effect of flood elimination works on the flood frequency of the river Wandle. Proc. Inst. Civil Engrs. 13: 317-338.

Nash, J. E. (1960a): A unit hydrograph study, with particular reference to British catchments. Proc. Inst. Civil Engrs. 17: 249-282.

Nash, J. E. (1960b): A note on an investigation into two aspects of the relation between rainfall and storm runoff. IASH Pub. 51: 567-578.

O'Kelly, J. J. (1955): The employment of unit hydrographs to determine the flows of the Irish arterial drainage channels. J. Inst. Civil Engrs. 4: 365-445.

Palmer, J. R. (1969): An improved procedure for orthogonalizing the search vectors in Rosenbrock's and Swaan's direct search optimiation methods. Computer J. 12: 69-71.

Philip, J. R. (1957): The theory of infiltration: 1. the infiltration equation and its solution. Soil Science 83: 345-357.

Philip, J. R. (1968): The theory of infiltration. Advances in Hydrosciences 5: 215-296.

Rodda, J. C. (1967): The rainfall measurement problem. IASH Pub. 78: 215-223.

Rodda, J. C. (1970): On the questions of rainfall measurment and representativeness. Proc. IASH Symposium on World Water Balance, p. 174-186, Reading, England.

Rode, A. A. (1965): The theory of soil moisture. Vol. 1 (Translated from Russian), published for U.S.D.A. and N.S.F. by the Israel Program for Scientific Translation, Jerusalem (1969).

Rosenbrock, H. H. (1960): An automatic method for finding the greatest or least value of a function. Computer J. 4: 175-184.

Shelburne, K. L. and Singh, V. P. (1976): Studies on rainfall-runoff modeling: 4. estimation of parameters of two mathematical models of surface runoff. WRRI Report No. 076, 96 p., New Mexico Water Resources Research Institute, New Mexico State University, Las Cruces, New Mexico.

Sherman, B. and Singh, V. P. (1976a): A distributed converging overland flow model: 1. mathematical solutions. WRR 12(5): 889-896.

Sherman, B. and Singh, V. P. (1976b): A distributed converging overland flow model: 2. effect of infiltration. WRR 12(5): 897-901.

Singh, K. P. (1964): Nonlinear instantaneous unit hydrograph theory. J. Hydraulics Division, Proc. ASCE 90(2): 313-347.

Singh, V. P. (1975a): A laboratory investigation of surface runoff. J. Hydrology 25: 187-200.

Singh, V. P. (1975b): Hybrid formulation of kinematic wave models of watershed runoff. J. Hydrology 27: 33-50.

Singh, V. P. (1975c): Kinematic wave modeling of watershed surface runoff: a hybrid approach. IASH Pub. 117: 255-264.

Singh, V. P. (1975d): A distributed approach to kinematic wave modeling of watershed surface runoff. Proc. National Symposium on Urban Hydrology and Sediment Control, pp. 227-236, University of Kentucky, Lexington, Kentucky.

Singh, V. P. (1975e): Estimation and optimization of kinematic wave parameters. Water Resources Bulletin 11(6): 1091-11002.

Singh, V. P. and Birsoy, Y. K. (1975): Studies on rainfall-runoff modeling: 1. estimation of mean areal rainfall. WRRI Report 061, 70 p., New Mexico Water Resources Research Institute, New Mexico State University, Las Cruces, New Mexico.

Singh, V. P. (1976a): Studies on rainfall-runoff modeling: 2. a distributed kinematic wave model of watershed surface runoff. WRRI Report No. 65, 153 p., New Mexico Water Resources Research Institute, New Mexico State University, Las Cruces, New Mexico.

Singh, V. P. (1976b): Studies on rainfall-runoff modeling: 3. converging overland flow. WRRI Report No. 73, 290 p., New Mexico Water Resources Research Institute, New Mexico State University, Las Cruces, New Mexico.

Singh, V. P. (1976c): Mathematical aspects of surface runoff. Proc. Symposium on Inland Waterways for Navigation, Flood Control and Water Diversions 1: 773-792, Colorado State University, Fort Collins, Colorado.

Singh, V. P. (1976d): A distributed converging overland flow model: 3. application to natural watersheds. WRR 12(5): 902-908.

Singh, V. P. and Woolhiser, D. A. (1976): Sensitivity of linear and nonlinear surface runoff models to input erros. J. Hydrology 29: 243-249.

Smith, R. E. and Woolhiser, D. A. (1971): Overland flow on an infiltrating surface. WRR 7(4): 899-913.

Sutcliffe, J. V. (1966): The assessment of random erros in areal rainfall estimation. IASH Bull. 11(3): 35-42.

USDA (1963): Hydrologic data for experimental agricultural watersheds in the United States. U. S. Govt. Printing Office, Washington, D. C.

Wooding, R. A. (1965a): A hydraulic model for the catchment-stream problem: 1. kinematic wave theory. J. Hydrology 3(3): 254-267.

Wooding, R. A. (1965b): A hydraulic model for the catchment-stream problem: 2. numerical solutions. J. Hydrology 3(3): 268-282.

Wooding, R. A. (1966): A hydraulic model for the catchment-stream problem: 3. comparison with runoff observations. J. Hydrology 4(1): 21-37.

Woolhiser, D. A. (1969): Overland flow on a converging surface. Trans. ASAE 12(4): 460-462.

Woolhiser, D. A., Hansen, C. L. and Kuhlman, A. R. (1970): Overland flow on rangeland watersheds. J. Hydrology (N.Z.) 9(2): 336-356.